CHAPTER I

INTRODUCTION

1.1 Research Background

Income is a fundamental necessity for every individual to maintain their livelihood and well-being. During their productive years, income is obtained through employment. However, every individual will eventually reach retirement age, when work activity must cease, and the source of income supporting personal and family welfare is interrupted. One solution to guarantee income continuity post-retirement is participation in a pension fund program, designed to plan for the future and provide financial benefits for employees after they are no longer actively working (Izzati and Kartikasari, 2022).

According to the President of the Republic of Indonesia Law No. 11 of 1992 concerning Pension Funds, a Pension Fund is a legal entity that manages and operates a program that promises pension benefits, with payments tied to a specified age. As per Law No. 11 of 1992, the pension program aims to improve the welfare of workers in retirement and is managed by a Pension Fund through structured and timely fund collection. This data is often referred to as the normal cost. The Normal Cost is the annual payment made by members of a pension fund program to help meet future pension expenses, while the pension benefit is the amount that must be accumulated by a certain time to ensure the future payment of pension benefits (Nursal, 2023).

Pension fund programs have been implemented by nearly all institutions and companies as an employment benefit. One major company that implements such a program is PT Pertamina (Persero), under Badan Usaha Milik Negara (BUMN). The pension scheme used by BUMN is categorized into two types: the Financial Institution Pension Fund (DPLK) scheme, which collaborates with financial institutions such as banks or insurance companies, and the Employer Pension Fund (DPPK) scheme, which is managed by the employer. In the DPPK scheme, the employer acts as the pension fund manager and is fully responsible for fulfilling future promised pension benefits.

Based on the Pertamina Pension Fund Practical Guidelines, PT Pertamina, as a BUMN, manages its pension fund using the DPPK scheme through a Defined Benefit Pension Program (PPMP) provided directly by the Pertamina Pension Fund (DP Pertamina). This scheme places PT Pertamina as the primary risk bearer, where the company has the full obligation to ensure the availability of funds to meet all promised future pension benefits. Although this scheme provides PT Pertamina with flexibility in formulating optimal pension fund management policies, in practice, various challenges persist. According to Ginting (2023), these issues stem from several factors, such as founder negligence, limited human resource capacity, ineffective investment governance, and contribution arrears due to irregular payments from the employer.

However, sustainable pension fund management is not free from the threat of uncertainty. One major challenge that can destabilize funding stability is the phenomenon of early retirement. There are several reasons why individuals choose to retire early, categorized into three main groups: (1) Individual and Family Reasons, (2) Organizational Reasons divided into three factors: first, changes in organizational context and structure; second, changes in work conditions, especially with increased psychosocial risks; and third, career opportunities, social recognition, and personal development and (3) Social Reasons (Annisa and Annisa, 2017).

A change in retirement age (early retirement) can lead to a liability shock, where the obligation to pay pension benefits, which should be distributed over the long term, becomes concentrated and must be paid earlier. This has the potential to cause actuarial liability insufficiency and worsen the pension fund's funding status if not properly anticipated.

In addition to the assumption of changing retirement age, optimization of the pension fund program is necessary to ensure that pension planning and management can meet its long-term obligations. One crucial aspect of pension fund management is actuarial valuation calculation, used to determine the amount of liability that must be reserved to meet future pension benefits. In pension fund programs, an important aspect to consider is the actuarial valuation method (actuarial cost method). This method is used to determine the normal contribution amount and the actuarial liability that must be fulfilled in a pension fund program (Rembet and Sari, et al., 2023). According to the Pension Fund Actuarial Practice Standard (SPA-DP) No. 3.02, actuarial valuation methods are generally divided into two main groups: Accrued Benefit Cost Method and Projected Benefit Cost Method. Methods within the Accrued Benefit Cost Method group include the Projected Unit Credit, while the Projected Benefit Cost Method group includes the Attained Age Normal, Entry Age Normal, and Individual Level Premium methods (PAI, 2019).

Various previous studies have extensively compared these two groups of methods. Researchers such as Wilandari, et al. (2016), Permana and et al. (2016) have compared the Projected Unit Credit method with various methods from the Projected Benefit Cost group. Consistent findings from these studies, including Izzati & Kartikasari (2022), conclude that methods in the Projected Benefit Cost group are considered superior.

The actuarial valuation process relies on several projective assumptions, including participant age, salary projections, and the crucial factor of the interest rate. The role of the interest rate as a discount factor is essential in measuring the present value of a pension fund's long-term liabilities. A problem arises when, as in the case of PT Pertamina, the actuarial interest rate is set too high, making it difficult to achieve commensurate investment returns. This situation underscores the urgency of selecting an accurate interest rate model to produce reliable actuarial projections that reflect real conditions.

Actuarial valuation calculations in various previous studies often rely on fixed (deterministic) interest rate assumptions, an approach that contradicts the reality of dynamic financial markets where interest rates constantly fluctuate. The limitations of this methodology provide a strong basis for adopting a stochastic interest rate approach, which can accommodate future interest rate uncertainty and volatility. Therefore, this research proposes the application of the Cox-Ingersoll-Ross (CIR) Model, a stochastic model widely recognized in finance for its ability to represent the mean-reverting property of interest rates (Cox, 1985). This model is chosen for its simplicity and its capacity to capture the tendency of interest rates to move toward a long-term average level, making it suitable for projecting pension liabilities. Furthermore, to test the resilience of the pension fund under extreme conditions, this research does not rely solely on normal scenarios but also introduces an early retirement assumption. By simulating a scenario where a number of employees retire earlier than expected, this analysis aims to measure the impact of liability shock on the funding status, projected under various interest rate paths generated by the Cox-Ingersoll-Ross Model. The integration of demographic pressure (early retirement) and financial pressure (stochastic interest rates) is expected to provide a far more comprehensive and realistic picture of the risks faced by the pension fund.

Based on the background explanation and previous research, the researcher is interested in conducting a study entitled "Application of the Cox-Ingersoll-Ross Stochastic Interest Rate Model in Calculating the Supplemental Cost of a Pension Fund Program with Early Retirement Assumptions: A Case Study of PT Pertamina (Persero)." This research is expected to contribute to pension fund management and decision-making in optimizing pension fund management strategies and improving the accuracy of pension benefit calculations for PT Pertamina (Persero).

1.2 Problem Statements

1. What is the accuracy level of the stochastic interest rate simulation using the Cox-Ingersoll-Ross model for determining the actuarial discount rate in pension fund valuation?
2. What are the differences in the calculated results for Normal Cost and Actuarial Liability under the assumptions of normal retirement age and early retirement age, using the Accrued Benefit Cost Method (Projected Unit Credit) and the Entry Age Normal Method, for PT Pertamina (Persero) when applying the stochastic interest rate from the Cox-Ingersoll-Ross model?
3. What are the calculated results for the Unfunded Actuarial Liability (UAL) and Supplemental Cost (SC) arising from early retirement in the pension funding program of PT Pertamina (Persero)?

