DAFTAR PUSTAKA

Alaoui, M. A. Aziz dan M. Daher Okiye. 2003. "Boundedness and Global Stability for a Predator-Prey Model with Modified Leslie-Gower and Holling-Type II Schemes". *Applied Mathematics Letter*, 1069-1075.

Aly, Shaban, Imbunm Kim dan Dongwoo Sheen. 2011. "Turing Instability for a Ratio-Dependent Predator-Prey Model with Diffusion". *Applied Mathematics and Computation 217*, 7265-7281.

Baker, Ruth, 2016. *Mathematical Modeling in Biology Lecture Notes*. Trinity Term.

Berestycki, Henri dan alessandro Zilio. 2019. "Predator-Prey Models with Competition: The Emergence of Territoriality". *The American Naturalist*, vol.193, no.3: 436-446.

Boyce, W. E. dan Richard C. DiPrima. 2012. *Elementary Differential Equations and Boundary Value Problems*. New York: WileyPLUS.

Cai Yongli, Caidi Zhao, Weiming Wang dan Jinfeng Wang. 2014. "Dynamics of a Leslie-Gower Predator-Prey Model with Additive Allee Effect". *Appl. Math. Modelling* (2014) : 1 – 33.

Davis, Mark. 2010. *Finite Difference Methods*. Imperial Collage London, 11.

E, Liznarova dan Pekar S. 2013. "Dangerous Prey is Associated with a Type IV Functional Response in Spiders". *Ani Behav*, 85:1183 – 90.

Esfandiari, Ramin S. 2017. *Numerical Methods for Engineers and Scientists Using MATLAB Second Edition*. CRC Press Taylor & Francis Group.

Gao, Wenjing, Yihui Tong, Lihua Zhai, Ruizhi Yang dan Leiyu Tang. 2019. "Turing Instability and Hopf Bifurcation in a Predator Prey Model with Delay and Predator Harvesting". *Advances in Difference Equation*. Springer, 270.

Gao, Xiaoyan, Sadia Ishag, Shengmao Fu, Wanjun Li, dan Weiming Wang. 2020. "Bifurcation and Turing Pattern Formation in a Diffusive Ratio-dependent Predator Prey Model with Predator Harvesting". *Nonlinear Analysis: Real Word Applications 51*, 102962 : 1-28.

Gunawan, Putu Harry. 2016. *Pengantar Persamaan Diferensial Parsial untuk Sains dan Teknik*. Jakarta: www.nulisbuku.com.

Gupta, R.P. dan Peeyush Chandra. 2013. “Bifurcation Analysis of a Modified Leslie-Gower Predator-Prey Model with Michaelis-Menten Type Prey Harvesting”. *J. Math. Anal. Appl*, 398: 278 – 295.

Gupta, Rajesh Kumar. 2019. *Numerical Methods Fundamental and Applications*. Cambridge University Press.

Huang, Jicai, Xiaojing Xia dan Xinan Zhang. 2016. “Bifurcation of Codimension 3 in a Predator-Prey System of Leslie Type with Simplified Holling Type IV Functional Response”. *International Jurnal of Bifurcation and Chaos*, vol.26 no. 2:1650034-11.

Huang, Tousheng, Huayong Zhang, Zhengran Hu, Ge Pan, Shengnan Ma, Xiumin Zhang dan Zichun Gao. 2019. “Predator-Prey Pattern Formation Driven by Population Diffusion Based on Moore Neighborhood Structure”. *Advances in Difference Equation*, 399.

Kasbawati, 2011. “Analisis Numerik Model Epidemiologi SIR dengan Faktor Difusi”. JMSK Vol. 7 No. 2: 98-107.

Khajanchi, Subhas. 2017. “Modeling the Dynamics of Stage-Structure Predator-Prey System with Monod-Haldane Type Response Function”. *Applied Mathematics and Computations*, 302:122–143.

Leslie, P. H. 1948. “Some Further Notes on The Use of Matrices in Population Mathematics*”. Biometrika,* vol. XXXV.

Leveque, 2007. *Finite Difference Methods for Ordinary and Partial Differential Equations: Steady State and Time-Dependent Problems*. Washington: Society for Industrial and Applied Mathematics.

Perthame, Benoit. 2018. *Mathematics Model in Biology.* Paris: Sorbonne University.

Mishra, P., S.N. Raw dan B. Tiwari. 2019. “Study of a Leslie-Gower Predator-Prey Model with Prey Defense and Mutual Interference of Predators”. *Chaos, Solitons and Fractals*, 120: 1–16.

Murray, J.D. 2003. *Mathematical Biology II: Spatial Models and Biomedical Applications*. Third Edition. New York: Springer-Verlag.

