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LAMPIRAN

Lampiran 1. Data Penelitian

Kabupaten/Kota	Y1	Y2	X1	X2	X3	X4	X5
Kep. Selayar	7	8	73.7	74.96	80.26	68.63	75.11
Bulukumba	4	51	80.5	81.71	77.92	73.31	86.35
Bantaeng	3	20	30.6	83.59	68.36	98.13	102.23
Jeneponto	7	69	93	71.35	76.55	74.58	94.55
Takalar	6	28	97.8	70.86	39.92	94.48	100.19
Gowa	15	41	65.7	83.74	79.45	93.29	99.51
Sinjai	4	54	72.5	74.41	85.34	87.78	97.17
Maros	4	22	79.4	72.56	64.89	91.59	94.72
Pangkep	6	58	62.1	75.04	75.56	89.09	90.82
Barru	3	10	120.2	99.17	58.52	91.98	98.18
Bone	7	57	72.3	69.78	75.21	94.12	95.47
Soppeng	3	31	67.6	76.91	81.03	77.03	87.27
Wajo	4	30	75.5	73.29	71.33	91.89	97.12
Sidrap	6	22	131.4	81.14	66.28	84.56	103.42
Pinrang	5	27	90.6	76.35	72.25	98.43	97.23
Enrekang	5	42	88.1	49.54	65.7	53.67	66.08
Luwu	10	44	81.5	68.7	73.83	78.17	86.92
Tana Toraja	3	16	83.2	76.51	58.8	84.91	94.34
Luwu Utara	5	37	60.4	71.9	86.27	82.18	94.6
Luwu Timur	6	13	81.2	78.62	67.01	88.34	92.98
Toraja Utara	5	15	66.1	72.79	60.1	71.61	74.37
Makassar	12	43	97.9	67	76.68	92.38	92.14
Pare-Pare	2	5	63.2	72.64	57.07	68.16	76.57
Palopo	1	11	67	73.82	31	86.4	93.32

Keterangan:

Y_1 : Jumlah Kematian Ibu

Y_2 : Jumlah Kematian Bayi

X_1 : Persentase penanganan komplikasi kebidanan

X_2 : Persentase peserta KB aktif

X_3 : Persentase pemberian ASI ekslusif

X_4 : Persentase cakupan pelayanan K4 ibu hamil

X_5 : Persentase persalinan yang ditolong tenaga kesehatan

Lampiran 2. Penurunan Fungsi *Likelihood* BGPR

$$Q = \ln L = \ln \prod_{i=1}^{n} \mu_0, \mu_{1i}, \mu_{2i} \exp\{-(\mu_0 + \mu_{1i} + \mu_{2i}) - y_{1i}\alpha_1 - y_{2i}\alpha_2\}$$

$$\sum_{k=0}^{min(y_{1i},y_{2i})} \frac{(\mu_{1i} + (y_{1i} - k)\alpha_1)^{y_{1i}-k-1}}{(y_{1i} - k)!} \frac{(\mu_{2i} + (y_{2i} - k)\alpha_2)^{y_{2i}-k-1}}{(y_{2i} - k)!}$$

$$\times \frac{(\mu_0 + k\alpha_0)^{k-1}}{k!} \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$$

Turunan parsial pertama dari fungsi ln *likelihood* terhadap μ_0,

$$\frac{\partial Q}{\partial \mu_0} = \frac{n}{\mu_0} - \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x_i^T \beta_1}} - \mu_0\right)} - \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x_i^T \beta_2}} - \mu_0\right)} + n + \sum_{i=1}^{n} \frac{1}{W_i} \frac{\partial W_i}{\partial \mu_0}$$

W_i diturunkan terhadap μ_0 dimana,

$$\frac{\partial W_i}{\partial \mu_0} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \left\{\frac{\partial W_{1i}}{\partial \mu_0} W_{2i} + \frac{\partial W_{2i}}{\partial \mu_0} W_{1i}\right\} \quad (1)$$

Untuk W_{1i} terhadap μ_0:

$$\frac{\partial W_{1i}}{\partial \mu_0} = \frac{-(y_{1i} - k - 1)\left(\left(e^{\mathbf{x_i^T \beta_1}} - \mu_0\right) + (y_{1i} - k)\alpha_1\right)^{y_{1i}-k-2}}{(y_{1i} - k)!} \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right) \quad (2)$$

$$\frac{\partial W_{1i}}{\partial \mu_0} = \frac{-(y_{1i} - k - 1)\left(\left(e^{\mathbf{x_i^T \beta_1}} - \mu_0\right) + (y_{1i} - k)\alpha_1\right)^{y_{1i}-k-2}}{(y_{1i} - k)!} \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$$

