DAFTAR PUSTAKA

Adiatma, A., Tohari, A., Faisal, A., & Alam, S. (2021). *Altitude Factors Affect Dengue Fever Cases in South Sulawesi: A Study using Poisson Inverse Gaussian Regression Model*. *Diversity: Jurnal Disease Preventive of Research Integrity*, 1(2), 58-63.

Amalia, A. T. R. (2020). *Estimasi Komponen Variansi Pada Rancangan Faktorial Model Campuran Menggunakan Metode Maksimum Likelihood*. Skripsi. Makassar: Universitas Hasanuddin.

Andriani, A., Edison, E., & Gracediani, L. (2013). Implementasi Pelayanan Ibu Hamil (K4) oleh Bidan Berdasarkan SPM di Puskesmas Silungkang. *Jurnal Kesehatan Masyarakat Andalas*, 8(1), 21-26.

Cheon, S., Song, S. H., & Jung, B. C. (2009). *Tests for Independence in a Bivariate Negative Binomial Model*. *Journal of the Korean Statistical Society*, 38(2), 185-190.

Dinas Kesehatan Provinsi Sulawesi Selatan. 2020. Profil Dinas Kesehatan Provinsi Sulawesi Selatan Tahun 2020.

Dinas Kesehatan Provinsi Sulawesi Selatan. 2021. Profil Dinas Kesehatan Provinsi Sulawesi Selatan Tahun 2021.

Famoye, F. (2010). *On The Bivariate Negative Binomial Regression Model*. *Journal of Applied Statistics*, 37(6), 969-981.

Fitriyanti, W., & Kurniawan, U. (2019). Regresi Negatif Binomial Bivariat untuk Mengatasi Overdispersi Regresi Poisson Bivariat. *Jurnal Statistika Universitas Muhammadiyah Semarang*, 7(1), 39-45.

Hariadi, W., & Sulantari, S. (2019). Estimasi Model Regresi Binomial Negatif Bivariat (BNBR) pada Penderita Kusta di Jawa Timur. *Unisda Journal of Mathematics and Computer Science (UJMC)*, *5*(2), 29-38.

Hilbe, J. M. (2011). *Negative Binomial Regression*. *Cambridge University Press*.

Hogg, R. V., & Craig, A. T. (2005). *Introduction to Mathematical Statistics. (Sixth Edition). Englewood Hills, New Jersey*.

Keswari, N. M. R., Sumarjaya, I. W., & Suciptawati, N. L. P. (2014). Perbandingan Regresi Binomial Negatif dan Regresi Generalisasi Poisson dalam Mengatasi Overdispersi. *E-Jurnal Matematika*, 3(3), 107-115.

Kurniawan, U. (2015). *Penaksiran Dan Pengujian Hipotesis Parameter Model Regresi Binomial Negatif Bivariat Studi Kasus: Jumlah Kematian Bayi dan Kematian Ibu di Propinsi Jawa Timur Tahun 2013*. Tesis. Surabaya: Institut Teknologi Sepuluh Nopember.

Kusuma, W., Komalasari, D., & Hadijati, M. (2013). Model Regresi *Zero Inflated Poisson* pada Data *Overdispersion*. *Jurnal Matematika*, 3(2), 71-85.

Lestari, W. dan Sari, D. F. (2019). Peran Tenaga Kesehatan dan Kualitas Kunjungan Nifas pada Ibu yang Memiliki Bayi Usia 2-12 Bulan. *Jurnal Kesehatan Mercusuar*, 2(1), 1-8.

Mardalena, S., Purnomo, J. T. D., & Prastyo, D. D. (2021). *Bivariate Poisson Inverse Gaussian Regression Model with Exposure Variable: Infant and Maternal Death Case Study*. *Journal of Physics: Conference Series*, 1752(1), 1-8.

Murtiyarini, I., Suryanti, Y., & Wuryandari, A. G. (2020). Pemberdayaan Kader dalam Deteksi Dini Komplikasi Masa Nifas di Desa Penyengat Olak Kec. Jambi Luar Kota Kab. Muaro Jambi Tahun 2019. *Jurnal BINAKES*, 1(1), 5-9.

Oktarin, S. A. (2018). *Model Regresi Conditional Bivariate Poisson pada Data Berat Badan Bayi Rendah dan Kematian Bayi (Studi Kasus: Di Kota Surabaya Tahun 2015)*. Tesis. Surabaya: Institut Teknologi Sepuluh Nopember.

Permatasari, R. J. (2018). Analisis Pengaruh Pendidikan dan Pelatihan, Motivasi dan Kompensasi Terhadap Kinerja Karyawan (Pada Citibank Pondok Indah

Jakarta Selatan). *JENIUS (Jurnal Ilmiah Manajemen Sumber Daya Manusia)*, 1(3), 187.

Profita, A. C. (2018). Beberapa Faktor yang Berhubungan dengan Keaktifan Kader Posyandu di Desa Pengadegan Kabupaten Banyumas. *Jurnal Administrasi Kesehatan Indonesia*, 6(2), 68-74.

Rahayu, A. (2020). Model-Model Regresi untuk Mengatasi Masalah Overdipersi pada Regresi Poisson. *Journal Peqguruang: Conference Series*, 2(1), 1-5.

Rahmadeni, R., & Jannah, F. F. (2019). Pemodelan *Generalized Poisson Regression* (GPR) pada Kasus Kematian Neonatal di Provinsi Riau. *Jurnal Sains Matematika dan Statistika*, 5(2), 43-49.

Sauddin, A., Auliah, N. I., & Alwi, W. (2020). Pemodelan Jumlah Kematian Ibu di Provinsi Sulawesi Selatan Menggunakan Regresi Binomial Negatif. *Jurnal MSA (Matematika dan Statistika serta Aplikasinya)*, 8(2), 42-47.

Sekarmini, N. M., Sukarsa I. K. G., & Srinadi, I. G. A. M. (2013). Penerapan *Regresi Zero Inflated Negative Binomial* (ZINB) untuk Pendugaan Kematian Anak Balita. *E-Jurnal Matematika*, 2(4), 11.

LAMPIRAN

Lampiran 1. Data Penelitian

Kabupaten/Kota	Y_1	Y_2	X_1	X_2	X_3	X_4	X_5	X_6
Kepulauan Selayar	7	8	73.7	68.63	53.63	69.8	75.88	14
Bulukumba	4	51	80.5	73.31	21.06	85.66	85.34	20
Bantaeng	3	20	30.6	98.13	82.68	102.23	0	14
Jeneponto	7	69	93	74.58	71.38	94.04	92.97	19
Takalar	6	28	97.8	94.48	35.28	100.19	99.02	15
Gowa	15	41	65.7	93.29	41.17	99.36	89.89	26
Sinjai	4	54	72.5	87.78	96.94	96.39	97.2	16
Maros	4	22	79.4	91.59	44.58	94.65	94.69	14
Pangkajene dan Kepulauan	6	58	62.1	89.09	61.43	89.43	90.64	23
Barru	3	10	120.2	91.98	82.4	96.24	95.87	12
Bone	7	57	72.3	94.12	83.4	90.77	95.32	38
Soppeng	3	31	67.6	77.03	77.44	87.21	87.27	17
Wajo	4	30	75.5	91.89	53.81	96.91	95.3	23
Sidenreng Rappang	6	22	131.4	84.56	94.19	103.31	98.94	14

(lanjutan)

Kabupaten/Kota	Y_1	Y_2	X_1	X_2	X_3	X_4	X_5	X_6
Pinrang	5	27	90.6	98.43	86.29	97.23	96.33	17
Enrekang	5	42	88.1	53.67	68.95	65.62	66.17	14
Luwu	10	44	81.5	78.17	43.02	85.36	86.37	22
Tana Toraja	3	16	83.2	84.91	54.43	91.42	0	21
Luwu Utara	5	37	60.4	82.18	65.38	92.12	90.64	14
Luwu Timur	6	13	81.2	88.34	96.76	92.76	0	17
Toraja Utara	5	15	66.1	71.61	53.29	74.37	0	26
Kota Makassar	12	43	97.9	92.38	100	92.14	88.38	46
Kota Parepare	2	5	63.2	68.16	100	76.21	76.63	7
Kota Palopo	1	11	67	86.4	52.35	93.32	94.05	12

Lampiran 2. *Output* Nilai Korelasi Pearson dengan *Software* SPSS

Correlations

		Y1	Y2
Y1	Pearson Correlation	1	.414*
	Sig. (2-tailed)		.044
	N	24	24
Y2	Pearson Correlation	.414*	1
	Sig. (2-tailed)	.044	
	N	24	24

*. Correlation is significant at the 0.05 level (2-tailed).

Lampiran 3. *Output* Uji Overdispersi dengan *Software* SPSS

Goodness of Fit[a]

	Value	df	Value/df
Deviance	20.112	17	1.183
Scaled Deviance	20.112	17	
Pearson Chi-Square	20.269	17	1.192
Scaled Pearson Chi-Square	20.269	17	
Log Likelihood[b]	-51.391		
Akaike's Information Criterion (AIC)	116.782		
Finite Sample Corrected AIC (AICC)	123.782		
Bayesian Information Criterion (BIC)	125.028		
Consistent AIC (CAIC)	132.028		

Dependent Variable: Y1

Model: (Intercept), X1, X2, X3, X4, X5, X6

a. Information criteria are in smaller-is-better form.

b. The full log likelihood function is displayed and used in computing information criteria.

(lanjutan)

Goodness of Fit[a]

	Value	df	Value/df
Deviance	97.653	17	5.744
Scaled Deviance	97.653	17	
Pearson Chi-Square	92.980	17	5.469
Scaled Pearson Chi-Square	92.980	17	
Log Likelihood[b]	-109.953		
Akaike's Information Criterion (AIC)	233.905		
Finite Sample Corrected AIC (AICC)	240.905		
Bayesian Information Criterion (BIC)	242.152		
Consistent AIC (CAIC)	249.152		

Dependent Variable: Y2

Model: (Intercept), X1, X2, X3, X4, X5, X6

a. Information criteria are in smaller-is-better form.

b. The full log likelihood function is displayed and used in computing information criteria.

Lampiran 4. *Output* Uji Multikolinieritas dengan *Software* SPSS

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	-.360	5.577		-.065	.949		
	X1	.009	.032	.057	.274	.787	.753	1.328
	X2	-.063	.119	-.225	-.532	.602	.182	5.503
	X3	-.017	.026	-.126	-.675	.509	.942	1.061
	X4	.068	.132	.213	.514	.614	.189	5.289
	X5	.011	.018	.119	.600	.556	.833	1.200
	X6	.249	.075	.672	3.308	.004	.791	1.265

a. Dependent Variable: Y1

Lampiran 5. Fungsi ln *likelihood*

Diketahui:

$$L = \prod_{i=1}^{n} P(Y_{i1}, Y_{i2})$$

$$= \prod_{i=1}^{n} \left[\prod_{t=1}^{2} \binom{m_t^{-1} + y_{it} - 1}{y_{it}} \left(\frac{\mu_{it}}{m_t^{-1} + \mu_{it}} \right)^{y_{it}} \left(\frac{m_t^{-1}}{m_t^{-1} + \mu_{it}} \right)^{m_t^{-1}} [1 \right.$$

$$\left. + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)] \right]$$

Fungsi ln *likelihood*-nya adalah sebagai berikut:

Misalkan:

$$A = \binom{m_t^{-1} + y_{it} - 1}{y_{it}}$$

$$= \frac{(m_t^{-1} + y_{it} - 1)!}{y_{it}!(m_t^{-1} - 1)!}$$

$$\ln A = \ln \left(\frac{(m_t^{-1} + y_{it} - 1)!}{y_{it}!(m_t^{-1} - 1)!} \right)$$

$$= \ln\big((m_t^{-1} + y_{it} - 1)!\big) - \ln(y_{it}!\,(m_t^{-1} - 1)!)$$

$$= \ln\big((m_t^{-1} + y_{it} - 1)!\big) - \ln(y_{it}!) - \ln\big((m_t^{-1} - 1)!\big)$$

$$= \ln\big((m_t^{-1} + y_{it} - 1)!\big) - \ln\big((m_t^{-1} - 1)!\big) - \ln(y_{it}!)$$