Rai, Vikas. 2013. *Spatial Ecology Patterns and Processes.* India : Bentham e Books.

Rodrigues, Daiana, Luis Paulo Barra, Marcello Labosco dan Flavia Bastos. 2014. Analysis of Turing Instability for Biological Models. *Springer International Publishing Switzerlan,* 576-591.

Ruan, Shigui dan Dongmei Xiao. 2001. ”Global Analysis in A Predator-Prey System with Nonmonotonic Functional Response”. *SIAM J. APPL. MATH*, vol.61 No.4:1445 –1472.

Shen, Chunxia. 2007. “Permanence and Global Attractivity of Food-Chain System with Holling IV Type Functional Response”. *Applied Mathematics and Computations,* 194:179 –185.

Sokol, W., dan J.A. Howel. 1987. “The Kinetics of Phenol Oxidation by Washed Cells”. *Biotechnology and Bioengineering*, 30: 921 – 927.

Song, Jie, Yonghui Xia, Yuzhen Bai, Yaoxiong Cai dan D. O’Regan. 2019. “A Non-Autonomous Leslie-Gower Model with Holling Type IV Functional Response and Harvesting Complexity”. *Advances in Difference Equation,* 299.

Upadhyay, R. K., N. Kumari dan V. Rai. 2008. “Wave of Chaos and Pattern Formation in Spatial Predator-Prey systems with Holling Type IV Predator Response”. *Math. Model. Nat. Phenom,* Vol.3 No.4: 71-95.

Upadhyay, R. K., W. Wang dan N. K. Thakur. 2010. “Spatiotemporal Dynamics in a Spatial Plankton System”. *Math. Model. Nat. Phenom*. Vol.5 No.5: 102-122.

Vejchodsky, Tomas. 2013. *Pattern Formation – Turing Instability. Centre for Mathematical Biology Mathematical.* Institute:University of Oxford.

Yan, X.P. dan Cun-Hua Zhang. 2014. “Stability and Turing Instability in a Diffusive Predator-Prey System with Beddington-DeAngelis Functional Response”. *Nonlinear Analysis: Real Word Applications*, 20: 1-13.

Yue, Qin, 2016. “Dynamics of Modified Leslie-Gower Model Predator-Prey Model with Holling Type II Schemes and a Prey Refuge”. *Springer Plus*, 5 : 461

Lampiran 1. Titik Kesetimbangan

Titik kesetimbangan dari Model (4.1) terjadi jika

$$\frac{du}{dt} = ru - bu^2 - \frac{m_1uv}{h_1 + u^2} = 0,$$
$$\frac{dv}{dt} = sv - \frac{m_2v^2}{e+u} = 0,$$

substitusi persamaan di atas pada laju perubahan populasi mangsa $\frac{du}{dt}$, diperoleh:

$$\frac{du}{dt} = ru - bu^2 - \frac{m_1uv}{h_1 + u^2} = 0,$$
$$\left(ru - bu - \frac{m_1v}{h_1 + u^2} \right) u = 0.$$

Jadi, diperoleh:

$$\bar{u} = 0 \quad \text{atau} \quad \begin{aligned} ru - bu - \frac{m_1v}{h_1+u^2} &= 0, \\ (r - bu)(h_1 + u^2) - m_1v &= 0. \end{aligned}$$

- Untuk $u = 0$, substitusi ke dalam laju perubahan populasi pemangsa $\frac{dv}{dt}$, diperoleh:

$$\frac{dv}{dt} = sv - \frac{m_2v^2}{e+u} = 0,$$
$$\left(s - \frac{m_2v}{e} \right) v = 0,$$
$$(se - m_2v) v = 0.$$

Sehingga, $\bar{v} = 0$ atau $\bar{v} = \frac{se}{m_2}$.

Jadi, diperoleh titik kesetimbangan $T_1 = (0,0)$ dan $T_2 = \left(0, \frac{se}{m_2} \right)$.

- Untuk $\bar{v}=0$ substitusi ke dalam laju perubahan populasi mangsa $\frac{du}{dt}$, diperoleh:

$$\frac{du}{dt}=ru-bu^2-\frac{m_1uv}{h_1+u^2}=0,$$
$$ru-bu^2-\frac{m_1u.0}{h_1+u^2}=0,$$
$$ru-bu^2=0,$$
$$r-bu=0,$$
$$\bar{u}=\frac{r}{b}.$$

Jadi, diperoleh titik kesetimbangan $T_3=\left(\frac{r}{b},0\right)$.