Untuk W_{2i} terhadap μ_0:

$$\frac{\partial W_{2i}}{\partial \mu_0} = u'v + uv'$$

dimana:

$$u = \frac{\left(\left(e^{\mathbf{x_i^T \beta_2}} - \mu_0\right) + (y_{2i} - k)\alpha_2\right)^{y_{2i}-k-1}}{(y_{2i} - k)!}$$

$$u' = \frac{-(y_{2i} - k - 1)\left(\left(e^{\mathbf{x_i^T \beta_2}} - \mu_0\right) + (y_{2i} - k)\alpha_2\right)^{y_{2i}-k-2}}{(y_{2i} - k)!}$$

$$v = \frac{(\mu_0 + k\alpha_0)^{k-1}}{k!}$$

$$v' = \frac{(k - 1)(\mu_0 + k\alpha_0)^{k-1}}{(y_{2i} - k)!}$$

Lampiran 2. Lanjutan

$$\frac{\partial W_{2i}}{\partial \mu_0} = \left[\frac{-(y_{2i}-k-1)\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-1}}{(y_{2i}-k)!}\frac{(\mu_0+k\alpha_0)^{k-1}}{k!}\right]$$

$$+\left[\frac{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-2}}{(y_{2i}-k)!}\frac{(k-1)(\mu_0+k\alpha_0)^{k-1}}{(y_{2i}-k)!}\right]$$

$$= \left[\frac{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-1}}{(y_{2i}-k)!}\frac{(\mu_0+k\alpha_0)^{k-1}}{k!}\right]+\left[\frac{-(y_{2i}-k-1)}{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)}+\frac{(k-1)}{(\mu_0+k\alpha_0)}\right] \quad (3)$$

Persamaan (2) dan (3) disubstitusikan kedalam persamaan (1):

$$\frac{\partial W_i}{\partial \mu_0} = \sum_{k=0}^{\min(y_{1i},y_{2i})}\frac{-(y_{1i}-k-1)\left(\left(e^{x_i^T\beta_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)^{y_{1i}-k-2}}{(y_{1i}-k)!}\exp\left(k(\alpha_1+\alpha_2-\alpha_0)\right)$$

$$\frac{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-1}}{(y_{2i}-k)!}\frac{(\mu_0+k\alpha_0)^{k-1}}{k!}+\frac{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-1}}{(y_{2i}-k)!}$$

$$\frac{(\mu_0+k\alpha_0)^{k-1}}{k!}\left[\frac{-(y_{2i}-k-1)}{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)}+\frac{(k-1)}{(\mu_0+k\alpha_0)}\right]$$

$$\frac{\left(\left(e^{x_i^T\beta_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}}{(y_{1i}-k)!}\exp\left(k(\alpha_1+\alpha_2-\alpha_0)\right)$$

$$= \sum_{k=0}^{\min(y_{1i},y_{2i})}\frac{\left(\left(e^{x_i^T\beta_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}}{(y_{1i}-k)!}\exp\left(k(\alpha_1+\alpha_2-\alpha_0)\right)$$

$$\frac{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-1}}{(y_{2i}-k)!}\frac{(\mu_0+k\alpha_0)^{k-1}}{k!}\left[\frac{-(y_{1i}-k-1)}{\left(\left(e^{x_i^T\beta_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)}+\left(\frac{-(y_{2i}-k-1)}{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)}+\right.\right.$$

$$\left.\left.\frac{(k-1)}{(\mu_0+k\alpha_0)}\right)\right]$$

Turunan parsial pertama dari Q terhadap μ_0 adalah

$$\frac{\partial Q}{\partial \mu_0} = \frac{n}{\mu_0}-\sum_{i=1}^{n}\frac{1}{\left(e^{x_i^T\beta_1}-\mu_0\right)}-\sum_{i=1}^{n}\frac{1}{\left(e^{x_i^T\beta_2}-\mu_0\right)}+n\sum_{i=1}^{n}\frac{1}{W_i}\frac{\partial W_i}{\partial \mu_0}$$

$$= \frac{n}{\mu_0}-\sum_{i=1}^{n}\frac{1}{\left(e^{x_i^T\beta_1}-\mu_0\right)}-\sum_{i=1}^{n}\frac{1}{\left(e^{x_i^T\beta_2}-\mu_0\right)}+n+\sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}\left\{\frac{-(y_{1i}-k-1)}{\left(\left(e^{x_i^T\beta_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)}+\right.$$

$$\left.\left(\frac{-(y_{2i}-k-1)}{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)}+\frac{(k-1)}{(\mu_0+k\alpha_0)}\right)\right\}$$

Lampiran 2. Lanjutan

Turunan parsial kedua dari Q terhadap μ_0 adalah

$$\frac{\partial^2 Q}{\partial \mu_0{}^2} = -\frac{n}{\mu_0{}^2} - \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - \mu_0\right)^2} - \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} - \mu_0\right)^2} + \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i}, y_{2i})} \left\{ \frac{-(y_{1i}-k-1)}{\left(\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^2} + \left(\frac{-(y_{2i}-k-1)}{\left(\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)^2} + \frac{(k-1)}{(\mu_0 + k\alpha_0)^2} \right) \right\}$$

Turunan parsial kedua dari Q terhadap μ_0 dan $\boldsymbol{\beta}_1^T$ adalah

$$\frac{\partial^2 Q}{\partial \mu_0 \, \partial \boldsymbol{\beta}_1^T} = -\sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - \mu_0\right)^2} \left(\exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_1\right) \mathbf{x}_i\right) + \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i}, y_{2i})} \frac{-(y_{1i}-k-1)\left(\exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_1\right) \mathbf{x}_i\right)}{\left(\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^2}$$

Turunan parsial kedua dari Q terhadap μ_0 dan $\boldsymbol{\beta}_2^T$ adalah

$$\frac{\partial^2 Q}{\partial \mu_0 \, \partial \boldsymbol{\beta}_2^T} = -\sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} - \mu_0\right)^2} \left(\exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_2\right) \mathbf{x}_i\right) + \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i}, y_{2i})} \frac{-(y_{2i}-k-1)\left(\exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_2\right) \mathbf{x}_i\right)}{\left(\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)^2}$$