Misalkan:

$$a = \ln\big((m_t^{-1} + y_{it} - 1)!\big) - \ln\big((m_t^{-1} - 1)!\big)$$

(lanjutan)

$= \ln\big((m_t^{-1} + y_{it} - 1)(m_t^{-1} + y_{it} - 1 - 1)(m_t^{-1} + y_{it} - 1 - 2)(m_t^{-1} + y_{it} - 1 - 3) \ldots (m_t^{-1})(m_t^{-1} - 1)(m_t^{-1} - 2)(m_t^{-1} - 3)(m_t^{-1} - 4) \ldots 3.2.1\big) - \ln\big((m_t^{-1} - 1)(m_t^{-1} - 1 - 1)(m_t^{-1} - 1 - 2)(m_t^{-1} - 1 - 3) \ldots 3.2.1.\big)$

$= \ln\big((m_t^{-1} + y_{it} - 1)(m_t^{-1} + y_{it} - 2)(m_t^{-1} + y_{it} - 3)(m_t^{-1} + y_{it} - 4) \ldots (m_t^{-1})(m_t^{-1} - 1)(m_t^{-1} - 2)(m_t^{-1} - 3)(m_t^{-1} - 4) \ldots 3.2.1\big) - \ln\big((m_t^{-1} - 1)(m_t^{-1} - 2)(m_t^{-1} - 3)(m_t^{-1} - 4) \ldots 3.2.1\big)$

$= \ln(m_t^{-1} + y_{it} - 1) + \ln(m_t^{-1} + y_{it} - 2) + \ln(m_t^{-1} + y_{it} - 3) + \ln(m_t^{-1} + y_{it} - 4) + \cdots + \ln(m_t^{-1}) + \ln(m_t^{-1} - 1) + \ln(m_t^{-1} - 2) + \ln(m_t^{-1} - 3) + \ln(m_t^{-1} - 4) + \cdots + \ln 3 + \ln 2 + \ln 1 - \ln(m_t^{-1} - 1) - \ln(m_t^{-1} - 2) - \ln(m_t^{-1} - 3) - \ln(m_t^{-1} - 4) - \cdots - \ln 3 - \ln 2 - \ln 1$

$= \ln(m_t^{-1} + y_{it} - 1) + \ln(m_t^{-1} + y_{it} - 2) + \ln(m_t^{-1} + y_{it} - 3) + \ln(m_t^{-1} + y_{it} - 4) + \cdots + \ln(m_t^{-1})$

$$= \sum_{j=0}^{y_{it}-1} \ln(m_t^{-1} + j)$$

$b = \ln(y_{it}!)$

$\ln A = a - b$

$$= \sum_{j=0}^{y_{it}-1} \ln(m_t^{-1} + j) - \ln(y_{it}!)$$

$$B = \left(\frac{\mu_{it}}{m_t^{-1} + \mu_{it}}\right)^{y_{it}} \left(\frac{m_t^{-1}}{m_t^{-1} + \mu_{it}}\right)^{m_t^{-1}}$$

$$= \frac{\mu_{it}^{y_{it}} m_t^{-1(m_t^{-1})}}{(m_t^{-1} + \mu_{it})^{y_{it} + m_t^{-1}}}$$

(lanjutan)

$$= \frac{\mu_{it}^{y_{it}} m_t^{-m_t^{-1}}}{(m_t^{-1} + \mu_{it})^{y_{it} + m_t^{-1}}}$$

$$\ln B = \ln\left(\frac{\mu_{it}^{y_{it}} m_t^{-m_t^{-1}}}{(m_t^{-1} + \mu_{it})^{y_{it} + m_t^{-1}}}\right)$$

$$= \ln\left(\mu_{it}^{y_{it}} m_t^{-m_t^{-1}}\right) - \ln\left((m_t^{-1} + \mu_{it})^{y_{it} + m_t^{-1}}\right)$$

$$= \ln\left(\mu_{it}^{y_{it}}\right) + \ln\left(m_t^{-m_t^{-1}}\right) - (y_{it} + m_t^{-1}) \ln(m_t^{-1} + \mu_{it})$$

$$= y_{it} \ln \mu_{it} - m_t^{-1} \ln m_t - (y_{it} + m_t^{-1}) \ln(m_t^{-1} + \mu_{it})$$

$$C = [1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]$$

$$\ln C = \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]$$

$$\ln L = \sum_{i=1}^{n} \left\{ \sum_{t=1}^{2} [\ln A + \ln B] + \ln C \right\}$$

$$= \sum_{i=1}^{n} \left\{ \sum_{t=1}^{2} \left[\sum_{j=0}^{y_{it}-1} \ln(m_t^{-1} + j) - \ln(y_{it}!) + y_{it} \ln \mu_{it} - m_t^{-1} \ln m_t \right.\right.$$

$$\left.\left. - (y_{it} + m_t^{-1}) \ln(m_t^{-1} + \mu_{it}) \right] + \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)] \right\}$$

$$= \sum_{i=1}^{n} \left\{ \sum_{t=1}^{2} \left[y_{it} \ln \mu_{it} - m_t^{-1} \ln m_t \right.\right.$$

$$\left. - (y_{it} + m_t^{-1}) \ln(\mu_{it} + m_t^{-1}) - \ln(y_{it}!) + \sum_{j=0}^{y_{it}-1} \ln(m_t^{-1} + j) \right]$$

$$\left. + \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)] \right\}$$

Lampiran 6. Turunan c_t terhadap Parameter-parameter

Turunan parsial pertama $\mathrm{c_t}$ terhadap $\mathrm{m_t}$

$$\frac{\partial \mathrm{c_t}}{\partial \mathrm{m_t}} = \frac{\partial}{\partial \mathrm{m_t}}\left((1 + d\mu_{it}m_t)^{-m_t^{-1}}\right)$$

$$= \frac{\partial}{\partial \mathrm{m_t}}\left(\left(e^{\ln(1+d\mu_{it}m_t)}\right)^{-m_t^{-1}}\right)$$

$$= \frac{\partial}{\partial \mathrm{m_t}}\left(e^{\ln(1+d\mu_{it}m_t)(-m_t^{-1})}\right)$$

$$= e^{\ln(1+d\mu_{it}m_t)(-m_t^{-1})}\left[\frac{d\mu_{it}}{1 + d\mu_{it}m_t}(-m_t^{-1}) + \ln(1 + d\mu_{it}m_t)m_t^{-2}\right]$$

$$= e^{(-m_t^{-1})\ln(1+d\mu_{it}m_t)}\left[\frac{d\mu_{it}}{1 + d\mu_{it}m_t}(-m_t^{-1}) + \ln(1 + d\mu_{it}m_t)m_t^{-2}\right]$$

$$= e^{\ln(1+d\mu_{it}m_t)^{-m_t^{-1}}}\left[\ln(1 + d\mu_{it}m_t)m_t^{-2} - \frac{d\mu_{it}m_t^{-1}}{1 + d\mu_{it}m_t}\right]$$

$$= (1 + d\mu_{it}m_t)^{-m_t^{-1}}m_t^{-1}\left[\ln(1 + d\mu_{it}m_t)m_t^{-1} - \frac{d\mu_{it}}{1 + d\mu_{it}m_t}\right]$$

$$= m_t^{-1}\left[m_t^{-1}\ln(1 + d\mu_{it}m_t) - \frac{d\mu_{it}}{1 + d\mu_{it}m_t}\right]c_t$$

Turunan parsial pertama $\mathrm{c_t}$ terhadap $\boldsymbol{\beta_{tj}}$

$$\frac{\partial c_t}{\partial \beta_{tj}} = \frac{\partial}{\partial \beta_{tj}}\left((1 + d\mu_{it}m_t)^{-\frac{1}{m_t}}\right)$$

$$= -m_t^{-1}(1 + d\mu_{it}m_t)^{-m_t^{-1}-1}dm_t\frac{\partial \mu_{it}}{\partial \beta_{tj}}$$

$$= \frac{-d(1+d\mu_{it}m_t)^{-m_t^{-1}}}{(1+d\mu_{it}m_t)}\mu_{it}x_j$$

$$= -\frac{dc_t\mu_{it}x_j}{(1+d\mu_{it}m_t)}$$

Turunan parsial kedua $\mathrm{c_t}$ terhadap $\mathrm{m_t}$ dan $\mathrm{m_t}$

$$\frac{\partial^2 c_t}{\partial m_t^2} = \frac{\partial}{\partial \mathrm{m_t}}\left(m_t^{-1}\left[m_t^{-1}\ln(1 + d\mu_{it}m_t) - \frac{d\mu_{it}}{1+d\mu_{it}m_t}\right]c_t\right)$$

(lanjutan)

Misalkan:

$$u = m_t^{-2}.\ln(1 + d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1 + d\mu_{it}m_t}$$

$$u' = -2mt^{-3}\ln(1 + d\mu_{it}m_t) + m_t^{-2}\frac{d\mu_{it}}{1+d\mu_{it}m_t} - \left(\frac{-d\mu_{it}m_t^{-2}(1+d\mu_{it}m_t) - d\mu_{it}m_t^{-1}d\mu_{it}}{(1+d\mu_{it}m_t)^2}\right)$$

$$= -2mt^{-3}\ln(1 + d\mu_{it}m_t) + m_t^{-2}\frac{d\mu_{it}}{1+d\mu_{it}m_t} - \left(\frac{-d\mu_{it}m_t^{-2} - d^2\mu_{it}^2m_t^{-1} - d^2\mu_{it}^2m_t^{-1}}{(1+d\mu_{it}m_t)^2}\right)$$

$$= -2mt^{-3}\ln(1 + d\mu_{it}m_t) + \frac{d\mu_{it}m_t^{-2}}{1 + d\mu_{it}m_t} - \frac{(-d\mu_{it}m_t^{-2} - 2d^2\mu_{it}^2m_t^{-1})}{(1 + d\mu_{it}m_t)^2}$$

$$= -2mt^{-3}\ln(1 + d\mu_{it}m_t) + \frac{d\mu_{it}m_t^{-2}}{1 + d\mu_{it}m_t} + \frac{d\mu_{it}m_t^{-2} + 2d^2\mu_{it}^2m_t^{-1}}{(1 + d\mu_{it}m_t)^2}$$

$$= -2mt^{-3}\ln(1 + d\mu_{it}m_t) + \frac{d\mu_{it}m_t^{-2}(1+d\mu_{it}m_t)}{(1+d\mu_{it}m_t)^2} + \frac{d\mu_{it}m_t^{-2} + 2d^2\mu_{it}^2m_t^{-1}}{(1+d\mu_{it}m_t)^2}$$

$$= -2mt^{-3}\ln(1 + d\mu_{it}m_t) + \frac{d\mu_{it}m_t^{-2} + d^2\mu_{it}^2m_t^{-1}}{(1+d\mu_{it}m_t)^2} + \frac{d\mu_{it}m_t^{-2} + 2d^2\mu_{it}^2m_t^{-1}}{(1+d\mu_{it}m_t)^2}$$

$$= -2mt^{-3}\ln(1 + d\mu_{it}m_t) + \frac{2d\mu_{it}m_t^{-2} + 3d^2\mu_{it}^2m_t^{-1}}{(1 + d\mu_{it}m_t)^2}$$

$$v = c_t$$

$$v' = m_t^{-1}\left[m_t^{-1}\ln(1 + d\mu_{it}m_t) - \frac{d\mu_{it}}{1 + d\mu_{it}m_t}\right]c_t$$

$$\frac{\partial^2 c_t}{\partial m_t^2} = \left(-2mt^{-3}\ln(1 + d\mu_{it}m_t) + \frac{2d\mu_{it}m_t^{-2} + 3d^2\mu_{it}^2m_t^{-1}}{(1+d\mu_{it}m_t)^2}\right)c_t + \left(m_t^{-2}\ln(1 + d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right)(m_t^{-1}\left[m_t^{-1}\ln(1 + d\mu_{it}m_t) - \frac{d\mu_{it}}{1+d\mu_{it}m_t}\right]c_t)$$

$$= \left\{\left(-2mt^{-3}\ln(1 + d\mu_{it}m_t) + \frac{2d\mu_{it}m_t^{-2} + 3d^2\mu_{it}^2m_t^{-1}}{(1+d\mu_{it}m_t)^2}\right) + \left(m_t^{-2}\ln(1 + d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right)^2\right\}c_t$$