1.3 Research Objectives

1. To determine the accuracy level of the stochastic interest rate simulation using the Cox-Ingersoll-Ross model for calculating the actuarial discount rate in valuation.
2. To identify the differences in the calculation results for Normal Cost and Actuarial Liability under the assumptions of normal retirement age and early retirement age, using the Accrued Benefit Cost Method (Projected Unit Credit) and the Entry Age Normal Method at PT Pertamina (Persero) with a stochastic interest rate based on the Cox-Ingersoll-Ross model.
3. To ascertain the calculation results for Unfunded Actuarial Liability (UAL) and Supplemental Cost (SC) resulting from early retirement in the pension funding program of PT Pertamina (Persero).

1.4 Scope and Limitations

1.4.1 Scope of the Study

The scope of this research encompasses the following key areas:

1) Data Sources and Actuarial Assumptions:
   Financial and Company Data**:** The primary company-specific data for PT Pertamina (Persero) will be sourced from its official 2024 Annual Report. This document will provide key financial, operational, and contextual information relevant to the pension fund analysis.
   Economic Assumption**:** The Cox-Ingersoll-Ross (CIR) stochastic interest rate model will be used for financial projections. This model will be calibrated using the Bank Indonesia (BI) rate from 2011–2020.
   Demographic Assumption**:** The 2023 Indonesian Mortality Table will serve as the primary basis for all mortality and life expectancy assumptions within the actuarial calculations.
2) Actuarial Valuation Methods. This study adopts a comparative actuarial framework, utilizing two primary valuation methods the Projected Unit Credit (PUC) metho and the Entry Age Normal (EAN) method, representing the Projected Benefit Cost approach. The application of these methods is explicitly focused on analysing the Defined Benefit Plan (Program Pensiun Manfaat Pasti) of PT Pertamina (Persero), which operates under the Employer Pension Fund (DPPK) scheme as its research object.
3) The core analysis of this research involves a comparative financial valuation conducted under two distinct demographic scenarios: a Baseline Scenario, which performs calculations using the normal retirement age stipulated in PT Pertamina's pension plan rules, and a Shock Scenario, which incorporates an assumed rate of early retirement to simulate and quantify the resulting liability shock on the fund.

4) Key Output Metrics, the financial impact will be measured and compared through specific actuarial liabilities and costs, including Normal Cost, Actuarial Liability (AL), Unfunded Actuarial Liability (UAL), and Supplemental Cost (SC)

1.4.2 Limitations of the Study

To maintain feasibility for an undergraduate thesis, this research acknowledges the following limitations:

1) Data specificity and granularity. While the PT Pertamina 2024 Annual Report provides authoritative, high-level financial data, it may not contain the granular, member-level census data (e.g., exact age, salary history, service years for each employee) ideal for a detailed actuarial valuation. The analysis will depend on the level of pension fund disclosure within the report and any available supplementary data and for the 2023 Indonesian Mortality Table, while current, is a generalized model and may not perfectly reflect the specific mortality profile of PT Pertamina's workforce.
2) Modelling and Assumption Constraints. This analysis acknowledges several methodological simplifications such as mortality rates are applied using a static, period-based table without incorporating explicit projections for future mortality improvements; furthermore, the calibration of the Cox-Ingersoll-Ross (CIR) interest rate model relies exclusively on historical Bank Indonesia data from 2011–2020, a dataset which may not fully represent all potential future economic conditions; additionally, the early retirement scenario is modelled as a simplified, uniform liability shock rather than a complex behavioural model based on individual or organizational factors
3) Analytical Focus. This thesis focuses primarily on the liability valuation and the impact of interest rates and retirement age. It does not include a comprehensive analysis of PT Pertamina's pension asset portfolio or a full Asset-Liability Management (ALM) study.
4) Temporal Limitation. The financial data is anchored to the 2024 reporting period. The study's projections are forward-looking simulations based on this snapshot in time, and their accuracy is subject to future economic and demographic shifts.

1.5 Theoretical Foundation

1.5.1 Pension Fund

Referring to Law Number 11 of 1992 concerning Pension Funds, a Pension Fund is a legal entity that manages and operates a program that promises pension benefits. According to this law, a pension fund must be established as a legal entity and is tasked with managing participant funds to provide benefits when participants enter retirement (Sanyoto, 2023). The purpose of a pension fund program is to provide social protection to employees after they enter retirement.

Based on the provisions of Article 1, point 1 of the Pension Fund Law, a pension fund holds the status of a legal entity authorized to manage a program committed to providing pension benefits for its participants. The legal recognition of pension funds as institutional legal entities has been in effect since the enactment of the related law, affirming the state's legitimacy over pension fund institutions in Indonesia (Marwa, 2020). As a legal entity, a pension fund functions to administer and operate a pension program aimed at providing guaranteed pension benefits. The fundamental mission of this institution is to create social and economic protection for workers when they enter retirement.

1.5.2 Types of Pension Fund Programs

Based on Law Number 11 of 1992 concerning Pension Funds, the pension funding system in Indonesia is classified into two main schemes, as outlined by Nuhung et al. (2020), with criteria based on the pattern and timing of contribution payments:

a) Defined Benefit Pension Program (Program Pensiun Manfaat Pasti - PPMP)
   The PPMP, also known as a defined benefit plan, is a pension program that establishes a formula to determine the amount of benefits a participant will receive upon reaching retirement age. In this mechanism, the value of the pension benefit is determined in advance using a formula that typically considers variables such as years of service and the participant's income level. An actuary then calculates the contributions that must be paid, where the employer's contribution is usually larger than the participant's contribution. Under this scheme, the company bears the primary responsibility for making contributions. All investment risks within this program are borne by the company as the responsible party.
b) Defined Contribution Pension Program (Program Pensiun Iuran Pasti)
   A defined contribution plan is a pension program that stipulates the amount of contributions paid by the participant (employee) and the company (employer). The pension benefit received by the participant is calculated based on the accumulation of paid contributions, plus the returns from its development or investment.

1.5.3 Types of Pension Benefits

According to Law Number 11 of 1992 concerning Pension Funds, a pension benefit is a periodic payment that must be provided by the administrator to a pension participant in accordance with the time and mechanisms stipulated in the pension fund regulations. There are four types of pension benefits, including (Miyasari, 2023):

a) Normal Pension Benefit: The benefit provided to a participant after reaching or exceeding the normal retirement age.

b) Early Pension Benefit: The pension benefit paid to a participant at a certain age before reaching the normal retirement age.
c) Disability Pension Benefit: The pension benefit provided to a participant who becomes disabled.
d) Deferred Pension Benefit: The benefit provided to a participant who ceases employment before reaching the normal retirement age. The benefit payment is made when the participant reaches the normal retirement age as stipulated in the pension fund regulations.