- Untuk $u\neq 0$, substitusi ke dalam laju perubahan populasi pemangsa $\frac{dv}{dt}$,

$$\frac{dv}{dt}=sv-\frac{m_2v^2}{e+u}=0,$$
$$sv\left(e+\bar{u}\right)-m_2v^2=0,$$
$$s\left(e+\bar{u}\right)-m_2v=0,$$

maka diperoleh

$$\bar{v}=\frac{s\left(e+\bar{u}\right)}{m_2},$$

Selanjutnya, substitusi nilai $\bar{v}$ ke dalam laju perubahan populasi mangsa $\frac{du}{dt}$, diperoleh:

$$\frac{du}{dt} = ru - bu^2 - \frac{m_1 uv}{h_1 + u^2} = 0,$$

$$r - bu - \frac{m_1 v}{h_1 + u^2} = 0,$$

$$r - bu - \frac{m_1 \frac{(s(e+\bar{u}))}{m_2}}{h_1 + u^2} = 0,$$

$$r - bu - \frac{m_1 s(e+\bar{u})}{m_2(h_1 + u^2)} = 0,$$

$$(r - bu)\left(m_2(h_1 + u^2)\right) - m_1 s(e+\bar{u}) = 0,$$

$$rm_2h_1 + rm_2u^2 - bm_2h_1u - bm_2u^3 - m_1se - m_1su = 0,$$

$$bu^3 - \frac{rm_2}{m_2}u^2 + \left(\frac{bm_2h_1}{m_2} + \frac{m_1 s}{m_2}\right)u - \frac{rm_2h_1}{m_2} + \frac{m_1 se}{m_2} = 0,$$

$$u^3 - \frac{r}{b}u^2 + \left(\frac{h_1bm_2 + m_1 s}{bm_2}\right)u + \frac{m_1se - rh_1m_2}{bm_2} = 0,$$

Persamaan di atas merupakan persamaan kubik yang dapat ditulis dalam bentuk:

$$f(u) = u^3 + a_1u^2 + a_2u + a_3 = 0, \quad (1)$$

dengan,

$$a_1 = -\frac{r}{b},$$

$$a_2 = \frac{h_1bm_2 + m_1s}{bm_2},$$

$$a_3 = \frac{m_1se - rh_1m_2}{bm_2}.$$

Jadi, diperoleh titik kesetimbangan T_4 dengan $\bar{u}$ adalah akar-akar positif dari Persamaan (1) dan $\bar{v} = \frac{s(e+\bar{u})}{m_2}$.

Lampiran 2. Titik Kesetimbangan Menggunakan Maple

> restart

> with(DEtools) :

> with(linalg) :

> $P1 := \left(r - b\cdot u - \frac{m1\cdot v}{h1 + u^2}\right)\cdot u$

$$P1 := \left(r - b\,u - \frac{m1\,v}{u^2 + h1}\right)u \quad (1)$$

>

> $P2 := \left(s - \frac{m2\cdot v}{e + u}\right)\cdot v$

$$P2 := \left(s - \frac{m2\,v}{e + u}\right)v \quad (2)$$

> $fixpoint := solve(\{P1, P2\}, \{u, v\})$

$$fixpoint := \{u = 0, v = 0\}, \left\{u = 0, v = \frac{s\,e}{m2}\right\}, \left\{u = \frac{r}{b}, v = 0\right\}, \left\{u = RootOf\left(b\,m2\,\_Z^3 - m2\,r\,\_Z^2 + (b\,h1\,m2 + m1\,s)\,\_Z + s\,e\,m1 - h1\,m2\,r\right), v = \frac{s\left(e + RootOf\left(b\,m2\,\_Z^3 - m2\,r\,\_Z^2 + (b\,h1\,m2 + m1\,s)\,\_Z + s\,e\,m1 - h1\,m2\,r\right)\right)}{m2}\right\} \quad (3)$$

>

> with(VectorCalculus) :

> with(LinearAlgebra) :

> $Jacobian(\langle P1, P2\rangle, [u, v])$

$$\begin{bmatrix} \left(-b + \frac{2\,m1\,v\,u}{\left(u^2 + h1\right)^2}\right)u + r - b\,u - \frac{m1\,v}{u^2 + h1} & -\frac{m1\,u}{u^2 + h1} \\ \frac{m2\,v^2}{(e + u)^2} & -\frac{2\,m2\,v}{e + u} + s \end{bmatrix} \quad (4)$$

Lampiran 3. Bukti Teorema 4.1.

a. Jika $j_1 > 0$ dan $|j_1| < |j_4|$ maka $Trace(J(T_4)) < 0$

Diketahui:

- $j_1 = r - 2b\bar{u} - \dfrac{m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2} > 0$,

- $Trace(J(T_4)) = j_1 + j_4$,

- $j_1 > 0$ sedemikian sehingga $|j_1| = j_1$,

- $j_4 < 0$ sedemikian sehingga $|j_4| = |-s| = s = -j_4$,

selanjutnya akan dibuktikan jika $j_1 > 0$, $j_4 < 0$ dan $|j_1| < |j_4|$ maka $Trace(J(T_4)) = j_1 + j_4 < 0$.