(lanjutan)

$$= \left\{\left(m_t^{-2}\ln(1+d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right)^2 - 2mt^{-3}\ln(1+d\mu_{it}m_t) + \frac{2d\mu_{it}m_t^{-2}+3d^2\mu_{it}^2m_t^{-1}}{(1+d\mu_{it}m_t)^2}\right\}c_t$$

Turunan parsial kedua $\mathbf{c_t}$ terhadap $\boldsymbol{\beta_{tj}}$ dan $\boldsymbol{\beta_{ts}}$

$$\frac{\partial c_t}{\partial\beta_{tj}\partial\beta_{ts}} = \frac{\partial}{\partial\beta_{tj}}\left(-\frac{dc_t\mu_{it}x_s}{(1+d\mu_{it}m_t)}\right)$$

Misalkan:

$$u = c_t.d\mu_{it}x_s$$

$$u' = \frac{\partial c_t}{\partial\beta_{ts}}d\mu_{it}x_s + c_t d\mu_{it}x_s x_j$$

$$v = 1 + d\mu_{it}m_t$$

$$v' = d\mu_{it}x_j m_t$$

$$\frac{\partial c_t}{\partial\beta_{tj}\partial\beta_{ts}} = -\frac{\left(\frac{\partial c_t}{\partial\beta_{ts}}d\mu_{it}x_s + c_t d\mu_{it}x_s x_j\right)(1+d\mu_{it}m_t) - c_t.d\mu_{it}x_s d\mu_{it}x_j m_t}{(1+d\mu_{it}m_t)^2}$$

$$= -\frac{\left(\frac{\partial c_t}{\partial\beta_{ts}}d\mu_{it}x_s\right)(1+d\mu_{it}m_t) + c_t d\mu_{it}x_s x_j(1+d\mu_{it}m_t) - c_t.d\mu_{it}x_s d\mu_{it}x_j m_t}{(1+d\mu_{it}m_t)^2}$$

$$= -\frac{\left(\frac{\partial c_t}{\partial\beta_{ts}}d\mu_{it}x_s\right)(1+d\mu_{it}m_t) + c_t d\mu_{it}x_s x_j + c_t d\mu_{it}x_s x_j d\mu_{it}m_t - c_t.d\mu_{it}x_s d\mu_{it}x_j m_t}{(1+d\mu_{it}m_t)^2}$$

$$= -\frac{\left(\frac{\partial c_t}{\partial\beta_{ts}}d\mu_{it}x_s\right)(1+d\mu_{it}m_t) + c_t d\mu_{it}x_s x_j}{(1+d\mu_{it}m_t)^2}$$

Turunan parsial kedua $\mathbf{c_t}$ terhadap $\boldsymbol{m_t}$ dan $\boldsymbol{\beta_{tj}}$

$$\frac{\partial^2 c_t}{\partial m_t\partial\beta_{tj}} = \frac{\partial}{\partial m_t}\left(-\frac{dc_t\mu_{it}x_j}{(1+d\mu_{it}m_t)}\right)$$

Misalkan:

$$u = c_t.d\mu_{it}x_j$$

(lanjutan)

$$u' = \left[m_t^{-2}\ln(1+d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right]c_t d\mu_{it}x_j$$

$$v = 1 + d\mu_{it}m_t$$

$$v' = d\mu_{it}$$

$$\frac{\partial^2 c_t}{\partial m_t \partial\beta_{tj}} = -\frac{\left[m_t^{-2}\ln(1+d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right]c_t d\mu_{it}x_j(1+d\mu_{it}m_t) - c_t.d\mu_{it}x_j d\mu_{it}}{(1+d\mu_{it}m_t)^2}$$

$$= -\left[m_t^{-2}\ln(1+d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right]\frac{c_t d\mu_{it}x_j(1+d\mu_{it}m_t)}{(1+d\mu_{it}m_t)^2} + \frac{c_t d\mu_{it}x_j d\mu_{it}}{(1+d\mu_{it}m_t)^2}$$

$$= -\left[m_t^{-2}\ln(1+d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right]\frac{c_t d\mu_{it}x_j}{1+d\mu_{it}m_t} + \frac{d^2\mu_{it}^2}{(1+d\mu_{it}m_t)^2}c_t x_j$$

$$= \left[m_t^{-2}\ln(1+d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right]\left(-\frac{c_t d\mu_{it}x_j}{1+d\mu_{it}m_t}\right) + \frac{d^2\mu_{it}^2}{(1+d\mu_{it}m_t)^2}c_t x_j$$

$$= \left[m_t^{-2}\ln(1+d\mu_{it}m_t) - \frac{d\mu_{it}m_t^{-1}}{1+d\mu_{it}m_t}\right]\frac{\partial c_t}{\partial\beta_{tj}} + \left(\frac{d\mu_{it}}{1+d\mu_{it}m_t}\right)^2 c_t x_j$$

Lampiran 7. Turunan Parsial Pertama Fungsi ln *likelihood* terhadap Masing-masing Parameter

Turunan parsial pertama fungsi ln *likelihood* terhadap λ

$$\frac{\partial \ln L}{\partial \lambda} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\sum_{t=1}^{2}\left[y_{it}\ln\mu_{it} - m_t^{-1}\ln m_t - \left(y_{it}+m_t^{-1}\right)\ln\left(\mu_{it}+m_t^{-1}\right) - \ln(y_{it}!) + \sum_{j=0}^{y_{it}-1}\ln\left(m_t^{-1}+j\right)\right] + \ln[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]\right\}\right)}{\partial \lambda}$$

Misalkan:

$$A = \ln[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]$$

$$A' = \frac{(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{n}\{A'\}$$

$$= \sum_{i=1}^{n}\frac{(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

Turunan parsial pertama fungsi ln *likelihood* terhadap $\mathbf{m_1}$

$$\frac{\partial \ln L}{\partial \mathrm{m}_1} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\sum_{t=1}^{2}\left[y_{it}\ln\mu_{it} - m_t^{-1}\ln m_t - \left(y_{it}+m_t^{-1}\right)\ln\left(\mu_{it}+m_t^{-1}\right) - \ln\left(y_{it}!\right) + \sum_{j=0}^{y_{it}-1}\ln\left(m_t^{-1}+j\right)\right] + \ln[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]\right\}\right)}{\partial \mathrm{m}_1}$$

Misalkan:

$$A = -m_1^{-1}.\ln(m_1)$$

$$A' = m_1^{-2}\ln(m_1) - m_1^{-1}\frac{1}{m_1}$$

$$= m_1^{-2}\ln(m_1) - m_1^{-2}$$

$$B = -(y_{i1}+m_1^{-1}).\ln(\mu_{i1}+m_1^{-1})$$

$$B' = m_1^{-2}\ln(\mu_{i1}+m_1^{-1}) - (y_{i1}+m_1^{-1})\left(\frac{-m_1^{-2}}{\mu_{i1}+m_1^{-1}}\right)$$

$$= m_1^{-2}\ln(\mu_{i1}+m_1^{-1}) + \frac{m_1^{-2}(y_{i1}+m_1^{-1})}{\mu_{i1}+m_1^{-1}}$$

(lanjutan)

$$C = \ln(m_1^{-1} + j)$$

$$C' = \frac{-m_1^{-2}}{m_1^{-1} + j}$$

$$D = \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]$$

$$= \ln[1 + \lambda(e^{-y_{i1}}e^{-y_{i2}} - e^{-y_{i1}}c_2 - e^{-y_{i2}}c_1 + c_1c_2)]$$

$$D' = \frac{\lambda(-e^{-y_{i2}} + c_2)\frac{\partial c_1}{\partial m_1}}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

$$= \frac{-\lambda(e^{-y_{i2}} - c_2)\frac{\partial c_1}{\partial m_1}}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

$$\frac{\partial \ln L}{\partial \mathrm{m}_1} = \sum_{i=1}^{n} \left\{ A' + B' + \sum_{j=0}^{y_{i2}-1} C' + D' \right\}$$

$$= \sum_{i=1}^{n} \left\{ m_1^{-2}\, ln(m_1) - m_1^{-2} + m_1^{-2}\, ln(\mu_{i1} + m_1^{-1}) + \frac{m_1^{-2}(y_{i1} + m_1^{-1})}{\mu_{i1} + m_1^{-1}} + \sum_{j=0}^{y_{i2}-1} \frac{-m_1^{-2}}{m_1^{-1} + j} + \frac{-\lambda(e^{-y_{i2}} - c_2)\frac{\partial c_1}{\partial m_1}}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \right\}$$

$$= \sum_{i=1}^{n} \left\{ \mathrm{m}_1^{-2} \ln(\mathrm{m}_1) + \mathrm{m}_1^{-2}[\ln(\mu_{i1} + m_1^{-1}) - 1] + \frac{m_1^{-2}(y_{i1} + m_1^{-1})}{\mu_{i1} + m_1^{-1}} - \sum_{j=0}^{y_{i2}-1} \frac{\mathrm{m}_1^{-2}}{\mathrm{m}_1^{-1} + j} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1} \right\}$$

Turunan parsial pertama fungsi ln *likelihood* terhadap $\mathbf{m_2}$

$$\frac{\partial \ln L}{\partial \mathrm{m}_2} = \frac{\partial\left(\sum_{i=1}^{n} \left\{ \sum_{t=1}^{2} \left[y_{it} \ln \mu_{it} - m_t^{-1} \ln m_t - (y_{it} + m_t^{-1}) \ln(\mu_{it} + m_t^{-1}) - \ln(y_{it}!) + \sum_{j=0}^{y_{it}-1} \ln(m_t^{-1} + j) \right] + \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)] \right\} \right)}{\partial \mathrm{m}_2}$$

Misalkan:

$$A = -m_2^{-1}.\ln(m_2)$$

(lanjutan)

$A' = m_2^{-2}\ln(m_2) - m_2^{-1}\frac{1}{m_2}$

$\quad = m_2^{-2}\ln(m_2) - m_2^{-2}$

$B = -(y_{i2} + m_2^{-1}).\ln(\mu_{i2} + m_2^{-1})$

$B' = m_2^{-2}\ln(\mu_{i2} + m_2^{-1}) - (y_{i2} + m_2^{-1})\left(\frac{-m_2^{-2}}{\mu_{i2}+m_2^{-1}}\right)$

$\quad = m_2^{-2}\ln(\mu_{i2} + m_2^{-1}) + \frac{m_2^{-2}(y_{i2}+m_2^{-1})}{\mu_{i1}+m_2^{-1}}$

$C = \ln(m_2^{-1} + j)$

$C' = \frac{-m_2^{-2}}{m_2^{-1}+j}$

$D = \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]$

$\quad = \ln[1 + \lambda(e^{-y_{i1}}e^{-y_{i2}} - e^{-y_{i1}}c_2 - e^{-y_{i2}}c_1 + c_1c_2)]$

$D' = \frac{\lambda(-e^{-y_{i1}}+c_1)\frac{\partial c_2}{\partial m_2}}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$

$\quad = \frac{-\lambda(e^{-y_{i1}}-c_1)\frac{\partial c_2}{\partial m_2}}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$

$$\frac{\partial \ln L}{\partial \mathrm{m}_1} = \sum_{i=1}^{n}\left\{A' + B' + \sum_{j=0}^{y_{it}-1} C' + D'\right\}$$

$$= \sum_{i=1}^{n}\left\{m_2^{-2}\ln(m_2) - m_2^{-2} + m_2^{-2}\ln(\mu_{i2} + m_2^{-1}) + \frac{m_2^{-2}(y_{i2} + m_2^{-1})}{\mu_{i2} + m_2^{-1}}\right.$$

$$\left. + \sum_{j=0}^{y_{i2}-1}\frac{-m_2^{-2}}{m_2^{-1} + j} + \frac{-\lambda(e^{-y_{i1}} - c_1)\frac{\partial c_2}{\partial m_2}}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right\}$$