Turunan parsial kedua dari Q terhadap μ_0 dan α_1 adalah

$$\frac{\partial^2 Q}{\partial \mu_0 \, \partial \alpha_1} = \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i}, y_{2i})} \frac{-(y_{1i}-k-1)}{\left(\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^2}$$

Turunan parsial kedua dari Q terhadap μ_0 dan α_2 adalah

$$\frac{\partial^2 Q}{\partial \mu_0 \, \partial \alpha_2} = \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i}, y_{2i})} \frac{-(y_{2i}-k-1)}{\left(\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)^2}$$

Turunan parsial kedua dari Q terhadap μ_0 dan α_0 adalah

$$\frac{\partial^2 Q}{\partial \mu_0 \, \partial \alpha_0} = \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i}, y_{2i})} \frac{(k-1)}{(\mu_0 + k\alpha_0)^2}$$

Turunan parsial pertama dari logaritma fungsi ln *likelihood* terhadap $\boldsymbol{\beta}_1$,

$$\frac{\partial Q}{\partial \boldsymbol{\beta}_1} = \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - \mu_0\right)} \left(\exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_1\right) \mathbf{x}_i\right) + \sum_{i=1}^{n} \left(\exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_1\right) \mathbf{x}_i\right) + \sum_{i=1}^{n} \frac{1}{W_i} \frac{\partial W_i}{\partial \boldsymbol{\beta}_1}$$

W_i diturunkan terhadap $\boldsymbol{\beta}_1$ dimana,

$$\frac{\partial W_i}{\partial \boldsymbol{\beta}_1} = \sum_{k=0}^{\min(y_{1i}, y_{2i})} \left\{ \frac{\partial W_{1i}}{\partial \boldsymbol{\beta}_1} W_{2i} + \frac{\partial W_{2i}}{\partial \boldsymbol{\beta}_1} W_{1i} \right\}$$

kemudian W_{1i} diturunkan terhadap $\boldsymbol{\beta}_1$,

$$\frac{\partial W_{1i}}{\partial \boldsymbol{\beta}_1} = \sum_{i=1}^{n} \frac{(y_{1i}-k-1)\left(\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-2} \left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\right) \mathbf{x}_i}{(y_{1i}-k)!} \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$$

kemudian W_{2i} diturunkan terhadap $\boldsymbol{\beta}_1$,

$$\frac{\partial W_{2i}}{\partial \boldsymbol{\beta}_1} = 0$$

sehingga $\frac{\partial W_i}{\partial \boldsymbol{\beta}_1} = \sum_{k=0}^{\min(y_{1i}, y_{2i})} \frac{\partial W_{1i}}{\partial \boldsymbol{\beta}_1} W_{2i}$.

Lampiran 2. Lanjutan

$$\frac{\partial W_i}{\partial \boldsymbol{\beta}_1} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{(y_{1i}-k-1)\left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)^{y_{1i}-k-2}\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i \exp(k(\alpha_1+\alpha_2-\alpha_0))}{(y_{1i}-k)!}$$

$$\times \frac{\left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-1}(\mu_0+k\alpha_0)^{k-1}}{(y_{2i}-k)!k!}$$

Maka didapatkan turunan parsial pertama dari Q terhadap $\boldsymbol{\beta}_1$

$$\frac{\partial Q}{\partial \boldsymbol{\beta}_1} = \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)}\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i + \sum_{i=1}^{n}\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i + \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}\frac{1}{W_i}\frac{\partial W_i}{\partial \boldsymbol{\beta}_1}$$

$$\frac{\partial Q}{\partial \boldsymbol{\beta}_1} = \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)}\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i + \sum_{i=1}^{n}\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i + \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}\frac{(y_{1i}-k-1)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i}{\left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)}$$

Turunan parsial kedua $\boldsymbol{\beta}_1$ terhadap $\boldsymbol{\beta}_i^T$ adalah

$$\frac{\partial^2 Q}{\partial \boldsymbol{\beta}_1\,\partial \boldsymbol{\beta}_i^T} = -\sum_{i=1}^{n}\frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)}\mathbf{x}_i\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T + \sum_{i=1}^{n}\mathbf{x}_i\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T + \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})} u'v + uv'$$

dimana

$$u = (y_{1i}-k-1)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i$$

$$u' = (y_{1i}-k-1)\mathbf{x}_i\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T$$

$$v = \left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)$$

$$v' = \left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T$$

$$\frac{\partial^2 Q}{\partial \boldsymbol{\beta}_1\,\partial \boldsymbol{\beta}_i^T} = -\sum_{i=1}^{n}\frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)}\mathbf{x}_i\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T + \sum_{i=1}^{n}\mathbf{x}_i\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T$$

$$+\sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}(y_{1i}-k-1)\mathbf{x}_i\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T\left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)+$$

$$(y_{1i}-k-1)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i\left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T$$

$$= \sum_{i=1}^{n}\mathbf{x}_i\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i^T\Bigg[-\frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)}+1+\sum_{k=0}^{\min(y_{1i},y_{2i})}(y_{1i}-k-1)\left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+\right.$$

$$\left.(y_{1i}-k)\alpha_1\right)+(y_{1i}-k-1)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)\Bigg]$$