1.5.4 Stochastic Differential Equation

A stochastic process is defined as a collection of random variables $\{X(t), t \in T\}$, where t represents time and $X(t)$ denotes the state of the process at time t (Taylor and Karlin, 1998).

Definition 1: A stochastic process $\{W_t, t \geq 0\}$ is called a Brownian motion with drift θ if the following conditions are met (Medikasari, 2020):

a) $W(0) = 0$
b) $\{W_t, t \geq 0\}$ has stationary and independent increments.
c) W_t is normally distributed with mean θt and variance t.

Let $\{r_t, t \geq 0\}$ be a stochastic process and $\{W_t, t \geq 0\}$ be a Wiener process. Then:

$$dr_t = \theta(r_t)dt + \sigma(r_t, t)dW_t \tag{1}$$

This equation is a stochastic differential equation (SDE), where $\theta(r_t, t)$ is termed the drift coefficient and $\sigma(r_t, t)$ is the diffusion coefficient (Brigo and Mercurio, 2006).

The stochastic differential equation in (1) can be expressed in integral form as:

$$r_t = r_0 + \int_0^t \theta(r(s), s)ds + \int_0^t \sigma(r(s), s)dW_s \tag{2}$$

with initial value r_0, and where W_t is a Wiener process (Brigo & Mercurio, 2006).

1.5.5 Itô's Integral

The Itô integral is employed to solve stochastic differential equations, thereby facilitating the derivation of the expectation and variance for interest rates under the Cox-Ingersoll-Ross model.

For a simple process, the Itô integral $\int_0^T r_t \, dW_t$ is defined as follows (Brigo & Mercurio, 2006):

$$\int_0^T r_t \, dW_t = \sum_{i=0}^{n-1} r_i \, [W(t_{i+1}) - W(t_i)] \tag{3}$$

The Itô integral for simple processes possesses the following properties (Brigo & Mercurio, 2006):

1. Linearity**:** If r_t and Y_t are simple processes and a, b are constants, then:

$$\int_0^T (ar_t + bY_t) dW_t = a \int_0^T r_t \, dW_t + b \int_0^T Y_t \, dW_t \tag{4}$$

2. Zero Expectation: The expectation of the Itô stochastic integral is zero:

$$E\left[\left(\int_0^T r_t \, dW_t\right)\right] = 0 \tag{5}$$

3. Itô Isometry**:** The Itô stochastic integral satisfies the isometry property:

$$E\left[\left(\int_0^T r_t \; dW_t\right)^2\right] = \int_0^T E[r_t^2] dt \tag{6}$$

$$for, t \in [0, T]$$

1.5.6 Cox-Ingersoll-Ross (CIR) Model

The Cox-Ingersoll-Ross (CIR) model is a type of model that describes the behavior of interest rates. It is characterized by mean reversion, meaning the interest rate tends to revert to a long-term average level, and it ensures that projected interest rates remain non-negative (Cox JC, 1985).

The economic rationale underpinning the mean reversion concept posits that high interest rates tend to slow economic activity, thereby reducing the demand for credit from borrowers. This situation eventually pulls interest rates back toward their equilibrium level. Conversely, low interest rates stimulate borrowing demand. The mean reversion theory is considered apt for analyzing interest rate behavior because, in its absence, interest rates could risk increasing perpetually, similar to stock price movements. In reality, however, interest rates are not permanent and always fluctuate within specific cycles (Kiley, 2019).

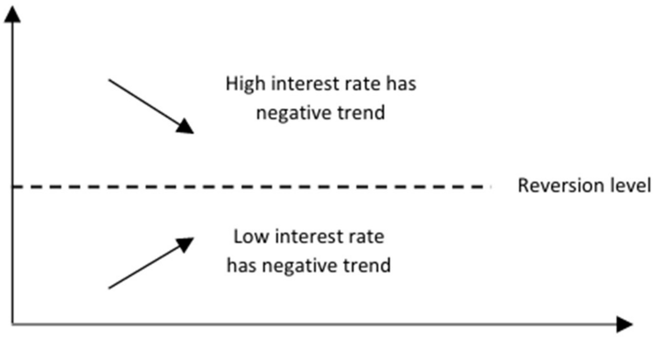

Figure 1. Mean reversion illustration

Figure 1 illustrates the concept of mean reversion. When interest rates are high, economic activity tends to slow, which in turn drives interest rates back toward their equilibrium level. Conversely, when interest rates are low, demand for funds from borrowers typically increases, causing interest rates to adjust upward toward equilibrium.

This model was introduced in 1985 by John C. Cox, Jonathan E. Ingersoll, Jr., and Stephen A. Ross. The general form of the CIR model is as follows (Cox, 1985):

$$dr_t = c(\theta - r_t)dt + \sigma\sqrt{r_t}\, dW_t \tag{7}$$

Where:

- c : Speed of mean reversion
- θ : Long-term interest rate level
- σ : Volatility
- dW_t: Increment of the Wiener process.

To derive the expected value, let $\mu(t) = E[r_t]$. Differentiate both sides of the CIR equation with respect to t:

$$\frac{d\mu}{dt} = c(\theta - \mu)$$

Multiply both sides by the integrating factor e^{ct}:

$$e^{ct}\frac{d\mu}{dt} + ce^{ct}\mu = c\theta e^{ct}$$

Observe that the left side of the equation is the derivative of μe^{ct} with respect to time:

$$\frac{d}{dt}(\mu e^{ct}) = c\theta e^{ct}$$

Integrate both sides with respect to t from 0 to t:

$$\int_0^t \frac{d}{ds}(\mu e^{cs})ds = \int_0^t c\theta\, e^{cs}ds$$
$$\mu(t)e^{ct} - r_0 = \theta(e^{ct} - 1)$$
$$\mu(t) = r_0 e^{-ct} + \theta(1 - e^{-ct})$$

Thus, the expectation for the CIR model is:

$$E(r_t) = r_0 e^{-ct} + \theta(1 - e^{-ct}) \quad (8)$$

To derive the variance, calculate $E[r_t^2]$. Define $\phi(t) = E[r_t^2]$. Apply Itô's lemma to the function $f(r_t) = r_t^2$, which is defined as:

$$df(r_t) = f'(r_t)dr_t + \frac{1}{2}f''(r_t)(dr_t)^2$$

By applying Itô's lemma and the CIR model, the variance is obtained as:

$$Var\,[r_t] = r_0\frac{\sigma^2}{c}(e^{-ct} - e^{-2ct}) + \frac{\theta\sigma^2}{2c}\,(1 - e^{-ct})^2 \quad (9)$$

1.5.7 Parameter Estimation for the Cox-Ingersoll-Ross Model Using Ordinary Least Squares (OLS)

The method used to estimate the parameters is Ordinary Least Squares (OLS). OLS is a method where the estimated parameter values are obtained by minimizing the sum of squared errors (Anita, et al., 2023). The parameters α, c, and σ̂ will be estimated using the OLS method as follows:

By discretizing Equation (7), the following equation is obtained (Bua, 2022):

$$r_{t+\Delta t} - r_t = c(\theta - r_t)\Delta t + \sigma\sqrt{r_t}\sqrt{\Delta t}\varepsilon_t \quad (10)$$

Divide both sides by $\sqrt{r_t}$ (assuming $r_t > 0$)

$$\frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}} = \frac{c\theta\Delta t}{\sqrt{r_t}} - c\Delta t\sqrt{r_t} + \sigma\sqrt{\Delta t}\varepsilon_t \quad (11)$$

Assume $Y_t = \frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}}$, $\alpha = c\theta$, $\beta = -c$, $X_{1t} = \frac{\Delta t}{\sqrt{r_t}}$, $X_{2t} = \Delta t\sqrt{r_t}$, dan $\eta_t = \sigma\sqrt{\Delta t}\varepsilon_t$, so that $\eta_t \sim N(0, \sigma^2\Delta t)$

Thus, the following equation is obtained:

$$Y_t = \alpha X_{1t} + \beta X_{2t} + \eta_t \quad (12)$$

In this case, given the time series data $\{r_1, r_2, \dots, r_n\}$ the number of observations $m = n - 1$ will be calculated. α and β will be estimated using OLS (Ordinary Least Squares), which is obtained by minimizing the sum of squared errors:

$$S = \sum_{t=1}^{n-1} (Y_t - \alpha X_{1t} - \beta X_{2t})^2 \tag{13}$$

Next, S will be partially differentiated with respect to α and β.

Partial derivative of S with respect to α**:**

$$\frac{\partial S}{\partial \alpha} = -2 \sum_{t=1}^{n-1} (Y_t - \alpha X_{1t} - \beta X_{2t}) X_{1t} = 0$$

$$\sum_{t=1}^{n-1} X_{1t} Y_t = \alpha \sum_{t=1}^{n-1} X_{1t}^2 + \beta \sum_{t=1}^{n-1} X_{1t} \sum_{t=1}^{n-1} X_{2t} \tag{14}$$

Partial derivative of S with respect to β**:**

$$\frac{\partial S}{\partial \beta} = -2 \sum_{t=1}^{n-1} (Y_t - \alpha X_{1t} - \beta X_{2t}) X_{2t} = 0$$

$$\sum_{t=1}^{n-1} X_{1t} Y_t = \alpha \sum_{t=1}^{n-1} X_{1t}^2 + \beta \sum_{t=1}^{n-1} X_{1t} \sum_{t=1}^{n-1} X_{2t} \tag{15}$$

With

$$\sum X_{1t} Y_t = \sum \left(\frac{\Delta t}{\sqrt{r_t}}\right)\left(\frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}}\right) = \Delta t \sum \frac{r_{t+\Delta t} - r_t}{r_t}$$

$$\sum X_{2t} Y_t = \sum \left(\Delta t \sqrt{r_t}\right)\left(\frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}}\right) = \Delta t \sum (r_{t+\Delta t} - r_t)$$

$$\sum X_{1t}^2 = \sum \frac{(\Delta t)^2}{r_t} = (\Delta t)^2 \sum \frac{1}{r_t}$$

$$\sum X_{2t}^2 = (\Delta t)^2 \sum r_t$$

$$\sum X_{1t} X_{2t} = \sum \left(\frac{\Delta t}{\sqrt{r_t}}\right) \left(\Delta t \sqrt{r_t}\right) = (\Delta t)^2 \sum 1 = (\Delta t)^2 (n-1)$$

Thus, the following equation is obtained:

$$\Delta t \sum \frac{r_{t+\Delta t} - r_t}{r_t} = \alpha (\Delta t)^2 \sum \frac{1}{r_t} + \beta (\Delta t)^2 (n-1) \tag{16}$$

$$\Delta t \sum (r_{t+\Delta t} - r_t) = \alpha (\Delta t)^2 (n-1) + \beta (\Delta t)^2 \sum r_t \tag{17}$$

Multiply both equations with $\frac{1}{\Delta t}$, so that:

$$\sum \frac{r_{t+\Delta t} - r_t}{r_t} = \alpha \Delta t \sum \frac{1}{r_t} + \beta \Delta t (n-1) \tag{18}$$

$$\sum (r_{t+\Delta t} - r_t) = \alpha \Delta t (n-1) + \beta \Delta t \sum r_t \tag{19}$$

Assume

$$S_1 = \sum \frac{r_{t+\Delta t} - r_t}{r_t} = \sum \frac{r_{t+\Delta t}}{r_t} - \sum 1 = \sum \frac{r_{t+\Delta t}}{r_t} - (n-1)$$

$$S_2 = \sum (r_{t+\Delta t} - r_t) = \sum r_{t+\Delta t} - \sum r_t$$

$$S_3 = \sum \frac{1}{r_t}$$

$$S_4 = \sum r_t$$

Thus, equations (17) and (18) become:

$$S_1 = \alpha \Delta t S_3 + \beta \Delta t (n-1) \tag{20}$$

$$S_2 = \alpha \Delta t (n-1) + \beta \Delta t S_4 \tag{21}$$

Based on equation (19), α is obtained as follows:

$$\alpha = \frac{S_1 - \beta \Delta t (n-1)}{\Delta t S_3} \tag{22}$$

Substitute equation (21) into equation (20) to obtain the value of β.

$$S_2 = \left(\frac{S_1 - \beta \Delta t (n-1)}{\Delta t S_3} \right) \Delta t (n-1) + \beta \Delta t S_4 \tag{23}$$

$$S_2 - \frac{S_1 (n-1)}{S_3} = \beta \Delta t \left(\frac{(n-1)^2}{S_3} + S_4 \right)$$

$$\beta = \frac{S_2 - \frac{S_1 (n-1)}{S_3}}{\Delta t \left(\frac{(n-1)^2}{S_3} + S_4 \right)}$$

$$\beta = \frac{S_2 S_3 - S_1 (n-1)}{\Delta t (S_3 S_4 - (n-1)^2)} \tag{24}$$

It is known that $\beta = -c, so -\beta = c$, therefore equation (23) becomes:

$$c = \frac{S_1 (n-1) - S_2 S_3}{\Delta t (S_3 S_4 - (n-1)^2)} \tag{25}$$

Thus, the equation for c is obtained as follows:

$$c = \frac{(n-1)^2 + \sum r_{t+\Delta t} \sum \frac{1}{r_t} - \sum r_t \sum \frac{1}{r_t} - (n-1) \sum \frac{r_{t+\Delta t}}{r_t}}{\Delta t \left((n-1)^2 - \sum r_t \sum \frac{1}{r_t} \right)} \tag{26}$$