(lanjutan)

$$= \sum_{i=1}^{n} \left\{ \mathrm{m}_2^{-2} \ln(m_2) + \mathrm{m}_2^{-2}[\ln(\mu_{i2} + m_2^{-1}) - 1] + \frac{m_2^{-2}(y_{i2} + m_2^{-1})}{\mu_{i2} + m_2^{-1}} \right.$$

$$\left. - \sum_{j=0}^{y_{i2}-1} \frac{\mathrm{m}_2^{-2}}{\mathrm{m}_2^{-1} + j} - \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_2}{\partial m_2} \right\}$$

Turunan parsial pertama fungsi ln *likelihood* terhadap $\boldsymbol{\beta}_{1j}$

$$\frac{\partial \ln L}{\partial \beta_{1j}} = \frac{\partial \left(\sum_{i=1}^{n} \left\{ \sum_{t=1}^{2} \left[y_{it} \ln \mu_{it} - m_t^{-1} \ln m_t - (y_{it} + m_t^{-1}) \ln(\mu_{it} + m_t^{-1}) - \ln(y_{it}!) + \sum_{j=0}^{y_{it}-1} \ln(m_t^{-1} + j) \right] + \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)] \right\} \right)}{\partial \beta_{1j}}$$

Misalkan:

$$A = y_{i1} \ln \mu_{i1}$$

$$A' = \frac{y_{i1}}{\mu_{i1}} \frac{\partial \mu_{i1}}{\partial \beta_{1j}}$$

$$B = -(y_{i1} + m_1^{-1}).\ln(\mu_{i1} + m_1^{-1})$$

$$B' = -\frac{y_{i1} + m_1^{-1}}{\mu_{i1} + m_1^{-1}} \frac{\partial \mu_{i1}}{\partial \beta_{1j}}$$

$$C = \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]$$

$$= 1 + \lambda(e^{-y_{i1}} e^{-y_{i2}} - e^{-y_{i1}} c_2 - e^{-y_{i2}} c_1 + c_1 c_2)$$

$$C' = \frac{\lambda(-e^{-y_{i2}} + c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \beta_{1j}}$$

$$= \frac{-\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \beta_{1j}}$$

$$\frac{\partial \ln L}{\partial \beta_{1j}} = \sum_{i=1}^{n} \left\{ \frac{y_{i1}}{\mu_{i1}} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{y_{i1} + m_1^{-1}}{\mu_{i1} + m_1^{-1}} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} + \frac{-\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \beta_{1j}} \right\}$$

(lanjutan)

$$= \sum_{i=1}^{n} \left\{ \frac{y_{i1}}{\mu_{i1}} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{(y_{i1} + m_1^{-1}) m_1}{(\mu_{i1} + m_1^{-1}) m_1} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_1}{\partial \beta_{1j}} \right\}$$

$$= \sum_{i=1}^{n} \left\{ \frac{y_{i1}}{\mu_{i1}} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{m_1 y_{i1} + 1}{m_1 \mu_{i1} + 1} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_1}{\partial \beta_{1j}} \right\}$$

$$= \sum_{i=1}^{n} \left\{ \frac{y_{i1}(1 + m_1 \mu_{i1})}{\mu_{i1}(1 + m_1 \mu_{i1})} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{\mu_{i1}(m_1 y_{i1} + 1)}{\mu_{i1}(1 + m_1 \mu_{i1})} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_1}{\partial \beta_{1j}} \right\}$$

$$= \sum_{i=1}^{n} \left\{ \frac{y_{i1} + y_{i1} m_1 \mu_{i1}}{\mu_{i1}(1 + m_1 \mu_{i1})} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{\mu_{i1} m_1 y_{i1} + \mu_{i1}}{\mu_{i1}(1 + m_1 \mu_{i1})} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_1}{\partial \beta_{1j}} \right\}$$

$$= \sum_{i=1}^{n} \left\{ \frac{y_{i1} - \mu_{i1}}{\mu_{i1}(1 + m_1 \mu_{i1})} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_1}{\partial \beta_{1j}} \right\}$$

Turunan parsial pertama fungsi ln *likelihood* terhadap β_{2j}

$$\frac{\partial \ln L}{\partial \beta_{2j}} = \frac{\partial \left(\sum_{i=1}^{n} \left\{ \sum_{t=1}^{2} \left[y_{it} \ln \mu_{it} - m_t^{-1} \ln m_t - (y_{it} + m_t^{-1}) \ln(\mu_{it} + m_t^{-1}) - \ln(y_{it}!) + \sum_{j=0}^{y_{it}-1} \ln(m_t^{-1} + j) \right] + \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)] \right\} \right)}{\partial \beta_{2j}}$$

Misalkan:

$A = y_{i2} \ln \mu_{i2}$

$A' = \frac{y_{i2}}{\mu_{i2}} \frac{\partial \mu_{i2}}{\partial \beta_{2j}}$

$B = -(y_{i2} + m_2^{-1}) \ln(\mu_{i2} + m_2^{-1})$

$B' = -\frac{y_{i2} + m_2^{-1}}{\mu_{i2} + m_2^{-1}} \frac{\partial \mu_{i2}}{\partial \beta_{2j}}$

(lanjutan)

$$C = \ln[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]$$

$$= 1 + \lambda(e^{-y_{i1}}e^{-y_{i2}} - e^{-y_{i1}}c_2 - e^{-y_{i2}}c_1 + c_1c_2)$$

$$C' = \frac{\lambda(-e^{-y_{i1}}+c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{2j}}$$

$$= \frac{-\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{2j}}$$

$$\frac{\partial \ln L}{\partial \beta_{1j}} = \sum_{i=1}^{n}\left\{\frac{y_{i2}}{\mu_{i2}}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{y_{i2}+m_2^{-1}}{\mu_{i2}+m_2^{-1}}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} + \frac{-\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{2j}}\right\}$$

$$= \sum_{i=1}^{n}\left\{\frac{y_{i2}}{\mu_{i2}}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{(y_{i2}+m_2^{-1})m_2}{(\mu_{i2}+m_2^{-1})m_2}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{2j}}\right\}$$

$$= \sum_{i=1}^{n}\left\{\frac{y_{i2}}{\mu_{i2}}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{m_2y_{i2}+1}{m_2\mu_{i2}+1}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{12}}\right\}$$

$$= \sum_{i=1}^{n}\left\{\frac{y_{i2}(1+m_2\mu_{i2})}{\mu_{i2}(1+m_2\mu_{i2})}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\mu_{i2}(m_2y_{i2}+1)}{\mu_{i2}(1+m_2\mu_{i2})}\frac{\partial \mu_{i1}}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{12}}\right\}$$

$$= \sum_{i=1}^{n}\left\{\frac{y_{i2}+y_{i2}m_2\mu_{i2}}{\mu_{i2}(1+m_2\mu_{i2})}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\mu_{i2}m_2y_{i2}+\mu_{i2}}{\mu_{i2}(1+m_2\mu_{i2})}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{12}}\right\}$$

$$= \sum_{i=1}^{n}\left\{\frac{y_{i2}-\mu_{i2}}{\mu_{i2}(1+m_2\mu_{i2})}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{12}}\right\}$$

Lampiran 8. Turunan Parsial Kedua Fungsi *ln likelihood*

Turunan parsial kedua fungsi ln *likelihood* terhadap λ dan λ

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = \frac{\partial\left(\sum_{i=1}^{n} \frac{(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right)}{\partial \lambda}$$

Misalkan:

$$A = \frac{(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}{1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

$$A' = \frac{0-(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}{[1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}$$

$$= -\frac{[(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}{[1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}$$

$$= -\left[\frac{(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}{1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right]^2$$

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = \sum_{i=1}^{n} \{A'\}$$

$$= \sum_{i=1}^{n} \left\{-\left[\frac{(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right]^2\right\}$$

$$= -\sum_{i=1}^{n} \left[\frac{(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right]^2$$

Turunan parsial kedua fungsi *ln likelihood* terhadap λ dan $\mathbf{m_1}$

$$\frac{\partial \ln \mathrm{L}}{\partial \lambda\, \partial \mathrm{m}_1} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\begin{matrix} \mathrm{m}_1^{-2} \ln(\mathrm{m}_1) + \mathrm{m}_1^{-2}[\ln(\mu_{i1} + m_1^{-1}) - 1] + \frac{m_1^{-2}(y_{i1} + m_1^{-1})}{\mu_{i1} + m_1^{-1}} - \sum_{j=0}^{y_{i2}-1} \frac{\mathrm{m}_1^{-2}}{\mathrm{m}_1^{-1} + j} \\ - \frac{\lambda(e^{-y_{i2}} - c_2)}{1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1} \end{matrix}\right\}\right)}{\partial \lambda}$$

Misalkan:

$$A = -\frac{\lambda(e^{-y_{i2}} - c_2)}{1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

Misalkan:

(lanjutan)

$$X = \frac{-\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$X' = \frac{-(e^{-y_{i2}}-c_2)\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)-\left(-\lambda(e^{-y_{i2}}-c_2)(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$= \frac{-(e^{-y_{i2}}-c_2)-\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)^2+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)^2}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$= \frac{-(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$Y = \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$Y' = 0$$

$$A' = X'Y + XY'$$

$$= \frac{-(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1} + 0$$

$$= \frac{-(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$\frac{\partial \ln \mathrm{L}}{\partial \lambda\, \partial \mathrm{m}_1} = \sum_{i=1}^{n}\{A'\}$$

$$= \sum_{i=1}^{n}\left\{\frac{-(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap λ dan $\mathbf{m_2}$

$$\frac{\partial \ln \mathrm{L}}{\partial \lambda\, \partial \mathrm{m}_2} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\begin{matrix}\mathrm{m}_2^{-2}\ln(\mathrm{m}_2)+\mathrm{m}_2^{-2}[\ln(\mu_{i2}+m_2^{-1})-1]+\frac{m_2^{-2}(y_{i2}+m_2^{-1})}{\mu_{i2}+m_2^{-1}}-\sum_{j=0}^{y_{i2}-1}\frac{\mathrm{m}_2^{-2}}{\mathrm{m}_2^{-1}+j}\\ -\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}\end{matrix}\right\},\right)}{\partial \lambda}$$

Misalkan:

$$A = -\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}$$

Misalkan:

(lanjutan)

$$X=-\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$X'=\frac{-(e^{-y_{i1}}-c_1)\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)-\left(-\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$=\frac{-(e^{-y_{i1}}-c_1)-\lambda(e^{-y_{i1}}-c_1)^2(e^{-y_{i2}}-c_2)+\lambda(e^{-y_{i1}}-c_1)^2(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$=\frac{-(e^{-y_{i1}}-c_1)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$Y=\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}$$

$$Y'=0$$

$$A'=X'Y+XY'$$

$$=\frac{-(e^{-y_{i1}}-c_1)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}+0$$

$$=\frac{-(e^{-y_{i1}}-c_1)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}$$

$$\frac{\partial \ln \mathrm{L}}{\partial\lambda\,\partial \mathrm{m}_2}=\sum_{i=1}^{n}\{A'\}$$

$$=\sum_{i=1}^{n}\left\{\frac{-(e^{-y_{i1}}-c_1)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}\right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap $\boldsymbol{\lambda}$ dan $\boldsymbol{\beta_{1j}}$

$$\frac{\partial \ln \mathrm{L}}{\partial\lambda\,\partial\beta_{1j}}=\frac{\partial\left(\sum_{i=1}^{n}\left\{\frac{y_{i1}-\mu_{i1}}{\mu_{i1}(1+m_1\mu_{i1})}\frac{\partial\mu_{i1}}{\partial\beta_{1j}}-\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial \mathrm{c}_1}{\partial\beta_{1j}}\right\}\right)}{\partial\lambda}$$