Turunan parsial kedua $\boldsymbol{\beta}_1$ terhadap $\boldsymbol{\beta}_2^T$ adalah

$$\frac{\partial^2 Q}{\partial \boldsymbol{\beta}_1\,\partial \boldsymbol{\beta}_2^T} = 0$$

Turunan parsial kedua $\boldsymbol{\beta}_1$ terhadap α_1 adalah

$$\frac{\partial^2 Q}{\partial \boldsymbol{\beta}_1\,\partial \alpha_1} = \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}\frac{-(y_{1i}-k)(y_{1i}-k-1)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)\mathbf{x}_i}{\left(\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)^2}$$

Lampiran 2. Lanjutan

Turunan parsial kedua $\boldsymbol{\beta_1}$ terhadap α_2 adalah

$\frac{\partial^2 Q}{\partial \boldsymbol{\beta}_1 \, \partial \alpha_2} = 0$

Turunan parsial kedua $\boldsymbol{\beta_1}$ terhadap α_0 adalah

$\frac{\partial^2 Q}{\partial \boldsymbol{\beta}_1 \, \partial \alpha_0} = 0$

Turunan parsial pertama dari logaritma fungsi ln *likelihood* terhadap $\boldsymbol{\beta_2}$,

$$\frac{\partial Q}{\partial \boldsymbol{\beta_2}} = \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right)} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x_i} + \sum_{i=1}^{n} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x_i} + \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{1}{W_i} \frac{\partial W_i}{\partial \boldsymbol{\beta_2}}$$

Lampiran 2. Lanjutan

W_i diturunkan terhadap $\boldsymbol{\beta_2}$dimana,

$$\frac{\partial W_i}{\partial \boldsymbol{\beta_2}} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \left\{ \frac{\partial W_{1i}}{\partial \boldsymbol{\beta_2}} W_{2i} + \frac{\partial W_{2i}}{\partial \boldsymbol{\beta_2}} W_{1i} \right\}$$

kemudian W_{1i} diturunkan terhadap $\boldsymbol{\beta_2}$,

$$\frac{\partial W_{1i}}{\partial \boldsymbol{\beta_2}} = 0$$

kemudian W_{2i} diturunkan terhadap $\boldsymbol{\beta_2}$,

$$\frac{\partial W_{2i}}{\partial \boldsymbol{\beta_2}} = \frac{(y_{2i} - k - 1)\left(\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (y_{2i} - k)\alpha_2\right)^{y_{2i}-k-2} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x_i}(\mu_0 + k\alpha_0)}{(y_{2i} - k)!\, k!}$$

sehingga $\frac{\partial W_i}{\partial \boldsymbol{\beta_2}} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{\partial W_{2i}}{\partial \boldsymbol{\beta_2}} W_{1i}$

$$\frac{\partial W_i}{\partial \boldsymbol{\beta_2}} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{(y_{2i}-k-1)\left(\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)^{y_{2i}-k-2} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x_i}(\mu_0 + k\alpha_0)}{(y_{2i}-k)!\,k!}$$

$$\times \frac{\left(\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_1}} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}}{(y_{1i}-k)!} \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$$

Maka didapatkan turunan parsial pertama dari Q terhadap $\boldsymbol{\beta_2}$

$$\frac{\partial Q}{\partial \boldsymbol{\beta}_2} = \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right)} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x_i} + \sum_{i=1}^{n} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x_i} + \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{1}{W_i} \frac{\partial W_i}{\partial \boldsymbol{\beta_2}}$$

$$\frac{\partial Q}{\partial \boldsymbol{\beta}_2} = \sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right)} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x_i} + \sum_{i=1}^{n} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x_i} + \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{(y_{2i}-k-1)\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x_i}}{\left(\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)}$$

Turunan parsial kedua $\boldsymbol{\beta_2}$ terhadap $\boldsymbol{\beta_2^T}$ adalah

$$\frac{\partial^2 Q}{\partial \boldsymbol{\beta}_2 \, \partial \boldsymbol{\beta}_2^{\mathbf{T}}} == -\sum_{i=1}^{n} \frac{1}{\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right)} \mathbf{x_i} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x}_i^T + \sum_{i=1}^{n} \mathbf{x_i} \left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right) \mathbf{x}_i^T \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i},y_{2i})} u'v + uv'$$

dimana

$u = (y_{2i} - k - 1)\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x_i}$

Lampiran 2. Lanjutan

$u' = (\mathrm{y_{2i}} - \mathrm{k} - 1)\mathbf{x_i}\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x}_i^T$

$v = \left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (\mathrm{y_{2i}} - \mathrm{k})\alpha_2\right)$

$v' = \left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (\mathrm{y_{2i}} - \mathrm{k})\alpha_2\right)\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x}_i^T$

$$\frac{\partial^2 Q}{\partial\boldsymbol{\beta_2}\,\partial\boldsymbol{\beta_2^T}} = -\sum_{i=1}^{n}\frac{1}{\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_1}} - \mu_0\right)}\mathbf{x_i}\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_1}}\right)\mathbf{x}_i^T + \sum_{i=1}^{n}\mathbf{x_i}\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_1}}\right)\mathbf{x}_i^T$$

$$+\sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}(\mathrm{y_{2i}} - \mathrm{k} - 1)\mathbf{x_i}\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x}_i^T\left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (\mathrm{y_{2i}} - \mathrm{k})\alpha_2\right) +$$

$$(\mathrm{y_{2i}} - \mathrm{k} - 1)\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x_i}\left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (\mathrm{y_{2i}} - \mathrm{k})\alpha_2\right)\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x}_i^T$$

$$= \sum_{i=1}^{n}\mathbf{x_i}\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x}_i^T\left[-\frac{1}{\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right)} + 1 + \sum_{k=0}^{\min(y_{1i},y_{2i})}(\mathrm{y_{2i}} - \mathrm{k} - 1)\left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) +\right.\right.$$

$$\left.\left.(\mathrm{y_{2i}} - \mathrm{k})\alpha_2\right) + (\mathrm{y_{2i}} - \mathrm{k} - 1)\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (\mathrm{y_{2i}} - \mathrm{k})\alpha_2\right)\right]$$