Bukti:

$$|j_1| < |j_4|,$$
$$j_1 < -j_4,$$
$$j_1 + j_4 < 0.$$

Jadi, terbukti jika $j_1 > 0$ dan $|j_1| < |j_4|$ maka $Trace(J(T_4)) = j_1 + j_4 < 0$.

b. Jika $j_1 > 0$ dan $|j_2 j_3| > |j_1 j_4|$ maka $\det(J(T_4)) > 0$

Diketahui:

- $j_1 = r - 2b\bar{u} - \dfrac{m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2} > 0$,

- $\det(J(T_4)) = j_1 j_4 - j_2 j_3$,

- $j_1 j_4 < 0$ sedemikian sehingga

$$
\begin{aligned}
|j_1 j_4| &= \left| r - 2b\bar{u} - \frac{m_1 \bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2} \right| |-s|, \\
&= \left| r - 2b\bar{u} - \frac{m_1 \bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2} \right| (s), \\
&= \left(r - 2b\bar{u} - \frac{m_1 \bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2} \right) (s), \\
&= \left(j_1 \right)(-j_4), \\
&= -j_1 j_4.
\end{aligned}
$$

- $j_2 j_3 < 0$ sedemikian sehingga

$$
\begin{aligned}
|j_2 j_3| &= \left| -\frac{m_1 m_2 \bar{u}\bar{v}^2}{\left(h_1 + \bar{u}^2\right)(e + \bar{u})^2} \right|, \\
&= \frac{m_1 m_2 \bar{u}\bar{v}^2}{\left(h_1 + \bar{u}^2\right)(e + \bar{u})^2}, \\
&= -j_2 j_3.
\end{aligned}
$$

selanjutnya akan dibuktikan jika $j_1 > 0$, $j_1 j_4 < 0$, $j_2 j_3 < 0$ dan $|j_2 j_3| > |j_1 j_4|$ maka $\det(J(T_4)) = j_1 j_4 - j_2 j_3 > 0$.

Bukti:

$$
\begin{aligned}
|j_2 j_3| &> |j_1 j_4|, \\
-j_2 j_3 &> -j_1 j_4, \\
-j_2 j_3 + j_1 j_4 &> 0, \\
j_1 j_4 - j_2 j_3 &> 0.
\end{aligned}
$$

Jadi, terbukti jika $j_1 > 0$ dan $|j_2 j_3| > |j_1 j_4|$ maka $\det(J(T_4)) = j_1 j_4 - j_2 j_3 > 0.$

c. Jika $\left(r-2b\bar{u}\right)\alpha^2 > m_1\bar{v}\beta$ maka $j_1 > 0$

Diketahui:

- $j_1 = r - 2b\bar{u} - \dfrac{m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2}$,

selanjutnya akan dibuktikan jika $\left(r-2b\bar{u}\right)\alpha^2 > m_1\bar{v}\beta$ maka $j_1 > 0$.

Bukti

$$
\begin{aligned}
j_1 &> 0,\\
r - 2b\bar{u} - \frac{m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2} &> 0,\\
\frac{\left(r-2b\bar{u}\right)(h_1 + \bar{u}^2)^2 - m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2} &> 0,\\
\frac{\left(r-2b\bar{u}\right)(h_1 + \bar{u}^2)^2}{(h_1 + \bar{u}^2)^2} &> \frac{m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2},\\
\left(r-2b\bar{u}\right)(h_1 + \bar{u}^2)^2 &> m_1\bar{v}(h_1 - \bar{u}^2),
\end{aligned}
$$

atau $\left(r-2b\bar{u}\right)\alpha^2 > m_1\bar{v}\beta$ dengan $\alpha = (h_1 + \bar{u}^2)$.

d. Jika $|j_1| < |j_4|$ maka $\left(r-2b\bar{u}-s\right)\alpha^2 < m_1\bar{v}\beta$

Diketahui

- $j_1 = r - 2b\bar{u} - \dfrac{m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2} > 0$ sedemikian sehingga

$$|j_1| = \frac{\left(r-2b\bar{u}\right)(h_1 + \bar{u}^2)^2 - m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2},$$

- $j_4 = -s < 0$ sedemikian sehingga $|j_4| = |-s| = s = -j_4$.

Selanjutnya akan dibuktikan jika $|j_1| < |j_4|$ maka $\left(r-2b\bar{u}-s\right)\alpha^2 < m_1\bar{v}\beta$.