Misalkan:

$$A=-\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial \mathrm{c}_1}{\partial\beta_{1j}}$$

Misalkan:

(lanjutan)

$$X = -\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$X' = \frac{-(e^{-y_{i2}}-c_2)\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)-\left(-\lambda(e^{-y_{i2}}-c_2)(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$= \frac{-(e^{-y_{i2}}-c_2)-\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)^2+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)^2}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$= \frac{-(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$Y = \frac{\partial c_1}{\partial \beta_{1j}}$$

$$Y' = 0$$

$$A' = X'Y + XY'$$

$$= \frac{-(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial c_1}{\partial \beta_{1j}} + 0$$

$$= \frac{-(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial c_1}{\partial \beta_{1j}}$$

$$\frac{\partial \ln L}{\partial \lambda\, \partial \beta_{1j}} = \sum_{i=1}^{n} A'$$

$$= \sum_{i=1}^{n} \frac{-(e^{-y_{i2}} - c_2)}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}\frac{\partial c_1}{\partial \beta_{1j}}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap λ dan β_{2j}

$$\frac{\partial \ln L}{\partial \lambda\, \partial \beta_{2j}} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\frac{y_{i2}-\mu_{i2}}{\mu_{i2}(1+m_2\mu_{i2})}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{2j}}\right\}\right)}{\partial \lambda}$$

Misalkan:

$$A = -\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial \beta_{2j}}$$

Misalkan:

(lanjutan)

$$X = -\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$X' = \frac{-(e^{-y_{i1}}-c_1)\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)-\left(-\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$= \frac{-(e^{-y_{i1}}-c_1)-\lambda(e^{-y_{i1}}-c_1)^2(e^{-y_{i2}}-c_2)+\lambda(e^{-y_{i1}}-c_1)^2(e^{-y_{i2}}-c_2)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$= \frac{-(e^{-y_{i1}}-c_1)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$Y = \frac{\partial \mathrm{c}_2}{\partial \beta_{2\mathrm{j}}}$$

$$Y' = 0$$

$$A' = X'Y + XY'$$

$$= \frac{-(e^{-y_{i1}}-c_1)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_2}{\partial \beta_{2\mathrm{j}}} + 0$$

$$= \frac{-(e^{-y_{i1}}-c_1)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_2}{\partial \beta_{2\mathrm{j}}}$$

$$\frac{\partial \ln \mathrm{L}}{\partial \lambda\, \partial \beta_{2\mathrm{j}}} = \sum_{\mathrm{i}=1}^{\mathrm{n}} A'$$

$$= \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{-(e^{-y_{i1}} - c_1)}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}\frac{\partial \mathrm{c}_2}{\partial \beta_{2\mathrm{j}}}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap $\mathbf{m_1}$ dan $\mathbf{m_1}$

$$\frac{\partial \ln \mathrm{L}}{\partial \mathrm{m}_1^2} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\begin{matrix}\mathrm{m}_1^{-2}\ln(\mathrm{m}_1)+\mathrm{m}_1^{-2}\left[\ln\left(\mu_{i1}+m_1^{-1}\right)-1\right]+\frac{m_1^{-2}\left(y_{i1}+m_1^{-1}\right)}{\mu_{i1}+m_1^{-1}}-\sum_{j=0}^{y_{i1}-1}\frac{\mathrm{m}_1^{-2}}{\mathrm{m}_1^{-1}+j}\\ -\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\end{matrix}\right\}\right)}{\partial \mathrm{m}_1}$$

Misalkan:

$$A = \mathrm{m}_1^{-2}.\ln(\mathrm{m}_1)$$

$$= -2m_1^{-3}\ln(m_1) + m_1^{-2}\frac{1}{m_1}$$

(lanjutan)

$$= -2m_1^{-3}\ln(m_1) + m_1^{-3}$$

$$B = \mathrm{m}_1^{-2}[\ln(\mu_{i1} + m_1^{-1}) - 1]$$

$$= \mathrm{m}_1^{-2}\ln(\mu_{i1} + m_1^{-1}) - \mathrm{m}_1^{-2}$$

$$B' = -2\mathrm{m}_1^{-3}\ln(\mu_{i1} + m_1^{-1}) + \mathrm{m}_1^{-2}\frac{-m_1^{-2}}{\mu_{i1} + m_1^{-1}} - (-2\mathrm{m}_1^{-3})$$

$$= -2\mathrm{m}_1^{-3}\ln(\mu_{i1} + m_1^{-1}) - \frac{m_1^{-4}}{\mu_{i1} + m_1^{-1}} + 2\mathrm{m}_1^{-3}$$

$$C = \frac{m_1^{-2}(y_{i1}+m_1^{-1})}{\mu_{i1}+m_1^{-1}}$$

$$= \frac{m_1^{-2}y_{i1} + m_1^{-3}}{\mu_{i1} + m_1^{-1}}$$

$$C' = \frac{(-2m_1^{-3}y_{i1}-3m_1^{-4})(\mu_{i1}+m_1^{-1})-\left((m_1^{-2}y_{i1}+m_1^{-3})(-m_1^{-2})\right)}{(\mu_{i1}+m_1^{-1})^2}$$

$$= \frac{-2m_1^{-3}y_{i1}\mu_{i1}-2m_1^{-4}y_{i1}-3m_1^{-4}\mu_{i1}-3m_1^{-5}-(-m_1^{-4}y_{i1}-m_1^{-5})}{\mu_{i1}^2+2\mu_{i1}m_1^{-1}+m_1^{-2}}$$

$$= \frac{-2m_1^{-3}y_{i1}\mu_{i1}-2m_1^{-4}y_{i1}-3m_1^{-4}\mu_{i1}-3m_1^{-5}+m_1^{-4}y_{i1}+m_1^{-5}}{\mu_{i1}^2+2\mu_{i1}m_1^{-1}+m_1^{-2}}$$

$$= \frac{-2m_1^{-3}y_{i1}\mu_{i1}-m_1^{-4}y_{i1}-3m_1^{-4}\mu_{i1}-2m_1^{-5}}{\mu_{i1}^2+2\mu_{i1}m_1^{-1}+m_1^{-2}} \cdot \frac{m_1^2}{m_1^2}$$

$$= \frac{-2m_1^{-1}y_{i1}\mu_{i1}-m_1^{-2}y_{i1}-3m_1^{-2}\mu_{i1}-2m_1^{-3}}{\mu_{i1}^2m_1^2+2\mu_{i1}m_1+1}$$

$$= \frac{-2m_1^{-1}y_{i1}\mu_{i1}-m_1^{-2}y_{i1}-3m_1^{-2}\mu_{i1}-2m_1^{-3}}{(1+m_1\mu_{i1})^2}$$

$$D = \frac{\mathrm{m}_1^{-2}}{\mathrm{m}_1^{-1}+j}$$

$$D' = \frac{-2\mathrm{m}_1^{-3}(\mathrm{m}_1^{-1}+j)-\left(\mathrm{m}_1^{-2}(-m_1^{-2})\right)}{(\mathrm{m}_1^{-1}+j)^2}$$

(lanjutan)

$$= \frac{-2\mathrm{m}_1^{-4} - 2\mathrm{m}_1^{-3} j + m_1^{-4}}{\left(\mathrm{m}_1^{-1} + j\right)^2}$$

$$= \frac{-m_1^{-4} - 2\mathrm{m}_1^{-3} j}{\left(\mathrm{m}_1^{-1} + j\right)^2}$$

$$= -\frac{m_1^{-4} + 2\mathrm{m}_1^{-3} j}{\left(\mathrm{m}_1^{-1} + j\right)^2}$$

$$E = \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

Misalkan:

$$u = \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

$$= \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} e^{-y_{i2}} - e^{-y_{i1}} c_2 - e^{-y_{i2}} c_1 + c_1 c_2)}$$

$$u' = \frac{0 - \left(\left(\lambda(e^{-y_{i2}} - c_2)\right)\left(-\lambda(e^{-y_{i2}} - c_2)\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\right)\right)}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}$$

$$= \frac{\lambda^2 (e^{-y_{i2}} - c_2)^2}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2} \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$= \left[\frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right]^2 \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$v = \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$v' = \frac{\partial^2 \mathrm{c}_1}{\partial \mathrm{m}_1^2}$$

$$E' = u'v + uv'$$

$$= \left[\frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right]^2 \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1} \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1} + \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial^2 \mathrm{c}_1}{\partial \mathrm{m}_1^2}$$

$$\frac{\partial \ln \mathrm{L}}{\partial \mathrm{m}_1^2} = \sum_{i=1}^{n} \left\{ A' + B' + C' - \sum_{j=0}^{y_{i1}-1} D' - E' \right\}$$

(lanjutan)

$$= \sum_{i=1}^{n}\Bigg\{-2m_1^{-3}\ln(m_1) + m_1^{-3} - 2\mathrm{m}_1^{-3}\ln(\mu_{i1} + m_1^{-1}) - \frac{m_1^{-4}}{\mu_{i1} + m_1^{-1}}$$

$$+ 2\mathrm{m}_1^{-3} - \frac{2m_1^{-1}y_{i1}\mu_{i1} + m_1^{-2}y_{i1} + 3m_1^{-2}\mu_{i1} + 2m_1^{-3}}{(1 + m_1\mu_{i1})^2}$$

$$+ \sum_{j=0}^{y_{i1}-1}\frac{m_1^{-4} + 2\mathrm{m}_1^{-3}j}{(\mathrm{m}_1^{-1} + j)^2} - \left[\frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right]^2\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$- \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\frac{\partial^2 \mathrm{c}_1}{\partial \mathrm{m}_1^2}\Bigg\}$$

$$= \sum_{i=1}^{n}\Bigg\{-2m_1^{-3}\ln(m_1) + 3m_1^{-3} - 2\mathrm{m}_1^{-3}\ln(\mu_{i1} + m_1^{-1}) - \frac{m_1^{-4}}{\mu_{i1} + m_1^{-1}}$$

$$- \frac{2m_1^{-1}y_{i1}\mu_{i1} + m_1^{-2}y_{i1} + 3m_1^{-2}\mu_{i1} + 2m_1^{-3}}{(1 + m_1\mu_{i1})^2} + \sum_{j=0}^{y_{i1}-1}\frac{m_1^{-4} + 2\mathrm{m}_1^{-3}j}{(\mathrm{m}_1^{-1} + j)^2}$$

$$- \left[\frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right]^2\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$- \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\frac{\partial^2 \mathrm{c}_1}{\partial \mathrm{m}_1^2}\Bigg\}$$

$$= \sum_{i=1}^{n}\Bigg\{m_1^{-3}\left[-2\ln(m_1) + 3 - 2\ln(\mu_{i1} + m_1^{-1}) - \frac{m_1^{-1}}{\mu_{i1} + m_1^{-1}}\right]$$

$$- \frac{2m_1^{-1}y_{i1}\mu_{i1} + m_1^{-2}y_{i1} + 3m_1^{-2}\mu_{i1} + 2m_1^{-3}}{(1 + m_1\mu_{i1})^2} + \sum_{j=0}^{y_{i1}-1}\frac{m_1^{-4} + 2\mathrm{m}_1^{-3}j}{(\mathrm{m}_1^{-1} + j)^2}$$

$$- \left[\frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\right]^2\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$- \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\frac{\partial^2 \mathrm{c}_1}{\partial \mathrm{m}_1^2}\Bigg\}$$

(lanjutan)

Turunan parsial kedua fungsi ln *likelihood* terhadap m_1 dan m_2

$$\frac{\partial^2 \ln L}{\partial \mathrm{m}_1\,\partial \mathrm{m}_2}=\frac{\partial\left(\sum_{i=1}^{n}\left\{\begin{array}{c}\mathrm{m}_2^{-2}\ln(\mathrm{m}_2)+\mathrm{m}_2^{-2}\left[\ln\left(\mu_{i2}+m_2^{-1}\right)-1\right]+\frac{m_2^{-2}\left(y_{i2}+m_2^{-1}\right)}{\mu_{i2}+m_2^{-1}}-\sum_{j=0}^{y_{i2}-1}\frac{\mathrm{m}_2^{-2}}{\mathrm{m}_2^{-1}+j}\\-\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}\end{array}\right\}\right)}{\partial \mathrm{m}_1}$$

