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LAMPIRAN

Lampiran 1 Penentuan Titik Kesetimbangan Endemik

Titik kesetimbangan endemik merupakan suatu keadaan dimana penyakit demam tifoid menyebar pada suatu populasi. Kondisi ini terjadi ketika ${I_c}^* > 0,\ I^* > 0,\ C^* > 0$ dan $B^* > 0$, sehingga dengan menggunakan persamaan $(4.3) - (4.8)$ diperoleh:

$$R^* = \frac{(L_1L_2 + \mu)S - \Lambda}{\varepsilon}\ , \qquad (4.9a)$$

$$S^* = \frac{L_3I_c}{L_1L_2\rho}, \qquad (4.9b)$$

$$I_c^* = \frac{L_4I - L_1L_2L_8S}{L_9}\ , \qquad (4.9c)$$

$$C = \frac{\beta I}{l_6}, \qquad (4.9d)$$

$$R^* = \frac{\phi I_c + L_{10}I + L_{11}C}{L_7}, \qquad (4.9e)$$

$$B^* = \frac{\eta_1 I_c + \eta_2 I + \eta_3 C}{\mu_b}. \qquad (4.9f)$$

Subtitusi persamaan $(4.9a)$ dan $(4.9d)$ ke persamaan $(4.9e)$ dan selesaikan untuk S diperoleh

$$\frac{(L_1L_2 + \mu)S - \Lambda}{\varepsilon} = \frac{\phi I_c + L_{10}I + L_{11}\left(\frac{\beta I}{l_6}\right)}{L_7}\ ,$$

$$S^* = \frac{\varepsilon L_6\phi I_c + \varepsilon(L_6L_{10} + \beta L_{11})I + L_6L_7\Lambda}{L_6L_7(L_1L_2 + \mu)}. \qquad (4.9g)$$

Subtitusi $(4.9b)$ ke $(4.9g)$ kemudian selesaikan untuk I_c diperoleh

$$\frac{L_3I_c}{L_1L_2\rho} = \frac{\varepsilon L_6\phi I_c + \varepsilon(L_6L_{10} + \beta L_{11})I + L_6L_7\Lambda}{L_6L_7(L_1L_2 + \mu)},$$

$$L_3L_6L_7(L_1L_2 + \mu)I_c = L_1L_2\rho(\varepsilon L_6\phi I_c + \varepsilon(L_6L_{10} + \beta L_{11})I + L_6L_7\Lambda),$$

$$(L_3L_6L_7(L_1L_2 + \mu) - L_1L_2\rho\varepsilon L_6\phi)I_c = L_1L_2\rho\varepsilon(L_6L_{10} + \beta L_{11})I + L_1L_2\rho L_6L_7\Lambda,$$

$$I_c^* = \frac{L_1L_2\rho\varepsilon(L_6L_{10} + \beta L_{11})I + L_1L_2\rho L_6L_7\Lambda}{(L_3L_6L_7(L_1L_2 + \mu) - L_1L_2\rho\varepsilon L_6\phi)}. \qquad (4.9h)$$

Subtitusi $(4.9b4.10)$ ke persamaan $(4.9c4.11)$

$$I_c^* = \frac{L_4 I - L_1 L_2 L_8 (\frac{L_3 I_c}{L_1 L_2 \rho})}{L_9},$$

$$I_c^* = \frac{L_4 \rho I}{L_9 \rho + L_8 L_3}. \qquad (4.9i)$$

Subtitusi $(4.9h)$ ke $(4.9i)$ dan selesaikan untuk (I) diperoleh

$$\frac{L_1 L_2 \rho \varepsilon (L_6 L_{10} + \beta L_{11}) I + L_1 L_2 \rho L_6 L_7 \Lambda}{(L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 \rho \varepsilon L_6 \phi)} = \frac{L_4 \rho I}{L_9 \rho + L_8 L_3},$$

$L_1 L_2 \rho \varepsilon (L_6 L_{10} + \beta L_{11})(L_9 \rho + L_8 L_3) I + L_1 L_2 \rho L_6 L_7 \Lambda (L_9 \rho + L_8 L_3) = L_4 \rho (L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 \rho \varepsilon L_6 \phi) I,$

$$I^* = \frac{L_1 L_2 L_6 L_7 \Lambda (L_9 \rho + L_8 L_3)}{L_4 L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 L_4 \rho \varepsilon L_6 \phi - L_1 L_2 \varepsilon (L_6 L_{10} + \beta L_{11})(L_9 \rho + L_8 L_3))}. \qquad (4.9j)$$

