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Lampiran

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> restart
> F1 :=  $\lambda_0^2 - \lambda_1^2 - \lambda_2^2 - \lambda_3^2 - 2 \cdot (\alpha_0 \lambda_0 - \alpha_1 \lambda_1 - \alpha_2 \lambda_2 - \alpha_3 \lambda_3) + (\alpha_0^2 - \alpha_1^2 - \alpha_2^2 - \alpha_3^2) - n(b_0^2 - b_1^2 - b_2^2 - b_3^2) = 0$ :
> F2 :=  $2 \cdot \lambda_0 \lambda_1 - 2 \cdot (\alpha_0 \lambda_1 + \alpha_1 \lambda_0) + 2 \cdot \alpha_0 \alpha_1 - 2 \cdot n \cdot b_0 \cdot b_1 = 0$ :
> F3 :=  $2 \cdot \lambda_0 \lambda_2 - 2 \cdot (\alpha_0 \lambda_2 + \alpha_2 \lambda_0) + 2 \cdot \alpha_0 \alpha_2 - 2 \cdot n \cdot b_0 \cdot b_2 = 0$ :
> F4 :=  $2 \cdot \lambda_0 \lambda_3 - 2 \cdot (\alpha_0 \lambda_3 + \alpha_3 \lambda_0) + 2 \cdot \alpha_0 \alpha_3 - 2 \cdot n \cdot b_0 \cdot b_3 = 0$ :
> G := solve( (F1, F2, F3, F4), [lambda0, lambda1, lambda2, lambda3] ) :
> GG := map(allvalues, (G))


$$\begin{aligned}
GG := & \left[ \left[ \lambda_0 = a_0 - b_0 \sqrt{n}, \lambda_1 = -\frac{n b_0 b_1 + a_1 (a_0 - b_0 \sqrt{n}) - a_0 a_1}{b_0 \sqrt{n}}, \lambda_2 = -\frac{n b_0 b_2 + a_2 (a_0 - b_0 \sqrt{n}) - a_0 a_2}{b_0 \sqrt{n}}, \lambda_3 = -\frac{n b_0 b_3 + a_3 (a_0 - b_0 \sqrt{n}) - a_0 a_3}{b_0 \sqrt{n}} \right] \middle| \lambda_0 \\ 
& = a_0 - \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}, \lambda_1 = -\frac{n b_0 b_1 + a_1 (a_0 - \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_1}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}}, \lambda_2 = -\frac{n b_0 b_2 + a_2 (a_0 - \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_2}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}}, \lambda_3 \\ 
& = -\frac{n b_0 b_3 + a_3 (a_0 - \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_3}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}} \right], \left[ \lambda_0 = a_0 - b_0 \sqrt{n}, \lambda_1 = -\frac{n b_0 b_1 + a_1 (a_0 - b_0 \sqrt{n}) - a_0 a_1}{b_0 \sqrt{n}}, \lambda_2 = \right. \\ 
& \left. -\frac{n b_0 b_2 + a_2 (a_0 - b_0 \sqrt{n}) - a_0 a_2}{b_0 \sqrt{n}}, \lambda_3 = -\frac{n b_0 b_3 + a_3 (a_0 - b_0 \sqrt{n}) - a_0 a_3}{b_0 \sqrt{n}} \right], \left[ \lambda_0 = a_0 + \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}, \lambda_1 \right. \\ 
& = \frac{n b_0 b_1 + a_1 (a_0 + \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_1}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}}, \lambda_2 = \frac{n b_0 b_2 + a_2 (a_0 + \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_2}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}}, \lambda_3 \\ 
& = \frac{n b_0 b_3 + a_3 (a_0 + \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_3}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}} \right], \left[ \lambda_0 = a_0 + b_0 \sqrt{n}, \lambda_1 = \frac{n b_0 b_1 + a_1 (a_0 + b_0 \sqrt{n}) - a_0 a_1}{b_0 \sqrt{n}}, \lambda_2 \right. \\ 
& = \frac{n b_0 b_2 + a_2 (a_0 + b_0 \sqrt{n}) - a_0 a_2}{b_0 \sqrt{n}}, \lambda_3 = \frac{n b_0 b_3 + a_3 (a_0 + b_0 \sqrt{n}) - a_0 a_3}{b_0 \sqrt{n}} \right], \left[ \lambda_0 = a_0 - \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}, \lambda_1 = \right. \\ 
& \left. -\frac{n b_0 b_1 + a_1 (a_0 - \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_1}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}}, \lambda_2 = -\frac{n b_0 b_2 + a_2 (a_0 - \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_2}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}}, \lambda_3 = \right. \\ 
& \left. -\frac{n b_0 b_3 + a_3 (a_0 - \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_3}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}} \right], \left[ \lambda_0 = a_0 + b_0 \sqrt{n}, \lambda_1 = \frac{n b_0 b_1 + a_1 (a_0 + b_0 \sqrt{n}) - a_0 a_1}{b_0 \sqrt{n}}, \lambda_2 \right. \\ 
& = \frac{n b_0 b_2 + a_2 (a_0 + b_0 \sqrt{n}) - a_0 a_2}{b_0 \sqrt{n}}, \lambda_3 = \frac{n b_0 b_3 + a_3 (a_0 + b_0 \sqrt{n}) - a_0 a_3}{b_0 \sqrt{n}} \right], \left[ \lambda_0 = a_0 + \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}, \lambda_1 \right. \\ 
& = \frac{n b_0 b_1 + a_1 (a_0 + \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_1}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}}, \lambda_2 = \frac{n b_0 b_2 + a_2 (a_0 + \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_2}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}}, \lambda_3 \\ 
& = \frac{n b_0 b_3 + a_3 (a_0 + \sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}) - a_0 a_3}{\sqrt{-b_1^2 n - b_2^2 n - b_3^2 n}} \right]
\end{aligned}$$


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Optimization Software:
www.balesio.com