Next, equation (20) will be used to formulate the equation for θ:

$$S_2 = c\Delta t(\theta(n-1) - S_4$$

$$\theta = \frac{S_2 + S_4 c\Delta t}{c\Delta t\,(n-1)}$$

$$\theta = \frac{1}{c\Delta t} \cdot \frac{S_2}{n-1} + \frac{S_4}{n-1} \tag{27}$$

It is known that:

$$c = \frac{S_2 S_3 - (n-1)S_1}{\Delta t((n-1)^2 - S_3 S_4)}$$

$$c\Delta t = \frac{S_2 S_3 - (n-1)S_1}{(n-1)^2 - S_3 S_4} \tag{28}$$

Substitute equation (27) into equation (26):

$$\theta = \frac{(n-1)^2 - S_3 S_4}{S_2 S_3 - (n-1)S_1} \cdot \frac{S_2}{n-1} + \frac{S_4}{n-1}$$

$$\theta = \frac{(n-1)S_2 - S_1 S_4}{S_2 S_3 - (n-1)S_1}$$

So that

$$\theta = \frac{(n-1)\sum r_{t+\Delta t} - \sum \frac{r_{t+\Delta t}}{r_t} \sum r_t}{\sum r_{t+\Delta t} \sum \frac{1}{r_t} - \sum r_t \sum \frac{1}{r_t} - (n-1)\sum \frac{r_{t+\Delta t}}{r_t} + (n-1)^2} \tag{29}$$

Next, the error variance will be estimated using the Standard Error of the Regression (SER), with the general formula:

$$\sigma = \sqrt{\frac{1}{n-k} \sum_{i=1}^{n} \eta_i^2} \tag{30}$$

It is known that $\eta_t = \sigma\sqrt{\Delta t}$, so that:

$$\frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}} = \frac{c\theta\Delta t}{\sqrt{r_t}} - c\Delta t\sqrt{r_t} + \sigma\sqrt{\Delta t}\varepsilon_t$$

$$\sigma\sqrt{\Delta t} = \frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}} - c\theta\Delta t \frac{1}{\sqrt{r_t}} + c\Delta t\sqrt{r_t} \tag{31}$$

Substitute equation (30) into equation (29) to obtain equation (31), and assume $\Delta t = 1$, so that $\eta_t = \sigma$. Based on equation (28) and equation (25), the formulas for estimating the parameters of the Cox-Ingersoll-Ross model using the Ordinary Least Squares (OLS) method are derived as follows (S. Artika, 2010):

$$\sigma = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n-1}\left(\frac{r_{t+1}-r_t}{\sqrt{r_t}} - \frac{c\theta}{\sqrt{r_t}} + c\sqrt{r_t}\right)^2} \tag{32}$$

$$\theta = \frac{(n-1)\sum r_{t+1} - \sum \frac{r_{t+1}}{r_t}\sum r_t}{\sum r_{t+1}\sum \frac{1}{r_t} - \sum r_t \sum \frac{1}{r_t} - (n-1)\sum \frac{r_{t+1}}{r_t} + (n-1)^2} \tag{29}$$

$$c = \frac{(n-1)^2 + \sum r_{t+1}\sum \frac{1}{r_t} - \sum r_t \sum \frac{1}{r_t} - (n-1)\sum \frac{r_{t+1}}{r_t}}{\Delta t\left((n-1)^2 + \sum r_t \sum \frac{1}{r_t}\right)} \tag{26}$$

1.5.8 Milstein Numerical Scheme

The Milstein method is one of the numerical methods used in mathematical computation to evaluate stochastic differential equations. Consider the following stochastic differential equation:

$$dr(t) = f\big(r(t)\big)dt + g\big(r(t)\big)dW(t),\ \ r(0) = r_0\ \ 0 \le t \le T \tag{33}$$

Then it has the following Milstein formula (Higham, 2001):

$$r_t = r_{t-1} + \Delta t f(r_{t-1}) + g(r_{t-1})\big(W(\tau_1) - W(\tau_{t-1})\big) + \frac{1}{2}g(r_{t-1})g'(r_{t-1})\left(\big(W(\tau_t) - W(\tau_{t-1})\big)^2 - \Delta t\right) \tag{34}$$

Where $t = 1, 2, \dots, L. \Delta t = \frac{T}{L}$. For $\tau_t = t\Delta t$ and $W(\tau_t) - W(\tau_{t-1}) = dW_t$ with $dW_t = \sqrt{\Delta t}\, N(0,1)$.

This simulation is based on the discretization approach of the Wiener process, which involves generating a series of random variables following a normal distribution. These random variables represent the approximation of the Wiener process $W(t)$ at discretized time points t.

1.5.9 Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error (MAPE) is an evaluation metric commonly used in statistics and machine learning to assess the prediction accuracy of regression models. MAPE calculates the average percentage difference between a model's predicted values and the actual observed data values.

Mathematically, MAPE can be calculated using the following equation (Nabillah and Ranggadara, 2020):

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{r_t - \widehat{r_t}}{r_t}\right| \tag{35}$$

Where:
r_t : observed value at time period t
$\widehat{r_t}$: forecast value for time period t
n : number of periods
The forecasting accuracy scale is as follows (Nabillah & Ranggadara, 2020):

Table 1. Range of MAPE values

Range of MAPE	Description
< 10%	Very Good Forecasting Model Competency
10% − 20%	Good Forecasting Model Competency
20%−50%	Reasonable Forecasting Model Competency
>50%	Poor Forecasting Model Competency

with the value of accuracy can be obtained with,

$$Accuracy = 1 - MAPE = 1 - \frac{1}{n}\sum_{t=1}^{n}\left|\frac{r_t - \widehat{r_t}}{r_t}\right| \tag{36}$$

1.5.10 Mean Squared Error (MSE)

Mean Squared Error (MSE) is an evaluation metric frequently used in statistics and machine learning to assess the accuracy of regression models in predicting numerical values. MSE calculates the average of the squared differences between the values predicted by the model and the actual observed data values. Mathematically, MSE can be calculated using the following equation (Suparman, 2012)

$$MSE = \frac{\sum_{i=1}^{n}(r_t - \widehat{r_t})^2}{n} \tag{37}$$

Where n is the total number of data samples, r_t is the actual value of the t-th data point, and $\widehat{r_t}$ is the predicted value for the i-th data point.

MSE computes the difference between the model's predicted values and the true values, then squares these differences to ensure all values are positive. These squared differences are summed and then divided by the total number of samples to obtain the average.

A smaller MSE value indicates better performance of the regression model in predicting numerical values. Therefore, MSE is well-suited for selecting the most accurate regression model.