Bukti:

$$|j_1| < |j_4|,$$
$$\frac{\left(r-2b\bar{u}\right)(h_1+\bar{u}^2)^2 - m_1\bar{v}(h_1-\bar{u}^2)}{(h_1+\bar{u}^2)^2} < s,$$
$$\left(r-2b\bar{u}\right)(h_1+\bar{u}^2)^2 - m_1\bar{v}(h_1-\bar{u}^2) < s(h_1+\bar{u}^2)^2,$$
$$\left(r-2b\bar{u}-s\right)(h_1+\bar{u}^2)^2 < m_1\bar{v}(h_1-\bar{u}^2).$$

Jadi, diperoleh kesimpulan bahwa jika $|j_1| < |j_4|$ maka $\left(r-2b\bar{u}-s\right)\alpha^2 < m_1\bar{v}\beta$, dengan $\alpha = (h_1+\bar{u}^2)$ dan $\beta = (h_1-\bar{u}^2)$.

e. Jika $j_1 > 0$ dan $|j_2 j_3| > |j_1 j_4|$ maka $m_1 m_2 \bar{u}\bar{v}^2 \alpha > s\gamma^2 \left[\left(r-2b\bar{u}\right)\alpha^2 - m_1\bar{v}\beta\right]$

Diketahui:

- $j_1 = r - 2b\bar{u} - \dfrac{m_1\bar{v}(h_1-\bar{u}^2)}{(h_1+\bar{u}^2)^2} > 0$,

- $j_2 j_3 < 0$ sedemikian sehingga $|j_2 j_3| = \dfrac{m_1 m_2 \bar{u}\bar{v}^2}{\left(h_1+\bar{u}^2\right)(e+\bar{u})^2}$,

- $j_1 j_4 < 0$ sedemikian sehingga

$$|j_1 j_4| = s\left(\frac{\left(r-2b\bar{u}\right)(h_1+\bar{u}^2)^2 - m_1\bar{v}(h_1-\bar{u}^2)}{(h_1+\bar{u}^2)^2}\right),$$

selanjutnya akan dibuktikan jika $j_1 > 0$ dan $|j_2 j_3| > |j_1 j_4|$ maka $m_1 m_2 \bar{u}\bar{v}^2 \alpha > s\gamma^2 \left[\left(r-2b\bar{u}\right)\alpha^2 - m_1\bar{v}\beta\right]$, Bukti:

$$|j_2 j_3| > |j_1 j_4|,$$

$$\frac{m_1 m_2 \bar{u}\bar{v}^2}{\left(h_1 + \bar{u}^2\right)(e + \bar{u})^2} > s\left(\frac{\left(r - 2b\bar{u}\right)(h_1 + \bar{u}^2)^2 - m_1\bar{v}(h_1 - \bar{u}^2)}{(h_1 + \bar{u}^2)^2}\right),$$

$$\left(m_1 m_2 \bar{u}\bar{v}^2\right)(h_1 + \bar{u}^2) > s(e + \bar{u})^2\left[\left(r - 2b\bar{u}\right)(h_1 + \bar{u}^2)^2 - m_1\bar{v}(h_1 - \bar{u}^2)\right].$$

Jadi, terbukti jika $j_1 > 0$ dan $|j_2 j_3| > |j_1 j_4|$ maka $m_1 m_2 \bar{u}\bar{v}^2\alpha > s\gamma^2\left[\left(r - 2b\bar{u}\right)\alpha^2 - m_1\bar{v}\beta\right]$, dengan $\alpha = (h_1 + \bar{u}^2)$, $\beta = (h_1 - \bar{u}^2)$ dan $\gamma = (e + u)$.

Lampiran 4. Syarat Batas dalam Ruang Dua Dimensi

Syarat batas dalam ruang dua dimensi diberikan sebagai berikut,

$$\frac{du}{dx}(1,y,t)=\frac{dv}{dx}(1,y,t)=0\,, \tag{2}$$

$$\frac{du}{dx}(Nx,y,t)=\frac{dv}{dx}(Nx,y,t)=0\,, \tag{3}$$

$$\frac{du}{dx}(x,1,t)=\frac{dv}{dx}(x,1,t)=0\,, \tag{4}$$

$$\frac{du}{dx}(x,Ny,t)=\frac{dv}{dx}(x,Ny,t)=0\,. \tag{5}$$

Persamaan (2) – (5) didiskritisasi menggunakan pendekatan beda pusat sehingga diperoleh model diskrit Persamaan (2) sebagai berikut:

$$\begin{gathered}\frac{u_{2,j,n}-u_{0,j,n}}{2\Delta x}=0 \text{ atau } u_{2,j,n}=u_{0,j,n}\,,\\ \frac{v_{2,j,n}-v_{0,j,n}}{2\Delta x}=0 \text{ atau } v_{2,n}=v_{0,n}\,.\end{gathered} \tag{6}$$