Misalkan:

$$A=\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}$$

$$A'=u'v+uv'$$

Misalkan

$$u=\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$u'=\frac{-\lambda\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)-\left(\lambda(e^{-y_{i1}}-c_1)\left(-\lambda(e^{-y_{i2}}-c_2)\cdot\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\right)\right)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$=\frac{-\lambda\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}-\lambda^2(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}+\lambda^2(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$=-\frac{\lambda}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}$$

$$v=\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}$$

$$v'=0$$

$$A'=u'v+uv'$$

$$=-\frac{\lambda}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}+0$$

$$=-\frac{\lambda}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1}\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}$$

$$\frac{\partial^2 \ln L}{\partial \mathrm{m}_1\,\partial \mathrm{m}_2}=\sum_{i=1}^{n}\{-A'\}$$

(lanjutan)

$$= \sum_{i=1}^{n} \left\{ \frac{\lambda}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2} \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1} \frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2} \right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap $\mathbf{m_1}$ dan $\boldsymbol{\beta_{1j}}$

$$\frac{\partial^2 \ln L}{\partial \mathrm{m}_1 \, \partial \beta_{1j}} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\frac{y_{i1}-\mu_{i1}}{\mu_{i1}(1+m_1\mu_{i1})} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)} \frac{\partial \mathrm{c}_1}{\partial \beta_{1j}}\right\}\right)}{\partial \mathrm{m}_1}$$

Misalkan:

$$A = \frac{y_{i1}-\mu_{i1}}{\mu_{i1}(1+m_1\mu_{i1})} \frac{\partial \mu_{i1}}{\partial \beta_{1j}}$$

Misalkan:

$$u = \frac{y_{i1}-\mu_{i1}}{\mu_{i1}(1+m_1\mu_{i1})}$$

$$u' = \frac{0-\left((y_{i1}-\mu_{i1})\mu_{i1}^2\right)}{\left(\mu_{i1}(1+m_1\mu_{i1})\right)^2}$$

$$= \frac{-(y_{i1}-\mu_{i1})\mu_{i1}^2}{\mu_{i1}^2(1+m_1\mu_{i1})^2}$$

$$= \frac{\mu_{i1}-y_{i1}}{(1+m_1\mu_{i1})^2} \cdot \frac{m_1^{-2}}{m_1^{-2}}$$

$$= \frac{m_1^{-2}(\mu_{i1}-y_{i1})}{m_1^{-2}(1+m_1\mu_{i1})(1+m_1\mu_{i1})}$$

$$= \frac{m_1^{-2}(\mu_{i1}-y_{i1})}{m_1^{-2}\left(1+2m_1\mu_{i1}+m_1^2\mu_{i1}^2\right)}$$

$$= \frac{m_1^{-2}(\mu_{i1}-y_{i1})}{m_1^{-2}+2m_1^{-1}\mu_{i1}+\mu_{i1}^2}$$

$$= \frac{m_1^{-2}(\mu_{i1}-y_{i1})}{\left(m_1^{-1}+\mu_{i1}\right)^2}$$

$$v = \frac{\partial \mu_{i1}}{\partial \beta_{1j}}$$

$$v' = 0$$

$$A' = u'v + uv'$$

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$$= \frac{m_1^{-2}(\mu_{i1}-y_{i1})}{\left(m_1^{-1}+\mu_{i1}\right)^2}\frac{\partial\mu_{i1}}{\partial\beta_{1j}} + 0$$

$$= \frac{m_1^{-2}(\mu_{i1}-y_{i1})}{\left(m_1^{-1}+\mu_{i1}\right)^2}\frac{\partial\mu_{i1}}{\partial\beta_{1j}}$$

$$B = \frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_1}{\partial\beta_{1j}}$$

Misalkan:

$$a = \frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$a' = \frac{u'v - uv'}{v^2}$$

$$= \frac{0-\left(\lambda(e^{-y_{i2}}-c_2)\left(-\lambda(e^{-y_{i2}}-c_2)\frac{\partial c_1}{\partial m_1}\right)\right)}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}$$

$$= \frac{\lambda^2(e^{-y_{i2}}-c_2)^2}{[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]^2}\frac{\partial c_1}{\partial m_1}$$

$$= \left[\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2\frac{\partial c_1}{\partial m_1}$$

$$b = \frac{\partial c_1}{\partial\beta_{1j}}$$

$$b' = \frac{\partial^2 c_1}{\partial m_1\,\partial\beta_{1j}}$$

$$B' = a'b + ab'$$

$$= \left[\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2\frac{\partial c_1}{\partial m_1}\frac{\partial c_1}{\partial\beta_{1j}} + \frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial^2 c_1}{\partial m_1\,\partial\beta_{1j}}$$

$$\frac{\partial^2 \ln L}{\partial m_1\,\partial\beta_{1j}} = \sum_{i=1}^{n}\{A' - B'\}$$

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$$= \sum_{i=1}^{n} \left\{ \frac{m_1^{-2}(\mu_{i1} - y_{i1})}{(m_1^{-1} + \mu_{i1})^2} \frac{\partial \mu_{i1}}{\partial \beta_{1j}} - \left[\frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \right]^2 \frac{\partial c_1}{\partial m_1} \frac{\partial c_1}{\partial \beta_{1j}} \right.$$

$$\left. - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial^2 c_1}{\partial m_1 \, \partial \beta_{1j}} \right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap $\mathbf{m_1}$ dan $\boldsymbol{\beta_{2j}}$

$$\frac{\partial^2 \ln L}{\partial m_1 \, \partial \beta_{2j}} = \frac{\partial \left(\sum_{i=1}^{n} \left\{ \frac{y_{i2} - \mu_{i2}}{\mu_{i2}(1 + m_2 \mu_{i2})} \frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_2}{\partial \beta_{2j}} \right\} \right)}{\partial m_1}$$

$$A = \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_2}{\partial \beta_{2j}}$$

Misalkan:

$$a = \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

$$a' = \frac{-\lambda \frac{\partial c_1}{\partial m_1}\left(1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)\right) - \left(\lambda(e^{-y_{i1}} - c_1)\left(\lambda(-e^{-y_{i2}} + c_2)\frac{\partial c_1}{\partial m_1}\right)\right)}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}$$

$$= \frac{-\lambda \frac{\partial c_1}{\partial m_1} - \lambda^2 (e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)\frac{\partial c_1}{\partial m_1} + \lambda^2 (e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)\frac{\partial c_1}{\partial m_1}}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}$$

$$= \frac{-\lambda}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2} \frac{\partial c_1}{\partial m_1}$$

$$b = \frac{\partial c_2}{\partial \beta_{2j}}$$

$$b' = 0$$

$$A = a'b + ab'$$

$$= \frac{-\lambda}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2} \frac{\partial c_1}{\partial m_1} \frac{\partial c_2}{\partial \beta_{2j}} + 0$$

$$= -\frac{\lambda}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2} \frac{\partial c_1}{\partial m_1} \frac{\partial c_2}{\partial \beta_{2j}}$$

$$\frac{\partial^2 \ln L}{\partial m_1 \, \partial \beta_{2j}} = \sum_{i=1}^{n} \{-A\}$$

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$$= \sum_{i=1}^{n} \left\{ \frac{\lambda}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2} \frac{\partial \mathrm{c}_1}{\partial \mathrm{m}_1} \frac{\partial \mathrm{c}_2}{\partial \beta_{2\mathrm{j}}} \right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap $\mathbf{m_2}$ dan $\mathbf{m_2}$

Turunan parsial kedua fungsi ln *likelihood* terhadap m_2 dan m_2 diperoleh dengan cara yang sama pada turunan parsial kedua fungsi ln *likelihood* terhadap m_1 dan m_1 sehingga diperoleh hasil sebagai berikut:

$$\frac{\partial^2 \ln L}{\partial \mathrm{m}_2^2} = \sum_{i=1}^{n} \left\{ m_2^{-3} \left[-2\ln(m_2) + 3 - 2\ln(\mu_{i2} + m_2^{-1}) - \frac{m_2^{-1}}{\mu_{i2} + m_2^{-1}} \right] - \right.$$

$$\frac{2m_2^{-1} y_{i2}\mu_{i2} + m_2^{-2} y_{i2} + 3m_2^{-2}\mu_{i2} + 2m_2^{-3}}{(1 + m_2\mu_{i2})^2} + \sum_{j=0}^{y_{i2}-1} \frac{m_2^{-4} + 2m_2^{-3} j}{(m_2^{-1} + j)^2}$$

$$- \left[\frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \right]^2 \frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2} \frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2} -$$

$$\left. \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial^2 \mathrm{c}_2}{\partial \mathrm{m}_2^2} \right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap $\mathbf{m_2}$ dan $\boldsymbol{\beta_{1j}}$

$$\frac{\partial^2 \ln L}{\partial \mathrm{m}_2 \, \partial \beta_{1\mathrm{j}}} = \frac{\partial \left(\sum_{i=1}^{n} \left\{ \frac{y_{i1} - \mu_{i1}}{\mu_{i1}(1 + m_1\mu_{i1})} \frac{\partial \mu_{i1}}{\partial \beta_{1\mathrm{j}}} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \beta_{1\mathrm{j}}} \right\} \right)}{\partial \mathrm{m}_2}$$

Misalkan:

$$A = \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial \mathrm{c}_1}{\partial \beta_{1\mathrm{j}}}$$

Misalkan:

$$a = \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

$$a' = \frac{-\lambda \frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2}\left(1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)\right) - \left(\lambda(e^{-y_{i2}} - c_2)\left(-\lambda(e^{-y_{i1}} - c_1)\frac{\partial \mathrm{c}_2}{\partial \mathrm{m}_2} \right) \right)}{[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}$$

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$$= \frac{-\lambda\frac{\partial c_2}{\partial m_2} - \lambda^2(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)\frac{\partial c_2}{\partial m_2} + \lambda^2(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)\frac{\partial c_2}{\partial m_2}}{[1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}$$

$$= \frac{-\lambda}{[1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}\frac{\partial c_2}{\partial m_2}$$

$$b = \frac{\partial c_1}{\partial \beta_{1j}}$$

$$b' = 0$$

$$A' = a'b + ab'$$

$$= \frac{-\lambda}{[1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}\frac{\partial c_2}{\partial m_2}\frac{\partial c_1}{\partial \beta_{1j}} + 0$$

$$= -\frac{\lambda}{[1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}\frac{\partial c_2}{\partial m_2}\frac{\partial c_1}{\partial \beta_{1j}}$$

$$\frac{\partial^2 \ln L}{\partial m_2\, \partial \beta_{1j}} = \sum_{i=1}^{n}\{-A'\}$$

$$= \sum_{i=1}^{n}\left\{\frac{\lambda}{[1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]^2}\frac{\partial c_2}{\partial m_2}\frac{\partial c_1}{\partial \beta_{1j}}\right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap $\mathbf{m_2}$ dan $\boldsymbol{\beta_{2j}}$

$$\frac{\partial^2 \ln L}{\partial m_2\, \partial \beta_{2j}} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\frac{y_{i2}-\mu_{i2}}{\mu_{i2}(1+m_2\mu_{i2})}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}} - c_1)}{1+\lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}\frac{\partial c_2}{\partial \beta_{2j}}\right\}\right)}{\partial m_2}$$

Misalkan:

$$A = \frac{y_{i2}-\mu_{i2}}{\mu_{i2}(1+m_2\mu_{i2})}\frac{\partial \mu_{i2}}{\partial \beta_{2j}}$$

Misalkan:

$$a = \frac{y_{i2}-\mu_{i2}}{\mu_{i2}(1+m_2\mu_{i2})}$$

$$a' = \frac{0-\left((y_{i2}-\mu_{i2})\mu_{i2}^2\right)}{\left(\mu_{i2}(1+m_2\mu_{i2})\right)^2}$$