1.5.11 Mortality Tables and Commutation Symbols

A mortality table is a concise representation that shows the probability of living or dying for individuals in a population at specific ages. Mortality tables (life tables) are used to analyze changes in mortality rates within a population, particularly in relation to social security, over time. In these tables, there is a difference in mortality risk between males and females. Males tend to have a higher mortality risk due to lifestyle patterns that are generally more susceptible to unhealthy habits compared to females. The formula for the mortality table is as follows (Futami, 1993).

$$d_x = l_x q_x \tag{38}$$

$$l_{x+1} = l_x - d_x \tag{39}$$

The probability that a person aged x will survive to reach age $x + n$ years can be expressed as (Futami, 1993):

$${}_n p_x = \frac{l_{x+n}}{l_x} \tag{40}$$

The probability that a person aged x will survive to reach age $x + n$ years can be expressed as (Futami, 1993):

$${}_n q_x = 1 - {}_n p_x \tag{41}$$

Where:

d_x : The number of people who died between age x and age $x + n$
l_x : The number of people who live exactly age x

Calculations in mortality tables are often simplified using commutation symbols. Some of the commutation symbols used include (Izzati, 2022):

$$D_x = v^x l_x \tag{42}$$

With $v = \frac{1}{1+i} = (1+i)^{-1}$

$$N_x = D_x + D_{x+1} + \cdots + D_w = \sum_{i=0}^{w-x} D_{x+i} \tag{43}$$

With w defined as the maximum age that will be obtained

1.5.12 Life Annuities

An annuity is a series of payments of a fixed amount made at regular intervals over a continuous period. A certain annuity is an annuity that is guaranteed to be paid for a specified term, while a life annuity is an annuity whose payments depend on the survival of an individual (Takashi, 1994). Annuity payments made at the beginning of an interval are called an annuity due, while those made at the end of an interval are called an annuity immediate (Miyasari, 2023). This research will focus on discrete annuities paid in advance (annuity due).

a) Discrete Whole Life Annuity Due
A discrete whole life annuity due is a series of payments made at the beginning of each period to a pension plan participant, which will continue until the participant's death (Takashi, 1994). The following is the equation for a discrete whole life annuity due paid at the beginning of the period (Rembet, Salsabila, Talarima & Unwaru, 2023):

$$\ddot{a}_x = \sum_{t=0}^{\omega} v^t \frac{l_{x+t}}{l_x} = \sum_{t=0}^{\omega} \frac{D_{x+t}}{D_x} = \frac{N_x}{D_x} \tag{44}$$

b) Discrete n-Year Temporary Life Annuity Due
A discrete n-year temporary life annuity due is a series of payments made at the beginning of each period to a pension plan participant, which continues for n years or until the participant dies, whichever occurs first.

$$\ddot{a}_{x:\overline{n}|} = \sum_{i=1}^{n-1} \frac{D_{x+i}}{D_x} = \frac{N_x - N_{x+n}}{D_x} \tag{45}$$

c) Discrete Whole Life Annuity Due Paid m Times per Year
If payments of whole life annuity are made at the beginning of each period m-times per year with a payment amount of one, the formula is as follows (Jordan, 1991):

$$\ddot{a}_r^{(m)} = \ddot{a}_r - \frac{m-1}{2m} \tag{46}$$

1.5.13 Actuarial Basis Functions for Pension Funds

1.5.13.1 Survival Function

A survival function is a function that describes the probability that an employee will continue working throughout their active service period until reaching the permissible retirement age. This survival probability is denoted as ${}_np_x$ with the formula (Islam et al., 2016):

$${}_np_x = \frac{l_{x+n}}{l_x} \tag{49}$$

1.5.13.2 Interest Rate Function

The interest rate function is a function used to determine the present value of a future payment by discounting it. The discount factor is denoted by v. To accumulate one unit of currency at the end of the year, a principal amount of v is invested at the beginning of the year with an annual interest rate i (Islam et al., 2016).

$$v^n = \frac{1}{(1+i)^n} \tag{50}$$

The interest rate function that fluctuates according to a stochastic model is expressed using the following equation (Winklevoss, 1993):

$$v^n = \prod_{t=1}^{n} \frac{1}{(1+r_t)} \tag{51}$$

1.5.13.3 Salary Function

In a pension program where benefits depend on the employee's salary, it is necessary to define salary notation and a procedure for projecting future salaries. The cumulative salary of an employee from the starting age of employment y up to age $r-1$ is denoted by Up_{total} (Miyasari, 2023):

$$Up_{total} = \sum_{x=y}^{r-1} Up_x \tag{52}$$

If it is assumed that the employee's annual salary increase rate is s, then the employee's final salary before retirement at age $r-1$, based on the salary at age x, can be calculated according to Aitken (1994) as follows:

$$Up_{r-1} = (1+s)^{r-1-x} Up_x \tag{53}$$

1.5.13.4 Benefit Function

The benefit function is used to determine the amount of pension benefits received by a participant in a pension program upon reaching retirement (Winklevoss, 1993). For example, b_x represents the amount of benefit that a participant aged x will receive if they continue to work for one more year. This amount is called the Benefit Accrual Function. B_x is the Accrued Benefit Function, which denotes the total pension benefit granted to a participant who has worked from the entry age y up to age $x-1$ (Caraka, 2016).

Using the Projected Unit Credit assumption, the pension benefit at age x is defined as follows:

$$B_r = \alpha(r-e)Up_{r-1} \tag{54}$$

Since B_x represents the monthly wage benefit, to obtain the annual benefit, it is multiplied by 12.

$$B_r = 12\alpha(r-e)Up_{r-1}$$

Furthermore, b_x is defined as the pension unit or a portion of the total pension benefit that will be received at age x.

$$b_x = \frac{B_r}{(r-e)} \tag{55}$$

So that,

$$b_x = \frac{\alpha(r-e)Up_x(1+s)^{r-1-x}}{(r-e)}$$

1.5.14 Normal Contribution (Normal Cost)

In general, pension contributions are payments made by pension fund participants to meet the cost of pension benefits (Andriani et al., 2009). Simply put, the normal cost is the present value of the pension benefit. The following are the formulas for calculating the normal cost using the Projected Unit Credit (PUC) and Entry Age Normal (EAN) methods.

1.5.14.1 Projected Unit Credit (PUC)

Projected Unit Credit, also referred to as Projected Accrued Benefit Cost by McGill and Grubbs, is a method very similar to the Traditional Unit Credit (TUC) method. However, this method incolDRorates the use of a salary scale into the calculation of pension benefits (Aitken, 1994). In this case, the pension benefit is determined based on years of service and salary. The normal cost at the beginning of each year is the pension benefit earned (accrued) during that year:

$$^{PUC}NC_x = b_x \frac{D_r^{(\tau)}}{D_x^{(\tau)}} \ddot{a}_r^{(12)} \tag{56}$$

Where b_x is the pension unit or a portion of the total pension benefit that will be received at age x, so that:

$$^{PUC}NC_x = \frac{D_r^{(\tau)}}{D_x^{(\tau)}} \left(\frac{\alpha\,(r-e)Up_x(1+s)^{r-1-x}}{(r-e)} \ddot{a}_r^{(12)} \right)$$

1.5.14.2 Entry Age Normal (Level Dollar)

In the Entry Age Normal (EAN) method, the normal contribution is a balance between the present value of the benefits to be received and the present value of future contributions.