Untuk model diskrit Persamaan (3) diberikan sebagai berikut:

$$\begin{gathered}\frac{u_{Nx+1,j,n}-u_{Nx-1,j,n}}{2\Delta x}=0 \text{ atau } u_{Nx+1,j,n}=u_{Nx-1,j,n}\,,\\ \frac{v_{Nx+1,j,n}-v_{Nx-1,j,n}}{2\Delta x}=0 \text{ atau } v_{2,n}=v_{0,n}\,.\end{gathered} \tag{7}$$

Selanjutnya, model diskrit Persamaan (4) diberikan sebagai berikut:

$$\begin{gathered}\frac{u_{i,2,n}-u_{i,0,n}}{2\Delta x}=0 \text{ atau } u_{i,2,n}=u_{i,0,n}\,,\\ \frac{v_{i,2,n}-v_{i,2,n}}{2\Delta x}=0 \text{ atau } v_{i,Ny+1,n}=v_{i,Ny-1,n}\,.\end{gathered} \tag{8}$$

Untuk model diskrit Persamaan (5) diberikan sebagai berikut:

$$\frac{u_{i,Ny+1,n}-u_{i,Ny-1,n}}{2\Delta x}=0 \text{ atau } u_{i,2,n}=u_{i,0,n},$$
$$\frac{v_{i,Ny+1,n}-v_{i,Ny-1,n}}{2\Delta x}=0 \text{ atau } v_{i,Ny+1,n}=v_{i,Ny-1,n}. \qquad (9)$$

Substitusi Persamaan (6) ke Persamaan (4.30) maka diperoleh model diskrit untuk syarat batas kiri sebagai berikut:

$$u_{n+1,1,j}=u_{n,1,j}+\Delta t\left(ru_{n,1,j}-bu_{n,1,j}^2-\frac{m_1u_{n,1,j}v_{n,1,j}}{h_1+u_{n,1,j}^2}\right)+$$
$$2\frac{Du\Delta t}{\Delta x^2}\left(u_{n,2,j}-u_{n,1,j}\right)+\frac{Du\Delta t}{\Delta y^2}\left(u_{n,1,j+1}-2u_{n,1,j}+u_{n,1,j-1}\right),$$
$$v_{n+1,1,j}=v_{n,1,j}+\Delta t\left(sv_{n,1,j}-\frac{m_2v_{n,1,j}^2}{e+u_{n,1,j}}\right)+2\frac{Dv\Delta t}{\Delta x^2}\left(v_{n,2,j}-v_{n,1,j}\right)+$$
$$\frac{Dv\Delta t}{\Delta y^2}\left(v_{n,1,j+1}-2v_{n,1,j}+v_{n,1,j-1}\right). \qquad (10)$$

Substitusi Persamaan (7) ke Persamaan (4.30) maka diperoleh model diskrit untuk syarat batas kanan sebagai berikut:

$$u_{n+1,Nx,j}=u_{n,Nx,j}+\Delta t\left(ru_{n,Nx,j}-bu_{n,Nx,j}^2-\frac{m_1u_{n,Nx,j}v_{n,Nx,j}}{h_1+u_{n,Nx,j}^2}\right)+$$
$$2\frac{Du\Delta t}{\Delta x^2}\left(u_{n,Nx-1,j}-u_{n,Nx,j}\right)+\frac{Du\Delta t}{\Delta y^2}\left(u_{n,Nx,j+1}-2u_{n,Nx,j}+u_{n,Nx,j-1}\right),$$
$$v_{n+1,1,j}=v_{n,1,j}+\Delta t\left(sv_{n,1,j}-\frac{m_2v_{n,1,j}^2}{e+u_{n,1,j}}\right)+2\frac{Dv\Delta t}{\Delta x^2}\left(v_{n,2,j}-v_{n,1,j}\right)+$$
$$\frac{Dv\Delta t}{\Delta y^2}\left(v_{n,1,j+1}-2v_{n,1,j}+v_{n,1,j-1}\right). \qquad (11)$$