Turunan parsial kedua $\boldsymbol{\beta_2}$ terhadap α_1 adalah

$$\frac{\partial^2 Q}{\partial\boldsymbol{\beta_2}\,\partial\alpha_1} = 0$$

Turunan parsial kedua $\boldsymbol{\beta_2}$ terhadap α_2 adalah

$$\frac{\partial^2 Q}{\partial\boldsymbol{\beta_2}\,\partial\alpha_2} = \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}\frac{-(\mathrm{y_{2i}} - \mathrm{k})(\mathrm{y_{2i}} - \mathrm{k} - 1)\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}}\right)\mathbf{x_i}}{\left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_2}} - \mu_0\right) + (\mathrm{y_{2i}} - \mathrm{k})\alpha_2\right)^2}$$

Turunan parsial kedua $\boldsymbol{\beta_2}$ terhadap α_0 adalah

$$\frac{\partial^2 Q}{\partial\boldsymbol{\beta_2}\,\partial\alpha_0} = 0$$

Turunan parsial pertama dari logaritma fungsi ln *likelihood* terhadap α_1,

$$\frac{\partial Q}{\partial\alpha_1} = -\sum_{i=1}^{n}\mathrm{y_{1i}} + \sum_{i=1}^{n}\frac{1}{\mathrm{W_i}}\frac{\partial\mathrm{W_i}}{\partial\alpha_1}$$

$\mathrm{W_i}$ diturunkan terhadap α_1 dimana

$$\frac{\partial\mathrm{W_i}}{\partial\alpha_1} = \sum_{k=0}^{\min(y_{1i},y_{2i})}\left\{\frac{\partial\mathrm{W_{1i}}}{\partial\alpha_1}\mathrm{W_{2i}} + \frac{\partial\mathrm{W_{2i}}}{\partial\alpha_1}\mathrm{W_{1i}}\right\}$$

Turunan $\mathrm{W_{1i}}$ terhadap α_1 yaitu

$$\frac{\partial\mathrm{W_{1i}}}{\partial\alpha_1} = \mathrm{u'v} + \mathrm{uv'}$$

dimana

$$\mathrm{u} = \frac{\left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_1}} - \mu_0\right) + (\mathrm{y_{1i}} - \mathrm{k})\alpha_1\right)^{\mathrm{y_{1i}} - \mathrm{k} - 1}}{(\mathrm{y_{1i}} - \mathrm{k})!}$$

$$\mathrm{u'} = \frac{(\mathrm{y_{1i}} - \mathrm{k})(\mathrm{y_{1i}} - \mathrm{k} - 1)\left(\left(\mathrm{e}^{\mathbf{x_i^T}\boldsymbol{\beta_1}} - \mu_0\right) + (\mathrm{y_{1i}} - \mathrm{k})\alpha_1\right)^{\mathrm{y_{1i}} - \mathrm{k} - 2}}{(\mathrm{y_{1i}} - \mathrm{k})!}$$

Lampiran 2. Lanjutan

$v = \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$

$v' = k\exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$

Sehingga diperoleh $\frac{\partial W_{1i}}{\partial \alpha_1}$ adalah

$$\frac{\partial W_{1i}}{\partial \alpha_1} = \frac{\left(\left(e^{x_i^T\beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-2}}{(y_{1i}-k-2)!}\exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$$

$$+\frac{\left(\left(e^{x_i^T\beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}}{(y_{1i}-k)!}k\exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$$

$$\frac{\partial W_{1i}}{\partial \alpha_1} = \left\{\frac{\left(\left(e^{x_i^T\beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}\exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)}{(y_{1i}-k)!}\right\}\left[\frac{(y_{1i}-k)(y_{1i}-k-1)}{\left(\left(e^{x_i^T\beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)} + k\right]$$

Kemudian turunan W_{2i} terhadap α_1 yaitu:

$$\frac{\partial W_{2i}}{\partial \alpha_1} = 0$$

Sehingga $\frac{\partial W_i}{\partial \alpha_1} = \sum_{k=0}^{\min(y_{1i},y_{2i})}\frac{\partial W_{1i}}{\partial \alpha_1}W_{21}$

$$\frac{\partial W_i}{\partial \alpha_1} = \sum_{k=0}^{\min(y_{1i},y_{2i})}\left\{\frac{\left(\left(e^{x_i^T\beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}\exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)}{(y_{1i}-k)!}\right\}\left[\frac{(y_{1i}-k)(y_{1i}-k-1)}{\left(\left(e^{x_i^T\beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)} + k\right]$$

$$\times\frac{\left(\left(e^{x_i^T\beta_2} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)^{y_{2i}-k-1}(\mu_0 + k\alpha_0)^{k-1}}{(y_{2i}-k)!k!}$$