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$$= -\frac{(y_{i2}-\mu_{i2})\mu_{i2}^2}{\mu_{i2}^2(1+m_2\mu_{i2})^2}$$

$$= \frac{(y_{i2}-\mu_{i2})}{(1+m_2\mu_{i2})^2}$$

$$= \frac{(y_{i2}-\mu_{i2})}{1+2m_2\mu_{i2}+m_2^2\mu_{i2}^2}\frac{m_2^{-2}}{m_2^{-2}}$$

$$= \frac{m_2^{-2}(y_{i2}-\mu_{i2})}{m_2^{-2}+2m_2^{-1}\mu_{i2}+\mu_{i2}^2}$$

$$= \frac{m_2^{-2}(y_{i2}-\mu_{i2})}{\left(m_2^{-1}+\mu_{i2}\right)^2}$$

$$b = \frac{\partial\mu_{i2}}{\partial\beta_{2j}}$$

$$b = 0$$

$$A' = a'b + ab'$$

$$= \frac{m_2^{-2}(y_{i2}-\mu_{i2})}{\left(m_2^{-1}+\mu_{i2}\right)^2}\frac{\partial\mu_{i2}}{\partial\beta_{2j}} + 0$$

$$= \frac{m_2^{-2}(y_{i2}-\mu_{i2})}{\left(m_2^{-1}+\mu_{i2}\right)^2}\frac{\partial\mu_{i2}}{\partial\beta_{2j}}$$

$$B = \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_2}{\partial\beta_{2j}}$$

Misalkan:

$$a = \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$a' = \frac{0-\left(\lambda(e^{-y_{i1}}-c_1)\left(-\lambda(e^{-y_{i1}}-c_1)\frac{\partial c_2}{\partial m_2}\right)\right)}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}$$

$$= \frac{\lambda^2(e^{-y_{i1}}-c_1)^2}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}\frac{\partial c_2}{\partial m_2}$$

$$= \left[\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2\frac{\partial c_2}{\partial m_2}$$

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$$b = \frac{\partial c_2}{\partial \beta_{2j}}$$

$$b' = \frac{\partial^2 c_2}{\partial m_2\, \partial \beta_{2j}}$$

$$B' = a'b + ab'$$

$$= \left[\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2 \frac{\partial c_2}{\partial m_2}\frac{\partial c_2}{\partial \beta_{2j}} + \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial^2 c_2}{\partial m_2\, \partial \beta_{2j}}$$

$$\frac{\partial^2 \ln L}{\partial m_2\, \partial \beta_{2j}} = \sum_{i=1}^{n}\{A' - B'\}$$

$$= \sum_{i=1}^{n}\left\{\frac{m_2^{-2}(y_{i2}-\mu_{i2})}{(m_2^{-1}+\mu_{i2})^2}\frac{\partial \mu_{i2}}{\partial \beta_{2j}} - \left[\frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2 \frac{\partial c_2}{\partial m_2}\frac{\partial c_2}{\partial \beta_{2j}} - \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial^2 c_2}{\partial m_2\, \partial \beta_{2j}}\right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap β_{1j} dan β_{1s}

$$\frac{\partial \log L}{\partial \beta_{1s}} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\sum_{t=1}^{2}\left[y_{it}\ln\mu_{it} - m_t^{-1}\ln m_t - (y_{it}+m_t^{-1})\ln(\mu_{it}+m_t^{-1}) - \ln(y_{it}!) + \sum_{j=0}^{y_{it}-1}\ln(m_t^{-1}+j)\right] + \ln[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]\right\}\right)}{\partial \beta_{1s}}$$

Misalkan:

$$A = y_{i1}\log\mu_{i1}$$

$$A' = \frac{y_{i1}}{\mu_{i1}}\frac{\partial \mu_{i1}}{\partial \beta_{1s}}$$

$$= \frac{y_{i1}}{\mu_{i1}}\mu_{i1}x_{1s}$$

$$= y_{i1}x_{1s}$$

$$B = (y_{i1}+m_1^{-1})\log(\mu_{i1}+m_1^{-1})$$

(lanjutan)

$$B' = \frac{y_{i1} + m_1^{-1}}{\mu_{i1} + m_1^{-1}} \frac{\partial \mu_{i1}}{\partial \beta_{1s}}$$

$$= \frac{(y_{i1} + m_1^{-1})\mu_{i1} x_{1s}}{\mu_{i1} + m_1^{-1}}$$

$$= \frac{y_{i1}\mu_{i1}x_{1s} + m_1^{-1}\mu_{i1}x_{1s}}{\mu_{i1} + m_1^{-1}}$$

$$C = \log[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]$$

$$C' = \frac{-\lambda(e^{-y_{i2}} - c_2)\frac{\partial c_1}{\partial \beta_{1s}}}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

$$= -\frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_1}{\partial \beta_{1s}}$$

$$\frac{\partial \log L}{\partial \beta_{1s}} = \sum_{i=1}^{n} \{A' - B' + C'\}$$

$$= \sum_{i=1}^{n} \left\{ y_{i1}x_{1s} - \frac{y_{i1}\mu_{i1}x_{1s} + m_1^{-1}\mu_{i1}x_{1s}}{\mu_{i1} + m_1^{-1}} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_1}{\partial \beta_{1s}} \right\}$$

$$\frac{\partial^2 \ln L}{\partial \beta_{1j}\, \partial \beta_{1s}} = \frac{\partial \left(\sum_{i=1}^{n} \left\{ y_{i1}x_{1s} - \frac{y_{i1}\mu_{i1}x_{1s} + m_1^{-1}\mu_{i1}x_{1s}}{\mu_{i1} + m_1^{-1}} - \frac{\lambda(e^{-y_{i2}} - c_2)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_1}{\partial \beta_{1s}} \right\} \right)}{\partial \beta_{1j}}$$

Misalkan:

$$A = \frac{y_{i1}\mu_{i1}x_{1s} + m_1^{-1}\mu_{i1}x_{1s}}{\mu_{i1} + m_1^{-1}}$$

Misalkan:

$$u = y_{i1}\mu_{i1}x_{1s} + m_1^{-1}\mu_{i1}x_{1s}$$

$$u' = (y_{i1}x_{1s} + m_1^{-1}x_{1s}) \frac{\partial \mu_{i1}}{\partial \beta_{1j}}$$

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$$= (y_{i1}x_{1s} + m_1^{-1}x_{1s})\mu_{i1}x_{1j}$$

$$= y_{i1}x_{1s}\mu_{i1}x_{1j} + m_1^{-1}x_{1s}\mu_{i1}x_{1j}$$

$$v = \mu_{i1} + m_1^{-1}$$

$$v' = \frac{\partial\mu_{i1}}{\partial\beta_{1j}}$$

$$= \mu_{i1}x_{1j}$$

$$A' = \frac{u'v - uv'}{v^2}$$

$$= \frac{(y_{i1}x_{1s}\mu_{i1}x_{1j}+m_1^{-1}x_{1s}\mu_{i1}x_{1j})(\mu_{i1}+m_1^{-1}) - [(y_{i1}\mu_{i1}x_{1s}+m_1^{-1}\mu_{i1}x_{1s})\mu_{i1}x_{1j}]}{(\mu_{i1}+m_1^{-1})^2}$$

$$= \frac{y_{i1}x_{1s}\mu_{i1}^2x_{1j}+y_{i1}x_{1s}\mu_{i1}x_{1j}m_1^{-1}+m_1^{-1}x_{1s}\mu_{i1}^2x_{1j}+m_1^{-2}x_{1s}\mu_{i1}x_{1j} - [y_{i1}\mu_{i1}^2x_{1s}x_{1j}+m_1^{-1}\mu_{i1}^2x_{1s}x_{1j}]}{\mu_{i1}^2+2\mu_{i1}m_1^{-1}+m_1^{-2}}$$

$$= \frac{y_{i1}x_{1s}\mu_{i1}x_{1j}m_1^{-1}+m_1^{-2}x_{1s}\mu_{i1}x_{1j}}{\mu_{i1}^2+2\mu_{i1}m_1^{-1}+m_1^{-2}}$$

$$= \frac{(y_{i1}\mu_{i1}m_1^{-1}+m_1^{-2}\mu_{i1})x_{1s}x_{1j}}{\mu_{i1}^2+2\mu_{i1}m_1^{-1}+m_1^{-2}} \cdot \frac{m_1^2}{m_1^2}$$

$$= \frac{(y_{i1}\mu_{i1}m_1+\mu_{i1})x_{1s}x_{1j}}{\mu_{i1}^2m_1^2+2\mu_{i1}m_1+1}$$

$$= \frac{\mu_{i1}(1+m_1y_{i1})x_{1s}x_{1j}}{(1+m_1\mu_{i1})^2}$$

$$B = \frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial c_1}{\partial\beta_{1s}}$$

Misalkan:

$$u = \frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}$$

$$u' = \frac{0-\left(\lambda(e^{-y_{i2}}-c_2)\left(-\lambda(e^{-y_{i2}}-c_2).\frac{\partial c_1}{\partial\beta_{1j}}\right)\right)}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}$$

$$= \frac{\lambda^2(e^{-y_{i2}}-c_2)^2}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}\frac{\partial c_1}{\partial\beta_{1j}}$$

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$$= \left[\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2 \frac{\partial c_1}{\partial \beta_{1j}}$$

$$v = \frac{\partial c_1}{\partial \beta_{1s}}$$

$$v' = \frac{\partial^2 c_1}{\partial \beta_{1j}\,\partial \beta_{1s}}$$

$$B' = u'v + uv'$$

$$= \left[\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2 \frac{\partial c_1}{\partial \beta_{1j}}\frac{\partial c_1}{\partial \beta_{1s}} + \frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial^2 c_1}{\partial \beta_{1j}\,\partial \beta_{1s}}$$

$$\frac{\partial^2 \ln L}{\partial \beta_{1j}\,\partial \beta_{1s}} = \sum_{i=1}^{n}\{-A-B\}$$

$$= \sum_{i=1}^{n}\left\{-\frac{\mu_{i1}(1+m_1 y_{i1})x_{1s}x_{1j}}{(1+m_1\mu_{i1})^2} - \left[\frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2 \frac{\partial c_1}{\partial \beta_{1j}}\frac{\partial c_1}{\partial \beta_{1s}} - \frac{\lambda(e^{-y_{i2}}-c_2)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\frac{\partial^2 c_1}{\partial \beta_{1j}\,\partial \beta_{1s}}\right\}$$

Turunan parsial kedua fungsi *ln likelihood* terhadap β_{1j} dan β_{2s}

$$\frac{\partial \log L}{\partial \beta_{2s}} = \frac{\partial\left(\sum_{i=1}^{n}\left\{\sum_{t=1}^{2}\left[y_{it}\ln\mu_{it} - m_t^{-1}\ln m_t - (y_{it}+m_t^{-1})\ln(\mu_{it}+m_t^{-1}) - \ln(y_{it}!) + \sum_{j=0}^{y_{it}-1}\ln(m_t^{-1}+j)\right] + \ln[1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)]\right\}\right)}{\partial \beta_{2s}}$$