Subtitusi $(4.9i)$ ke $(4.9j)$ dan selesaikan untuk (I_c) diperoleh

$$I_c^* = \frac{L_4 \rho (\frac{L_1 L_2 L_6 L_7 \Lambda (L_9 \rho + L_8 L_3)}{L_4 L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 L_4 \rho \varepsilon L_6 \phi - L_1 L_2 \varepsilon \left(L_6 L_{10} + \beta L_{11}\right)(L_9 \rho + L_8 L_3)})}{L_9 \rho + L_8 L_3},$$

$$I_c^* = \frac{L_4 \rho L_1 L_2 L_6 L_7 \Lambda}{L_4 L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 L_4 \rho \varepsilon L_6 \phi - L_1 L_2 \varepsilon \left(L_6 L_{10} + \beta L_{11}\right)(L_9 \rho + L_8 L_3)}. \qquad (4.9k)$$

Subtitusi $(4.9k)$ke $(4.9d)$ dan selesaikan untuk (C) diperoleh

$$C^* = \frac{\beta (\frac{L_1 L_2 L_6 L_7 \Lambda (L_9 \rho + L_8 L_3)}{L_4 L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 L_4 \rho \varepsilon L_6 \phi - L_1 L_2 \varepsilon \left(L_6 L_{10} + \beta L_{11}\right)(L_9 \rho + L_8 L_3))})}{l_6},$$

$$C^* = \frac{\beta L_1 L_2 \quad L_7 \Lambda (L_9 \rho + L_8 L_3)}{L_4 L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 L_4 \rho \varepsilon L_6 \phi - L_1 L_2 \varepsilon \left(L_6 L_{10} + \beta L_{11}\right)(L_9 \rho + L_8 L_3)}. \qquad (4.9l)$$

Subtitusi persamaan $(4.9k)$ ke persamaan $(4.9b)$ diperoleh

$$S^* = \frac{L_3 \left(\frac{L_4 \rho L_1 L_2 L_6 L_7 \Lambda}{L_4 L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 L_4 \rho \varepsilon L_6 \phi - L_1 L_2 \varepsilon \left(L_6 L_{10} + \beta L_{11}\right)(L_9 \rho + L_8 L_3)}\right)}{L_1 L_2 \rho},$$

$$S^* = \frac{L_3 L_4 L_7 \Lambda}{L_4 L_3 L_6 L_7 (L_1 L_2 + \mu) - L_1 L_2 L_4 \rho \varepsilon L_6 \phi - L_1 L_2 \varepsilon \left(L_6 L_{10} + \beta L_{11}\right)(L_9 \rho + L_8 L_3)}. \qquad (4.9m)$$

Subtitusi persamaan $(4.9m)$ ke persamaan $(4.9a)$ diperoleh

$$R^* = \frac{(L_1L_2+\mu)S-\Lambda}{\varepsilon}$$

$$R^* = \frac{L_3L_4L_7\Lambda(L_1L_2+\mu)}{\varepsilon L_4L_3L_6L_7(L_1L_2+\mu)-\varepsilon L_1L_2L_4\rho\varepsilon L_6\phi-\varepsilon L_1L_2\varepsilon\left(L_6L_{10}+\beta L_{11}\right)(L_9\rho+L_8L_3)}-\frac{\Lambda}{\varepsilon} \quad (4.9n)$$

Jadi diperoleh titik kesetimbangan endemik

$$E^* = (S^*, \quad I_c{}^*, \quad I^*, \quad C^*, \quad B^*, \quad R^*)$$

Dengan:

$$S^* = \frac{L_3L_4L_7\Lambda}{L_4L_3L_6L_7(L_1L_2+\mu)-L_1L_2L_4\rho\varepsilon L_6\phi-L_1L_2\varepsilon\left(L_6L_{10}+\beta L_{11}\right)(L_9\rho+L_8L_3)}$$

$$I_c^* = \frac{L_4\rho L_1L_2L_6L_7\Lambda}{L_4L_3L_6L_7(L_1L_2+\mu)-L_1L_2L_4\rho\varepsilon L_6\phi-L_1L_2\varepsilon(L_6L_{10}+\beta L_{11})(L_9\rho+L_8L_3)},$$

$$I^* = \frac{L_1L_2L_6L_7\Lambda(L_9\rho+L_8L_3)}{L_4L_3L_6L_7(L_1L_2+\mu)-L_1L_2L_4\rho\varepsilon L_6\phi-L_1L_2\varepsilon(L_6L_{10}+\beta L_{11})(L_9\rho+L_8L_3)},$$

$$C^* = \frac{\beta L_1L_2 \quad L_7\Lambda(L_9\rho+L_8L_3)}{L_4L_3L_6L_7(L_1L_2+\mu)-L_1L_2L_4\rho\varepsilon L_6\phi-L_1L_2\varepsilon(L_6L_{10}+\beta L_{11})(L_9\rho+L_8L_3)},$$