Maka didapatkan turunan parsial pertama dari Q terhadap α_1

$$\frac{\partial Q}{\partial \alpha_1} = -\sum_{i=1}^{n} y_{1i} + \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}\frac{1}{W_i}\frac{\partial W_i}{\partial \alpha_1}$$

$$\frac{\partial Q}{\partial \alpha_1} = -\sum_{i=1}^{n} y_{1i} + \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}\left(\frac{(y_{1i}-k)(y_{1i}-k-1)}{\left(\left(e^{x_i^T\beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)} + k\right)$$

Turunan parsial kedua α_1 terhadap α_1 adalah

$$\frac{\partial^2 Q}{\partial {\alpha_1}^2} = \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})}\left(-\frac{(y_{1i}-k)^2(y_{1i}-k-1)}{\left(\left(e^{x_i^T\beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^2}\right)$$

Turunan parsial kedua α_1 terhadap α_2 adalah

$$\frac{\partial^2 Q}{\partial \alpha_1 \partial \alpha_2} = 0$$

Turunan parsial kedua α_1 terhadap α_0 adalah

$$\frac{\partial^2 Q}{\partial \alpha_1 \partial \alpha_0} = 0$$

Lampiran 2. Lanjutan

Turunan pertama dari logaritma fungsi ln *likelihood* terhadap α_2,

$$\frac{\partial Q}{\partial \alpha_2} = -\sum_{i=1}^{n} y_{2i} + \sum_{i=1}^{n} \frac{1}{W_i}\frac{\partial W_i}{\partial \alpha_2}$$

W_i diturunkan terhadap α_2 dimana

$$\frac{\partial W_i}{\partial \alpha_2} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \left\{\frac{\partial W_{1i}}{\partial \alpha_2} W_{2i} + \frac{\partial W_{2i}}{\partial \alpha_2} W_{1i}\right\}$$

Turunan W_{1i} terhadap α_2 yaitu

$$\frac{\partial W_{1i}}{\partial \alpha_2} = 0$$

Kemudian W_{2i} terhadap α_2 yaitu

$$\frac{\partial W_{2i}}{\partial \alpha_2} = \frac{(y_{2i}-k)(y_{2i}-k-1)\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-2}(\mu_0+k\alpha_0)^{k-1}}{(y_{2i}-k)!k!}$$

Sehingga $\frac{\partial W_i}{\partial \alpha_2} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{\partial W_{2i}}{\partial \alpha_2} W_{1i}$

$$\frac{\partial W_i}{\partial \alpha_2} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{(y_{2i}-k)(y_{2i}-k-1)\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^{y_{2i}-k-2}(\mu_0+k\alpha_0)^{k-1}}{(y_{2i}-k)!k!}$$

$$\frac{\left(\left(e^{x_i^T\beta_1}-\mu_0\right)+(y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}}{(y_{1i}-k)!} \exp\left(k(\alpha_1+\alpha_2-\alpha_0)\right)$$

Maka didapatkan turunan parsial pertama dari Q terhadap α_2

$$\frac{\partial Q}{\partial \alpha_2} = -\sum_{i=1}^{n} y_{2i} + \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{1}{W_i}\frac{\partial W_i}{\partial \alpha_2}$$

$$\frac{\partial Q}{\partial \alpha_2} = -\sum_{i=1}^{n} y_{2i} + \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{\left((y_{2i}-k-1)(y_{2i}-k)\right)}{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)}$$

Turunan parsial kedua α_2 terhadap α_2 adalah

$$\frac{\partial^2 Q}{\partial {\alpha_2}^2} = \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})} - \frac{(y_{2i}-k-1)(y_{2i}-k)^2}{\left(\left(e^{x_i^T\beta_2}-\mu_0\right)+(y_{2i}-k)\alpha_2\right)^2}$$

Turunan parsial kedua α_2 terhadap α_0 adalah

$$\frac{\partial Q}{\partial \alpha_2\,\partial \alpha_0} = 0$$

Turunan parsial pertama dari logaritma fungsi ln *likelihood* terhadap α_0,

$$\frac{\partial Q}{\partial \alpha_0} = \sum_{i=1}^{n}\sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{1}{W_i}\frac{\partial W_i}{\partial \alpha_0}$$

W_i diturunkan terhadap α_0 dimana

$$\frac{\partial W_i}{\partial \alpha_0} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \left\{\frac{\partial W_{1i}}{\partial \alpha_0} W_{2i} + \frac{\partial W_{2i}}{\partial \alpha_0} W_{1i}\right\}$$

Lampiran 2. Lanjutan

Turunan W_{1i} terhadap α_0 yaitu

$$\frac{\partial W_{1i}}{\partial \alpha_0} = -\frac{\left(\left(e^{x_i^T \beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}}{(y_{1i}-k)!} k \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$$

Turunan W_{2i} terhadap α_0 yaitu

$$\frac{\partial W_{2i}}{\partial \alpha_0} = -\frac{\left(\left(e^{x_i^T \beta_2} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)^{y_{2i}-k-2} (k-1)k(\mu_0 + k\alpha_0)^{k-2}}{(y_{2i}-k-2)!k!}$$