Misalkan:

$$A = y_{i2}\log\mu_{i2}$$

$$A' = \frac{y_{i2}}{\mu_{i2}}\cdot\frac{\partial \mu_{i2}}{\partial \beta_{2s}}$$

$$= \frac{y_{i2}}{\mu_{i2}}\mu_{i2}x_{2s}$$

$$= y_{i2}x_{2s}$$

(lanjutan)

$$B = (y_{i2} + m_2^{-1}) \log(\mu_{i2} + m_2^{-1})$$

$$B' = \frac{y_{i2} + m_2^{-1}}{\mu_{i2} + m_2^{-1}} \frac{\partial \mu_{i2}}{\partial \beta_{2s}}$$

$$= \frac{(y_{i2} + m_2^{-1})\mu_{i2} x_{2s}}{\mu_{i2} + m_2^{-1}}$$

$$= \frac{y_{i2}\mu_{i2} x_{2s} + m_2^{-1}\mu_{i2} x_{2s}}{\mu_{i2} + m_2^{-1}}$$

$$C = \log[1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)]$$

$$C' = \frac{-\lambda(e^{-y_{i1}} - c_1)\frac{\partial c_2}{\partial \beta_{2s}}}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

$$= -\frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_2}{\partial \beta_{2s}}$$

$$\frac{\partial \log L}{\partial \beta_{2s}} = \sum_{i=1}^{n} \{A' - B' + C'\}$$

$$= \sum_{i=1}^{n} \left\{ y_{i2} x_{12} - \frac{y_{i2}\mu_{i2} x_{2s} + m_2^{-1}\mu_{i2} x_{2s}}{\mu_{i2} + m_2^{-1}} - \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_2}{\partial \beta_{2s}} \right\}$$

$$\frac{\partial^2 \ln L}{\partial \beta_{1j}\, \partial \beta_{2s}} = \frac{\partial \left(\sum_{i=1}^{n} \left\{ y_{i2} x_{12} - \frac{y_{i2}\mu_{i2} x_{2s} + m_2^{-1}\mu_{i2} x_{2s}}{\mu_{i2} + m_2^{-1}} - \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_2}{\partial \beta_{2s}} \right\} \right)}{\partial \beta_{1j}}$$

Misalkan:

$$A = \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)} \frac{\partial c_2}{\partial \beta_{2s}}$$

Misalkan:

$$u = \frac{\lambda(e^{-y_{i1}} - c_1)}{1 + \lambda(e^{-y_{i1}} - c_1)(e^{-y_{i2}} - c_2)}$$

(lanjutan)

$$u' = \frac{-\lambda\frac{\partial c_1}{\partial \beta_{1j}}\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)-\left(\lambda(e^{-y_{i1}}-c_1)\left(-\lambda(e^{-y_{i2}}-c_2)\frac{\partial c_1}{\partial \beta_{1j}}\right)\right)}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}$$

$$= \frac{-\lambda\frac{\partial c_1}{\partial \beta_{1j}}-\lambda^2(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\frac{\partial c_1}{\partial \beta_{1j}}+\lambda^2(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\frac{\partial c_1}{\partial \beta_{1j}}}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}$$

$$= \frac{-\lambda}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}\frac{\partial c_1}{\partial \beta_{1j}}$$

$$v = \frac{\partial c_2}{\partial \beta_{2s}}$$

$$v' = 0$$

$$A' = u'v + uv'$$

$$= \frac{-\lambda}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}\frac{\partial c_1}{\partial \beta_{1j}}\frac{\partial c_2}{\partial \beta_{2s}}$$

$$\frac{\partial^2 \ln L}{\partial \beta_{1j}\,\partial \beta_{2s}} = \sum_{i=1}^{n}\{-A\}$$

$$\frac{\partial^2 \ln L}{\partial \beta_{1j}\,\partial \beta_{2s}} = \sum_{i=1}^{n}\left\{\frac{\lambda}{\left(1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)\right)^2}\frac{\partial c_1}{\partial \beta_{1j}}\frac{\partial c_2}{\partial \beta_{2s}}\right\}$$

Turunan parsial kedua fungsi ln *likelihood* terhadap $\boldsymbol{\beta_{2j}}$ dan $\boldsymbol{\beta_{2s}}$

Turunan parsial kedua fungsi ln *likelihood* terhadap β_{2j} dan β_{2s} diperoleh dengan cara yang sama pada turunan parsial kedua fungsi ln *likelihood* terhadap β_{1j} dan β_{1s} sehingga diperoleh hasil sebagai berikut:

$$\frac{\partial^2 \ln L}{\partial \beta_{2j}\,\partial \beta_{2s}} = \sum_{i=1}^{n}\left\{\frac{-\mu_{i2}(1+m_2y_{i2})x_jx_s}{(1+m_2\mu_{i2})^2} - \frac{\lambda(e^{-y_{i1}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right.$$

$$\left.\frac{\partial^2 c_2}{\partial \beta_{2j}\,\partial \beta_{2s}} - \left[\frac{\lambda(e^{-y_{i2}}-c_1)}{1+\lambda(e^{-y_{i1}}-c_1)(e^{-y_{i2}}-c_2)}\right]^2\frac{\partial c_2}{\partial \beta_{2j}}\frac{\partial c_2}{\partial \beta_{2s}}\right\}$$

Lampiran 9. Penentuan Estimasi Awal Parameter-Parameter yang akan Ditaksir

Output model regresi binomial negatif univariat pada data jumlah kematian ibu dengan *software* R adalah sebagai berikut:

Melibatkan Variabel Prediktor		Tanpa Melibatkan Variabel Prediktor	
Parameter	**Estimasi**	**Parameter**	**Estimasi**
$\hat{\beta}_{10(0)}$	0.704209	$\hat{\beta}_{100(0)}$	1.7123
$\hat{\beta}_{11(0)}$	0.002270		
$\hat{\beta}_{12(0)}$	-0.012530		
$\hat{\beta}_{13(0)}$	-0.005304		
$\hat{\beta}_{14(0)}$	0.014344		
$\hat{\beta}_{15(0)}$	0.001698		
$\hat{\beta}_{16(0)}$	0.039305		
$\hat{m}_{1(0)} = 1$	1	$\hat{m}_{10(0)}$	1

Output model regresi binomial negatif univariat pada data jumlah kematian bayi dengan *software* R adalah sebagai berikut:

Melibatkan Variabel Prediktor		Tanpa Melibatkan Variabel Prediktor	
Parameter	**Estimasi**	**Parameter**	**Estimasi**
$\hat{\beta}_{20(0)}$	2.085610	$\hat{\beta}_{200(0)}$	3.4473
$\hat{\beta}_{21(0)}$	-0.011434		
$\hat{\beta}_{22(0)}$	-0.057045		
$\hat{\beta}_{23(0)}$	-0.001497		
$\hat{\beta}_{24(0)}$	0.060383		
$\hat{\beta}_{25(0)}$	0.008419		
$\hat{\beta}_{26(0)}$	0.050243		
$\hat{m}_{2(0)} = 1$	1	$\hat{m}_{20(0)}$	1

(lanjutan)

Estimasi awal dari λ diambil dari estimasi momen dengan menyamakan koefisien korelasi sampel dengan koefisien korelasi populasi.

Koefisien korelasi sampel

$$r_{y_1y_2} = \frac{s_{y_1y_2}}{s_{y_1}s_{y_2}}$$

dengan:

$$s_{y_1y_2} = \sum_{i=1}^{n} \frac{(y_{i1} - \bar{y}_1)(y_{i2} - \bar{y}_2)}{n - 1}$$

$$s_{y_1} = \sqrt{\sum_{i=1}^{n} \frac{(y_{i1} - \bar{y}_1)^2}{n - 1}}$$

$$s_{y_2} = \sqrt{\sum_{i=1}^{n} \frac{(y_{i2} - \bar{y}_2)^2}{n - 1}}$$

Koefisien korelasi populasi

$$\rho_{y_1y_2} = \frac{\sigma_{y_1y_2}}{\sigma_{y_1}\sigma_{y_2}}$$

dengan:

$$\sigma_{y_1y_2} = \lambda(c_1c_2A_1A_2)$$

$$\sigma_{y_t} = \sqrt{\frac{m_t^{-1}\theta_t}{(1 - \theta_t)^2}}$$

$$c_t = \left[\frac{1 - \theta_t}{1 - \theta_t e^{-1}}\right]^{m_t^{-1}}$$

$$A_t = \frac{m_t^{-1}\theta_t e^{-1}}{1 - \theta_t e^{-1}} - \frac{m_t^{-1}\theta_t}{1 - \theta_t}$$

$$\theta_t = \frac{\mu_{it}}{(m_t^{-1} + \mu_{it})}$$

Penyamaan koefisien korelasi sampel dan koefisien korelasi populasi

$$r_{y_1y_2} = \rho_{y_1y_2}$$

(lanjutan)

$$\frac{s_{y_1y_2}}{s_{y_1}s_{y_2}} = \frac{\sigma_{y_1y_2}}{\sigma_{y_1}\sigma_{y_2}}$$

$$\frac{s_{y_1y_2}}{s_{y_1}s_{y_2}} = \frac{\lambda(c_1c_2A_1A_2)}{\sigma_{y_1}\sigma_{y_2}}$$

$$\lambda(c_1c_2A_1A_2)s_{y_1}s_{y_2} = s_{y_1y_2}\sigma_{y_1}\sigma_{y_2}$$

$$\lambda = \frac{s_{y_1y_2}\sigma_{y_1}\sigma_{y_2}}{(c_1c_2A_1A_2)s_{y_1}s_{y_2}}$$

Nilai λ pada data jumlah kematian ibu dan bayi dengan melibatkan variabel prediktor

$$\lambda = \frac{23.5471(30.2272)(662.3277)}{(4.1736)(16.9551)(-4.5781)(-24.6929)(9.9112)(326.2536)}$$

$$= 0.0182$$

Nilai λ pada data jumlah kematian ibu dan bayi tanpa melibatkan variabel prediktor

$$\lambda = \frac{23.5471(6.1117)(72.8440)}{(2.2783)(6.0883)(-1.6957)(-7.5631)(9.9112)(326.2536)}$$

$$= 0.0182$$

Lampiran 10. *Output* Model Regresi Binomial Negatif Bivariat dengan *Software* R

Parameter	Estimasi	SE	W
$\hat{\beta}_{10}$	1.4365	0.8678	2.7398
$\hat{\beta}_{11}$	-0.0008	0.0054	0.0237
$\hat{\beta}_{12}$	-0.0161	0.0191	0.7141
$\hat{\beta}_{13}$	-0.0045	0.0041	1.1905
$\hat{\beta}_{14}$	0.0099	0.0217	0.2083
$\hat{\beta}_{15}$	0.0032	0.0027	1.4928
$\hat{\beta}_{16}$	0.0413	0.0110	14.1247
$\hat{\beta}_{20}$	2.3128	0.8553	7.3119
$\hat{\beta}_{21}$	-0.0113	0.0047	5.6311
$\hat{\beta}_{22}$	-0.0531	0.0166	10.2257
$\hat{\beta}_{23}$	-0.0002	0.0038	0.0035
$\hat{\beta}_{24}$	0.0539	0.0195	7.6035
$\hat{\beta}_{25}$	0.0090	0.0022	16.4455
$\hat{\beta}_{26}$	0.0447	0.0116	14.8900
$\widehat{m}_1$		0.0030	
$\widehat{m}_2$		0.1204	
$\hat{\lambda}$		1.6119	
$\ln L(\widehat{\Omega})$		-143.0911	
$\ln L(\widehat{\omega})$		-159.9705	
AIC		320.1821	

Lampiran 11. *Output* Model Regresi Binomial Negatif Bivariat dengan *Software* R tanpa Menyertakan Persentase Ibu Nifas Melaksanakan Program KF2.

Parameter	Estimasi	SE	W
$\hat{\beta}_{10}$	1.1812	0.9001	1.7221
$\hat{\beta}_{11}$	0.0049	0.0051	0.9165
$\hat{\beta}_{12}$	0.0030	0.0187	0.0266
$\hat{\beta}_{13}$	-0.0066	0.0041	2.6306
$\hat{\beta}_{14}$	-0.0045	0.0217	0.0439
$\hat{\beta}_{15}$	0.0359	0.0106	11.5833
$\hat{\beta}_{20}$	2.1484	1.0004	4.6116
$\hat{\beta}_{21}$	-0.0055	0.0056	0.9831
$\hat{\beta}_{22}$	-0.0556	0.0187	8.8087
$\hat{\beta}_{23}$	-0.0035	0.0041	0.7622
$\hat{\beta}_{24}$	0.0622	0.0214	8.4732
$\hat{\beta}_{25}$	0.0494	0.0129	14.7048
$\hat{m}_1$		0.0048	
$\hat{m}_2$		0.1787	
$\hat{\lambda}$		1.4622	
$\ln L(\hat{\Omega})$		-146.9819	
$\ln L(\hat{\omega})$		-159.9705	
AIC		323.9637	