$$R^* = \frac{L_3L_4L_7\Lambda(L_1L_2+\mu)}{\varepsilon L_4L_3L_6L_7(L_1L_2+\mu)-\varepsilon L_1L_2L_4\rho\varepsilon L_6\phi-\varepsilon L_1L_2\varepsilon\left(L_6L_{10}+\beta L_{11}\right)(L_9\rho+L_8L_3)}-\frac{\Lambda}{\varepsilon},$$

$$B^* = \frac{\eta_1 I^*{}_c+\eta_2 I^*+\eta_3 C^*}{\mu_b}.$$

Lampiran 2 **Listing Program** Perbandingan Solusi Sistem Tanpa Kontrol dan dengan Kontrol Optimal Menggunakan Matlab R2015a

Program Utama

```
clear
clc
clear all
global a1 a2 a3 w1 w2 w3 zeta epsilon nu phi eta1 eta2 eta3
alpha delta miu kappa rho tau miub theta beta

%Nilai parameter model
zeta=100; phi=0.0003; epsilon=0.000904 ;nu=0.9;
eta1=0.9; eta2=0.8; eta3=0.01; alpha=0.04; delta=0.0052;
miu=0.0417; kappa=50000; tau=0.0657;  miub=0.001; theta=0.2
;beta=0.04;
rho=0.3;

% Nilai awal state
a1=50; a2=50; a3=50; w1=9; w2=7; w3=10;
x10=3000;
x20=150;
x30=350;
x40=20;
x50=450;
x60=5000;

x0=[x10;x20;x30;x40;x50;x60];
%Nilai akhir Costate (syarat transversalitas)
nx=6;
lambdaT=zeros(nx,1);
%Interval waktu
Ntime=10000;
tf=10;
ti=linspace(0,tf,Ntime);
%Batas kontrol
M1=0;
M2=1;
nv=3;
Lb=M1.*ones(nv,Ntime);
Ub=M2.*ones(nv,Ntime);
%Parameter Sweep
test=-1;
deltaa=0.0001;
k=0;
%tebakan awal untuk fungsi kontrol u1, u2 dan u3
u=0*ones(nv,Ntime);
%solving sistem tanpa kontrol (u1=0 u2=0 u3=0)
options=odeset('AbsTol',1e-6,'RelTol',1e-6);
xc=ode45(@(t,x) syam_state2(t, x, u, ti),[0 tf],x0,options);
xc=deval(xc,ti);
%Awal Metode Sweep
```

```
x=zeros(nx,Ntime);
p=zeros(nx,Ntime);
while(test<0)
 k=k+1;
 oldx=x;
 oldp=p;
 oldu=u;
 %Forward Runge Kutta
 x=deval(ode45(@(t,x) syam_state2(t, x, u, ti), [0 tf],
x0),ti);
 %Backward Runge Kutta %yg digunakan nilai akhir lambdaT=0
 p=deval(ode45(@(t,p) syam_costate2(t, p, x, u, ti),[tf
0],lambdaT),ti);
 %menghitung nilai u dari syarat optimal sistem
 u1=syam_kontrol2(x,p); %menggunakan u dH/du=0
 %membuat u berada dalam interval yang diharapkan
 u1=syam_f_simplebounds2(u1,Lb,Ub);
 %mengupdate nilau u dalam metode sweep menggunakan
kombinasi konveks
 u=0.5*(u1+oldu); %uji Konvergensi u yang pertama
 %u=u1.*(1-c.^k)+oldu.*c^k;
 %menghitung nilai error
 temp1=deltaa*sum(abs(u))-sum(abs(oldu-u));
 temp2=deltaa*sum(abs(x))-sum(abs(oldx-x));
 temp3=deltaa*sum(abs(p))-sum(abs(oldp-p));

 test=min(temp1,min(temp2,temp3));
 %Buku Lenhart Hal:55
 %menghitung nilai fungsi tujuan menggunakan u akhir
 J(k)=syam_objektif2(x,u,ti);
 disp(['it:',num2str(k),',Test:',num2str(test)])
end
[m,n]=size(J);
Ju=syam_objektif2(x,u,ti);

XC2 = sum(xc(2,1:end-1)).*tf./Ntime;
disp (XC2)
XC3 = sum(xc(3,1:end-1)).*tf./Ntime;
disp (XC3)
XC4 = sum(xc(4,1:end-1)).*tf./Ntime;
disp (XC4)

XK2 = sum(x(2,1:end-1)).*tf./Ntime;
disp (XK2)
XK3 = sum(x(3,1:end-1)).*tf./Ntime;
disp (XK3)
XK4 = sum(x(4,1:end-1)).*tf./Ntime;
disp (XK4)

%menghitung nilai fungsi tujuan menggunakan u optimal
figure (1)
```

```
subplot (3,2,1)
plot(ti,xc(1,:),'R--',ti, x(1,:),'b-','LineWidth',2)
xlabel({'Waktu (Bulan)','(a)'})
ylabel('Populasi (S(t))')
legend('Tanpa Kontrol','Dengan Kontrol')
axis('tight')
grid on
hold on

subplot (3,2,2)
plot(ti,xc(2,:),'R--',ti, x(2,:),'b-','LineWidth',2)
xlabel({'Waktu (Bulan)','(b)'})
ylabel('Populasi (I_c(t))')
axis('tight')
grid on
hold on

subplot (3,2,3)
plot(ti,xc(3,:),'R--',ti, x(3,:),'b-','LineWidth',2)
xlabel({'Waktu (Bulan)','(c)'})
ylabel('Populasi (I(t))')
axis('tight')
grid on
hold on

subplot (3,2,4)
plot(ti,xc(4,:),'R--',ti, x(4,:),'b-','LineWidth',2)
xlabel({'Waktu (Bulan)','(d)'})
ylabel('Populasi (C(t))')
axis('tight')
grid on
hold on

subplot (3,2,5)
plot(ti,xc(5,:),'R--',ti, x(5,:),'b-','LineWidth',2)
xlabel({'Waktu (Bulan)','(e)'})
ylabel('Populasi (R(t))')
axis('tight')
grid on
hold on

subplot (3,2,6)
plot(ti,xc(6,:),'R--',ti, x(6,:),'b-','LineWidth',2)
xlabel({'Waktu (Bulan)','(f)'})
ylabel('Populasi (B(t))')
axis('tight')
grid on
hold on

figure (2)
```

```
plot(ti,u(1,:),'b-',ti,u(2,:),'black-',ti,u(3,:),'r-
','LineWidth',2)
legend('u_1 (Kampanye Kesehatan)','u_2 (Screening)','u_3
(Pengobatan)')
xlabel('Waktu (Bulan)')
ylabel('u_1(t),u_2(t),u_3(t)')
title('Fungsi Kontrol')
axis('tight')
grid on
```