Sehingga $\frac{\partial W_i}{\partial \alpha_0} = \sum_{k=0}^{\min(y_{1i},y_{2i})} \left\{\frac{\partial W_{1i}}{\partial \alpha_0} W_{2i} + \frac{\partial W_{2i}}{\partial \alpha_0} W_{1i}\right\}$

$$\frac{\partial W_i}{\partial \alpha_0} = \sum_{k=0}^{\min(y_{1i},y_{2i})} -\frac{\left(\left(e^{x_i^T \beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}}{(y_{1i}-k)!} k \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right) \times$$

$$\frac{\left(\left(e^{x_i^T \beta_2} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)^{y_{2i}-k} (\mu_0 + k\alpha_0)^{k-1}}{(y_{2i}-k)!k!} +$$

$$\frac{\left(\left(e^{x_i^T \beta_2} - \mu_0\right) + (y_{2i}-k)\alpha_2\right)^{y_{2i}-k-1} (k-1)k(\mu_0 + k\alpha_0)^{k-2}}{(y_{2i}-k-2)!k!} \times$$

$$\frac{\left(\left(e^{x_i^T \beta_1} - \mu_0\right) + (y_{1i}-k)\alpha_1\right)^{y_{1i}-k-1}}{(y_{1i}-k)!} \exp\left(k(\alpha_1 + \alpha_2 - \alpha_0)\right)$$

Maka didapatkan turunan parsial pertama dari Q terhadap α_0,

$$\frac{\partial Q}{\partial \alpha_0} = \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i},y_{2i})} \frac{1}{W_i} \frac{\partial W_i}{\partial \alpha_0}$$

$$\frac{\partial Q}{\partial \alpha_0} = \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i},y_{2i})} \left(-k + \frac{k(k-1)}{(\mu_0 + k\alpha_0)}\right)$$

Turunan parsial kedua α_0 terhadap α_0 adalah

$$\frac{\partial Q}{\partial {\alpha_0}^2} = \sum_{i=1}^{n} \sum_{k=0}^{\min(y_{1i},y_{2i})} \left(-\frac{k^2(k-1)}{(\mu_0 + k\alpha_0)^2}\right)$$

Lampiran 3. Syntax R Estimasi Parameter Model BGPR

```
BGPR=function(data,alfa0,maxit,epsilon)
{
  library(pracma)
  library(MASS)

  n=nrow(data)
  y1=as.matrix((data[,1]))
  y2=as.matrix((data[,2]))
  x=data[,-c(1,2,3,5,6,7)]

  #inisialisasi parameter
  f1=glm(formula=y1~x,family=quasipoisson(link="log"))
  f2=glm(formula=y2~x,family=quasipoisson(link="log"))
    beta10=f1$coefficients
    beta20=f2$coefficients
    x=as.matrix(cbind(rep(1,n),x))
    p=ncol(x)
    miu10=exp((x)%*%beta10)
    miu20=exp((x)%*%beta20)
    alfa1=summary(f1)$dispersion
    alfa2=summary(f2)$dispersion
    alfa012=as.matrix(c(alfa1,alfa2,alfa0))

    miu0=cov(y1,y2)

    rownames(alfa012)<-c('alfa1','alfa2','alfa0')
    start=as.matrix(c(beta10,beta20,miu0,alfa012))

    Q_BGPR=function(par)
    {
      beta1=as.matrix(par[1:p])
      beta2=as.matrix(par[(p+1):(2*p)])
      miu0=par[2*p+1]
      miu1=exp(x%*%beta1)
      miu2=exp(x%*%beta2)
      alfa0=par[2*p+2]
      alfa1=par[2*p+3]
      alfa2=par[2*p+4]
      A=matrix(nrow=n,ncol=1)
      for (i in 1:n)
      {
        A1=log(miu0*miu1[i]*miu2[i])+((-(miu0+miu1[i]+miu2[i])-
(y1[i]*alfa1)-(y2[i]*alfa2)))
        kk=min(y1[i],y2[i])
        B4=matrix(ncol=1,nrow=kk+1)
        for (k in 0:kk)
        {
          B1=(factorial(y1[i]-k))+log((factorial(y2[i]-
k))*(factorial(k)))
          B2=((y1[i]-k-1)*log(miu1[i]+(y1[i]-k)*alfa1))+((y2[i]-
k-1)*log(miu2[i]+(y2[i]-k)*alfa2))
          B3=((k-1)*log(miu0+k*alfa0)+((k*alfa1+alfa2-alfa0)))
          B4[k+1]=(B2+B3)-B1
        }
        A[i]=A1+sum(B4)
      }
```