Substitusi Persamaan (8) ke Persamaan (4.30) maka diperoleh model diskrit untuk syarat batas bawah sebagai berikut:

$$u_{n+1,i,1} = u_{n,i,1} + \Delta t\left(ru_{n,i,1} - bu_{n,i,1}^2 - \frac{m_1 u_{n,i,1} v_{n,i,1}}{h_1 + u_{n,i,1}^2} \right) + \frac{Du\Delta t}{\Delta x^2}\left(u_{n,i+1,1} - 2u_{n,i,1} + u_{n,i-1,1}\right) + 2\frac{Du\Delta t}{\Delta y^2}\left(u_{n,i,2} - u_{n,i,1}\right),$$

$$v_{n+1,i,1} = v_{n,i,1} + \Delta t\left(sv_{n,i,1} - \frac{m_2 v_{n,i,1}^2}{e + u_{n,i,1}} \right) + \frac{Dv\Delta t}{\Delta x^2}\left(v_{n,i+1,1} - 2v_{n,i,1} + v_{n,i-1,1}\right) + 2\frac{Dv\Delta t}{\Delta y^2}\left(v_{n,i,2} - v_{n,i,1}\right). \tag{12}$$

Substitusi Persamaan (9) ke Persamaan (4.30) maka diperoleh model diskrit untuk syarat batas atas sebagai berikut:

$$u_{n+1,i,Ny} = u_{n,i,Ny} + \Delta t\left(ru_{n,i,Ny} - bu_{n,i,Ny}^2 - \frac{m_1 u_{n,i,Ny} v_{n,i,Ny}}{h_1 + u_{n,i,Ny}^2} \right) + \frac{Du\Delta t}{\Delta x^2}\left(u_{n,i+1,Ny} - 2u_{n,i,Ny} + u_{n,i-1,Ny}\right) + 2\frac{Du\Delta t}{\Delta y^2}\left(u_{n,i,Ny-1} - u_{n,i,Ny}\right),$$

$$v_{n+1,i,Ny} = v_{n,i,Ny} + \Delta t\left(sv_{n,i,Ny} - \frac{m_2 v_{n,i,Ny}^2}{e + u_{n,i,Ny}} \right) + \frac{Dv\Delta t}{\Delta x^2}\left(v_{n,i+1,Ny} - 2v_{n,i,Ny} + v_{n,i-1,Ny}\right) + 2\frac{Dv\Delta t}{\Delta y^2}\left(v_{n,i,Ny-1} - v_{n,i,Ny}\right). \tag{13}$$

Berdasarkan uraian di atas, maka diperoleh model diskrit untuk syarat batas dalam ruang dua dimensi yang diberikan dalam Persamaan (10) – (13).