$$PVF({}^{EAN}NC) = PVF(Benefit) \tag{57}$$

$${}^{EAN}NC_e\, \ddot{a}^{(\tau)}_{e:\overline{r-e|}} = \ddot{a}^{(12)}_r \frac{D^{(\tau)}_r}{D^{(\tau)}_e}\,(B_r)$$

$${}^{EAN}NC_e = \left(\frac{D^{(\tau)}_r}{N^{(\tau)}_e - N^{(\tau)}_r}\right)(B_r)\ddot{a}^{(12)}_r$$

$${}^{EAN}NC_e = \alpha(r-e)(1+s)^{(r-1)-e}\ddot{a}^{(12)}_r \left(\frac{D^{(\tau)}_r}{N^{(\tau)}_e - N^{(\tau)}_r}\right) \tag{58}$$

In the EAN method, the normal cost remains constant for each individual throughout their working period, provided there are no changes to the benefit projections and basic actuarial assumptions (Aitken, 1994).

1.5.15 Actuarial Liability

Actuarial Liability (AL) is the amount of funds that a pension plan must hold to ensure that participants' pension benefits are secure and ready for disbursement upon retirement (Izzati & Kartikasari, 2022). Alternatively, AL can be defined as the estimated present value of all pension benefits accrued by a participant from the start of employment until retirement age. This AL value starts at zero when the participant first joins the plan. The following are the formulas for calculating actuarial liability using the Projected Unit Credit (PUC) and Entry Age Normal (EAN) methods:

1.5.15.1 Projected Unit Credit (PUC)

Under the PUC method, the Actuarial Liability at age x is defined as the difference between the Normal Cost at age x and the Normal Cost at age a, where $a = e$ represents the age at which an individual enters the pension program (assuming the pension entry age equals the age at which employment begins) (Aitken, 1994).

$${}^{PUC}AL_x = NC_x(x-e) \tag{59}$$

$${}^{PUC}AL_x = \frac{D^{(\tau)}_r}{D^{(\tau)}_x}\left(\frac{\alpha\,(r-e)Up_x(1+s)^{r-1-x}(x-e)}{(r-e)}\right)\ddot{a}^{(12)}_r$$

Or, when expressed in terms of ${}^{r}(PVFB)_x$ the formula for Actuarial Liability using the PUC method is as follows (Putri S.R., Susanti, & Riaman, 2024):

$${}^{PUC}AL_x = \frac{x-e}{r-e}\,{}^{r}(PVFB)_x$$

1.5.15.2 Entry Age Normal (EAN)

Under the Entry Age Normal (EAN) method, the Actuarial Liability is defined as the difference between the present value of future pension benefits and the present value of future premium contributions. A distinctive feature of the Entry Age Normal method is the assumption that the age at which pension funding begins is always equal to the age at which employment commences (a = e).

$$^{EAN}AL_x = {}^{r}(PVFB)_x - PVF\ (NC) \tag{60}$$

$$^{EAN}AL_x = B_r \frac{D_r^{(\tau)}}{D_x^{(\tau)}}\ \ddot{a}_r^{(12)} -\ NC_e \left(\frac{N_x^{(\tau)} - N_r^{(\tau)}}{D_x^{(\tau)}} \right) \tag{61}$$

1.5.16 Past Service Liability

According to Republic of Indonesia Minister of Finance Regulation No. 25/PMK.02/2013, Unfunded Past Service Liability refers to an outstanding past obligation for the Old Age Savings (Tabungan Hari Tua) program. This arises due to several conditions outlined in Article 2 of the same regulation, namely: (1) Changes to the benefit formula of the Old Age Savings Program; (2) Increases in the base salary table used for calculating Old Age Savings Program benefits; and (3) The addition of new participants whose enrolment date differs from their employment commencement date.

1.5.17 Unfunded Actuarial Liability (UAL)

Unfunded Actuarial Liability (UAL) is the excess of Actuarial Liability (AL) that occurs prior to retirement age (Aitken, 1996). Based on The Task Force on Local Government Services and Fiscal Stability (Final Report to the Governor, May 2006), UAL is defined as the amount of funding insufficiency required to pay pension benefits or other post-employment benefits to beneficiaries or participants of a pension funding program. The UAL value represents the shortfall that occurs when a pension fund's assets are insufficient to cover its actuarial liabilities.

UAL can be caused by several factors, such as: (1) Service periods prior to the pension funding, (2) Salary increases exceeding the assumed rate, and (3) Changes in the normal retirement age assumption (Riaman, et al., 2018). In the case of changes in the normal retirement age assumption, the actuarial liability shortfall occurs due to a change in the retirement age, often referred to as accelerated or early retirement. The emergence of a past service liability equal to the UAL increases the employer's obligation to provide additional contributions.

In the case of early retirement, let r be the projected initial/normal retirement age and let $k$be the assumed early retirement age. The equation is then formulated as follows:

1.4.17.1 Projected Unit Credit (PUC)

$$ {}^{PUC\ k}AL_x = \frac{D_k^{(\tau)}}{D_x^{(\tau)}}\left(\frac{\alpha(k-e)(1+s)^{(k-1)-x}Up_x}{(k-a)}\ddot{a}_k^{(12)}\right)(x-a) \quad (62) $$

1.4.17.2 Entry Age Normal

$$ {}^{EAN\ k}AL_x = \alpha\frac{D_k^{(\tau)}}{D_x^{(\tau)}}(k-e)Up_r\ddot{a}_k^{(12)} - NC_e\left(\frac{N_x^{(\tau)} - N_k^{(\tau)}}{D_x^{(\tau)}}\right) \quad (63) $$

Assuming that the previous period satisfies the condition $UAL_x^r = 0$, the UAL equation will be as follows:

$$ UAL_x^k = AL_x^k - AL_x^r > 0 $$

Therefore, the Past Service Liability (PSL) in this case is defined as:

$$ PSL = UAL_x^k = AL_x^k - AL_x^r > 0 \quad (64) $$

As previously explained, the PSL results in an additional obligation for the employer to contribute extra funds in order to cover the shortfall represented by the UAL.

1.5.18 Supplemental Cost

Supplemental Cost (SC) is an additional expense that the employer must pay to the pension fund when there is a funding deficit in the actuarial liability (Nurlatifah, 2015).