Lampiran 3. Lanjutan

```
Q=sum(A);
      print(Q)
      return(Q)
    }
    #Syntax Tampilan 1
    Koefisien=matrix(0,ncol=1,nrow=2*p+4)
    Std.Error=matrix(0,ncol=1,nrow=2*p+4)
Z.Value=matrix(0,ncol=1,nrow=2*p+4)
    P.Value=matrix(0,ncol=1,nrow=2*p+4)
    UjiSerentak=data.frame(matrix(0,ncol=1,nrow=9))

    #Optimasi
    fit=optim(par=start,fn=Q_BGPR,method="Nelder-
Mead",control=list(maxit=maxit,fnscale=-1,
        trace=0,REPORT=0,reltol=epsilon,abstol=epsilon),hessian=T)

    #Mengambil Nilai-Nilai Hasil Optimasi
    Koefisien=round(fit$par,4)
    hess=fit$hessian
    n.iteration=fit$diskrits[1]
    convergence=ifelse(fit$convergen==0,"Converged","Not-Converged")

    #Uji Parsial Koefisien
    inv.hess=diag(pinv(-hess))
    Std.Error=round(as.matrix(sqrt(abs(inv.hess))),4)
    Z.Value=round(Koefisien/Std.Error,4)
    P.Value=round(2*pnorm(abs(Z.Value),lower.tail=FALSE),4)

    #Syntax Tampilan 2
    rownames(Koefisien)=c(paste("Beta1",c(0:(p-1)),sep=" "),
                          paste("Beta2",c(0:(p-1)),sep=" "),
                          "Lamda0",paste("Alfa",c(0:2),sep=" "))
    rownames(Std.Error)=c(paste("Beta1",c(0:(p-1)),sep=" "),
                          paste("Beta2",c(0:(p-1)),sep=" "),
                          "Lamda0",paste("Alfa",c(0:2),sep=" "))
    rownames(Z.Value)=c(paste("Beta1",c(0:(p-1)),sep=" "),
                          paste("Beta2",c(0:(p-1)),sep=" "),
                          "Lamda0",paste("Alfa",c(0:2),sep=" "))
    rownames(P.Value)=c(paste("Beta1",c(0:(p-1)),sep=" "),
                          paste("Beta2",c(0:(p-1)),sep=" "),
                          "Lamda0",paste("Alfa",c(0:2),sep=" "))

    #Uji Serentak dengan G^2
    par0=as.matrix(rep(0,length(start)));

par0[c(1,(p+1),(2*p+1):(2*p+4))]=Koefisien[c(1,(p+1),(2*p+1):(2*p+4))]
    ln.H1=round(fit$value,3)
    ln.H0=round(Q_BGPR(par0),3)
    G2=round(-2*(ln.H0-ln.H1),4)
    v=2*(p-2)
    pvalF=round(pchisq((G2),v,lower.tail=FALSE),5)

    #Estimasi Y-hat BGPR
    y1hat=round((exp(x%*%as.matrix(Koefisien[1:p]))),3)
    y2hat=round((exp(x%*%as.matrix(Koefisien[(p+1):(2*p)]))),3)
```

Lampiran 3. Lanjutan

```
    #Estimasi Regresi (Untuk Pembanding)
    beta1.reg=as.matrix(lm(y1~x-1)$coef)
    beta2.reg=as.matrix(lm(y2~x-1)$coef)
    Y1.Reg=as.matrix(x)%*%beta1.reg
    Y2.Reg=as.matrix(x)%*%beta2.reg
#AIC
    error1=as.matrix(y1-y1hat)
    error2=as.matrix(y2-y2hat)
    E=cbind(error1,error2)
Sigma.d=(t(E)%*%E)/n
    detD=det(Sigma.d)
    aic=round((n*log(detD))-(2*2*p),3)
    aic.reg=round((n*log(det((t(cbind(as.matrix(y1-
round(Y1.Reg))
            ,as.matrix(y2-round(Y2.Reg))))%*%cbind(as.matrix(y1-
round(Y1.Reg)),
            as.matrix(y2-round(Y2.Reg))))/n)))-(2*2*p),3)
    aic.pois.reg=round((n*log(det((t(cbind(as.matrix(y1-
                round(miu10)),as.matrix(y2-
                round(miu20))))%*%cbind(as.matrix(y1
                -round(miu10)),as.matrix(y2-
                round(miu20))))/n)))-(2*2*p),3)

    #Syntax Tampilan 3

UjiSerentak=data.frame(cbind(n.iteration,convergence,ln.H1,ln.H0
,
                G2,pvalF,aic,aic.reg,aic.pois.reg),row.names =
NULL)
    UjiSerentak=rbind(n.iteration,convergence,ln.H1,ln.H0,
                    G2,pvalF,aic,aic.reg,aic.pois.reg)
    colnames(UjiSerentak)=list(c("Number of
Iteration","Converged/Not","lm.H1","ln.H0","G^2","P.Value of F",
                            "AIC BGPR","AIC Regression","AIC
Poisson Regression"))
    colnames(UjiSerentak)="Value"
    UjiSerentak=noquote(UjiSerentak)


Hasil=data.frame(cbind(y1,round(Y1.Reg),round((y1hat),3),rep("|"
,nrow(x)),y2,round(Y2.Reg),round((y2hat),3)))

colnames(Hasil)=(c('Y1','Y1.Reg','Y1.BGPR','|','Y2','Y2.Reg','Y2
.BGPR'))


UjiParsial=data.frame(cbind(Koefisien,Std.Error,Z.Value,P.Value)
,row.names=NULL)


colnames(UjiParsial)=c('Koefisien','Std.Error','Z.Value','P.Valu
e')
    rownames(UjiParsial)=c(paste("Beta1",c(0:(p-1)),sep=" "),
                           paste("Beta2",c(0:(p-1)),sep=" "),
                           "Lamda0",paste("Alfa",c(0:2),sep="
"))
```

Lampiran 3. Lanjutan

```
print(UjiParsial)
    print(UjiSerentak)
    print(Hasil)


list(Y1.hat=y1hat,y2=y2hat,Hasil=Hasil,Koefisien=Koefisien,Std.E
rror=Std.Error,

Z.Value=Z.Value,P.Value=P.Value,UjiSerentak=UjiSerentak,AIC=aic,
Error1=error1,
         Error2=error2)
}
```