Lampiran 5. Kode Program Simulasi Menggunakan Matlab

```
clear all
clc

%parameter
r = 0.5;
b = 0.09;
m1 = 0.7;
h1 = 7;
s = 0.35;
m2 = 0.5;
e = 5;
Du = 0.001;
Dv = 1;

%fixedpoint
upolinomial = [b -r (h1*b*m2 + m1*s)/m2 (m1*s*e - r*h1*m2)/m2];
akar_u = roots(upolinomial);
akarreal_u = akar_u(imag(akar_u)==0);
ubar = akarreal_u;
vbar = s*(e+ubar)/m2;

%SpatialDomain
delta_x=1;
delta_y=1;
delta_t=1/4;

Nx = 100/delta_x;
Ny = 100/delta_y;
Nt = 100/delta_t;

u=zeros(Nt,Nx,Ny);
v=zeros(Nt,Nx,Ny);

%Syarat Awal
for i=1:Nx
    for j=1:Ny
          u(1,i,j) = ubar + 0.1*cos(10*i).^2*cos(10*j).^2;
          v(1,i,j) = vbar + 0.1*cos(10*i).^2*cos(10*j).^2;
    end
end

for n = 1:Nt-1
    for i = 2:Nx-1
        for j = 2:Ny-1
             % Fungsi u dan v
             f_u = r.*u(n,i,j) - b.*u(n,i,j).^2 -
(m1.*u(n,i,j).*v(n,i,j)./(h1+u(n,i,j).^2));
             f_v = s.*v(n,i,j) - (m2.*v(n,i,j).^2./(e+u(n,i,j)));

             %Domain x dan y
            uxx = u(n,i+1,j) - 2*u(n,i,j) + u(n,i-1,j)./delta_x^2;
            vxx = v(n,i+1,j) - 2*v(n,i,j) + v(n,i-1,j)./delta_x^2;
```

```
                uyy = u(n,i,j+1) - 2*u(n,i,j) + u(n,i,j-1)./delta_y^2;
                vyy = v(n,i,j+1) - 2*v(n,i,j) + v(n,i,j-1)./delta_y^2;

                spatial_u = uxx + uyy;
                spatial_v = vxx + vyy;

               u(n+1,i,j) = u(n,i,j) + delta_t*(f_u + (Du*spatial_u));
               v(n+1,i,j) = v(n,i,j) + delta_t*(f_v + (Dv*spatial_v));
            end

%Syarat Batas

u(n+1,i,1) = u(n,i,1) + delta_t.*(r.*u(n,i,1)-b.*u(n,i,1).^2 -
(m1.*u(n,i,1).*v(n,i,1)./(h1+u(n,i,1).^2)) + ...
              Du.*((2*u(n,i,2)-2*u(n,i,1))/(delta_y.*delta_y) +
(u(n,i+1,1)+u(n,i-1,1)-2*u(n,i,1))/(delta_x.*delta_x)));
u(n+1,i,Ny) = u(n,i,Ny) + delta_t.*(r.*u(n,i,Ny)-b.*u(n,i,Ny).^2 -
(m1.*u(n,i,Ny).*v(n,i,Ny)./(h1+u(n,i,Ny).^2)) + ...
                Du.*((2*u(n,i,Ny-1)-
2*u(n,i,Ny))/(delta_y.*delta_y) + (u(n,i+1,Ny)+u(n,i-1,Ny)-
2*u(n,i,Ny))/(delta_x.*delta_x)));
v(n+1,i,1) = v(n,i,1) + delta_t.*(s.*v(n,i,1) -
(m2.*v(n,i,1).^2./(e+u(n,i,1))) + ...
                Dv.*((2*v(n,i,2)-2*v(n,i,1))/(delta_y.*delta_y) +
(v(n,i+1,1)+v(n,i-1,1)-2*v(n,i,1))/(delta_x.*delta_x)));
v(n+1,i,Ny) = v(n,i,Ny) + delta_t.*(s.*v(n,i,Ny) -
(m2.*v(n,i,Ny).^2./(e+u(n,i,Ny))) + ...
                Dv.*((2*v(n,i,Ny-1)-
2*v(n,i,Ny))/(delta_y.*delta_y) + (v(n,i+1,Ny)+v(n,i-1,Ny)-
2*v(n,i,Ny))/(delta_x.*delta_x)));

    end

    for j = 2:Ny-1

u(n+1,1,j) = u(n,1,j) + delta_t.*(r.*u(n,1,j)-b.*u(n,1,j).^2 -
(m1.*u(n,1,j).*v(n,1,j)./(h1+u(n,1,j).^2)) + ...
          Du.*((u(n,1,j+1)+u(n,1,j-1)-
2*u(n,1,j))/(delta_y.*delta_y) + (2*u(n,2,j)-
2*u(n,1,j))/(delta_x.*delta_x)));
u(n+1,Nx,j) = u(n,Nx,j) + delta_t.*(r.*u(n,Nx,j)-b.*u(n,Nx,j).^2 -
(m1.*u(n,Nx,j).*v(n,Nx,j)./(h1+u(n,Nx,j).^2)) + ...
          Du.*((u(n,Nx,j+1)+u(n,Nx,j-1)-
2*u(n,Nx,j))/(delta_y.*delta_y) + (2*u(n,Nx-1,j)-
2*u(n,Nx,j))/(delta_x.*delta_x)));
v(n+1,1,j) = v(n,1,j) + delta_t.*(s.*v(n,1,j) -
(m2.*v(n,1,j).^2./(e+u(n,1,j))) + ...
          Dv.*((v(n,1,j+1)+v(n,1,j-1)-
2*v(n,1,j))/(delta_y.*delta_y) + (2*v(n,2,j)-
2*v(n,1,j))/(delta_x.*delta_x)));
v(n+1,Nx,j) = v(n,Nx,j) + delta_t.*(s.*v(n,Nx,j) -
(m2.*v(n,Nx,j).^2./(e+u(n,Nx,j))) + ...
          Dv.*((v(n,Nx,j+1)+v(n,Nx,j-1)-
2*v(n,Nx,j))/(delta_y.*delta_y) + (2*v(n,Nx-1,j)-
2*v(n,Nx,j))/(delta_x.*delta_x)));
```

```
    end

    disp(n);
end

uplot(Nx,Ny) = 0;
vplot(Nx,Ny) = 0;

for i = 1:Nx
    for j = 1:Ny
        uplot(i,j) = u(n,i,j);
        u0plot(i,j) = u(1,i,j);
        vplot(i,j) = v(n,i,j);
        v0plot(i,j) = v(1,i,j);
    end
end

figure(1)
contourf(uplot,'LineColor','none');
colormap('jet')
title('Mangsa')
xlabel('x')
ylabel('y')
colorbar

figure(2)
contourf(vplot,'LineColor','none');
colormap('jet')
title('Pemangsa')
xlabel('x')
ylabel('y')
colorbar

figure(3)
contourf(u0plot,'LineColor','none');
colormap('jet')
title('Mangsa')
xlabel('x')
ylabel('y')
colorbar

figure(4)
contourf(v0plot,'LineColor','none');
colormap('jet')
title('Pemangsa')
xlabel('x')
ylabel('y')
colorbar
```