This supplemental cost is used to bridge the gap between the liability and the established pension benefits. It can be a one-time payment equal to the unfunded liability created during a given year, or it can be spread over several years. Supplemental costs, at least in theory, can follow any pattern over any period, or they can be aligned with the normal cost pattern corresponding to the actuarial cost method used (Howard et al., 1977). According to Aitken, the PSL equation is:

$$ PSL = SC_x\ddot{a}_{x:\overline{n|}} $$

Thus, the additional contribution at age x is calculated as follows:

$$ SC_x = \frac{PSL}{\ddot{a}_{x:\overline{n|}}} = \frac{AL_x^k - AL_x^r}{\ddot{a}_{x:\overline{n|}}} \quad (65) $$

With $n = k - x$

CHAPTER II

RESEARCH METHODOLOGY

2.1 Research Design and Approach

This study adopts a quantitative research design utilizing actuarial and financial modelling techniques. The primary analytical framework is based on the Projected Unit Credit (PUC) and Entry Age Normal (EAN) actuarial cost methods. These methods are used to calculate key pension liability metrics specifically, the Unfunded Actuarial Liability (UAL) and Supplemental Cost (SC) under an early retirement scenario. A critical component of the design is the incoIDRoration of stochastic modelling, whereby the Cox-Ingersoll-Ross (CIR) model is applied to simulate future interest rate paths, moving beyond deterministic assumptions. The focus is on the numerical analysis of collected data to model financial outcomes and evaluate liability implications under varying conditions.

2.2 Research Time and Location

The research has been conducted since November 2025. This study utilizes literature sources available directly from the official library and mathematics laboratory of Hasanuddin University, as well as several resources available online.

2.3 Research Object

The objects of this research are the employee salary data of PT Pertamina (Persero) and the monthly interest rate data from Bank Indonesia.

2.4 Research Procedure

The research procedure will be detailed as follows:

1) Data Collection
   The researcher gathers data from various previously published literature. This literature includes books, journals, articles, and other scientific sources relevant to the research topic. Furthermore, data is also obtained from official online literature sources corresponding to the research object.
2) Determination of Research Specifications
   After data collection, the research specifications are determined. In this case, the specification is the application of the stochastic Cox-Ingersoll-Ross (CIR) model to calculate the Unfunded Actuarial Liability (UAL) and Supplemental Cost (SC) in the pension funding of PT Pertamina (Persero). This research is preceded by calculating the Normal Cost and Actuarial Liability of PT Pertamina's pension fund.

3) Data Analysis
    c) Estimating the parameters of the Cox-Ingersoll-Ross model using the Ordinary Least Squares (OLS) method.
    d) Simulating the movement of the interest rate using the Cox-Ingersoll-Ross model.
    e) Calculating the accuracy of the Cox-Ingersoll-Ross interest rate movement simulation using Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE).
    f) Determining the Benefit Value (B_x) and Unit Benefit Value (b_x) for pension funding.
    g) Calculating the Normal Cost using the Projected Unit Credit (PUC) and Entry Age Normal (EAN) methods.
    h) Calculating the Actuarial Liability using the Projected Unit Credit (PUC) and Entry Age Normal (EAN) methods.
    i) Comparing the actuarial calculation results from the PUC and EAN methods for the normal retirement case with the early retirement assumption.
    j) Calculating the Past Service Liability using the Projected Unit Credit (PUC) and Entry Age Normal (EAN) methods.
    k) Calculating the Supplemental Cost using the Projected Unit Credit (PUC) and Entry Age Normal (EAN) methods.
4) Drawing Conclusions

2.5 Research Workflow

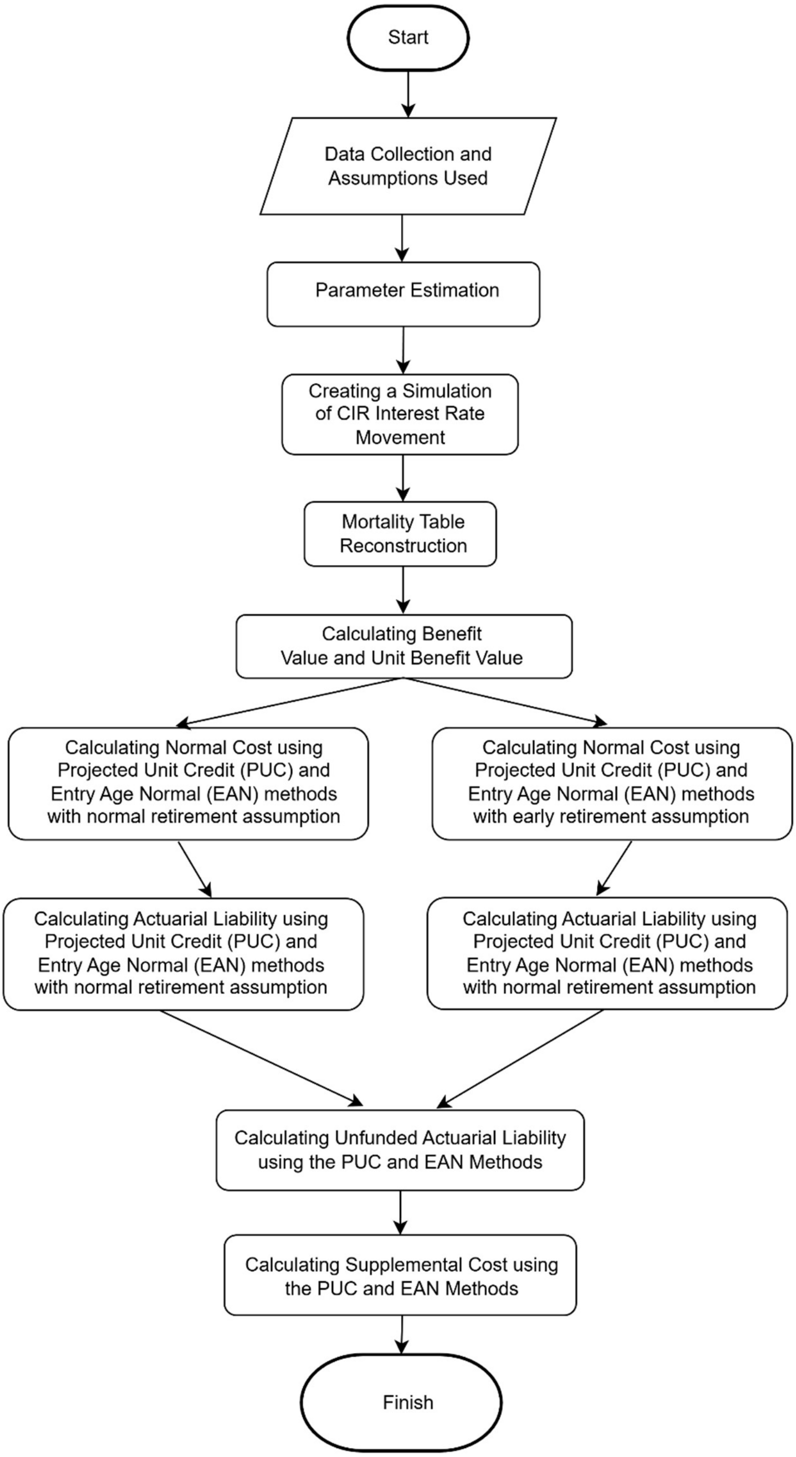

Figure 2. Research workflow