_State

```
function dx=syam_state2(t, x, u, ti) %ti adalah inputan
%t,x,u adalah variabel
global zeta epsilon tau nu phi eta1 eta2 eta3 alpha delta
miu kappa rho  miub theta beta
x1=x(1);
x2=x(2);
x3=x(3);
x4=x(4);
x5=x(5);
x6=x(6);

u1=u(1,:);
u1=interp1(ti,u1',t);
u2=u(2,:);
u2=interp1(ti,u2',t);
u3=u(3,:);
u3=interp1(ti,u3',t);

dx=zeros(6,1);

dx(1)=zeta+epsilon.*x5-miu.*x1-(((1-
u1).*x1.*nu.*x6)./(kappa+x6));
dx(2)=((1-u1).*rho.*nu.*x6.*x1./(kappa+x6))-
(phi+u2+theta+eta1+miu).*x2;
dx(3)=(1-u2).*theta.*x2-
(tau+u3+beta+eta2+delta+miu).*x3+(((1-u1).*(1-
rho).*nu.*x6.*x1)./(kappa+x6));
dx(4)=beta.*x3-(alpha+u3+eta3+miu).*x4;
dx(5)=phi.*x2+(tau+u3).*x3+(alpha+u3).*x4-(epsilon+miu).*x5;
dx(6)=eta1.*x2+eta2.*x3+eta3.*x4-miub.*x6;
end
```

_costate

```
function dp=syam_costate2(t, p, x, u, ti)
global a1 a2 a3 alpha nu miu rho  phi delta tau eta1 eta2
eta3 epsilon miub kappa theta beta
x = interp1(ti,x',t);

x1 = x(1);
```

```
x6 = x(6);

u1 = u(1,:);
u2 = u(2,:);
u3 = u(3,:);

u1 = interp1(ti,u1',t);
u2 = interp1(ti,u2',t);
u3 = interp1(ti,u3',t);

p1=p(1,:);
p2=p(2,:);
p3=p(3,:);
p4=p(4,:);
p5=p(5,:);
p6=p(6,:);

dp=zeros(6,1);
dp(1)=p1.*miu+(((p1-p3).*nu.*x6+(p3-p1).*nu.*u1.*x6+(p3-
p2).*rho.*nu.*x6+(p2-p3).*rho.*nu.*u1.*x6)./(kappa+x6));
dp(2)=(p2-p3).*theta+(p2-p5).*phi+(p2-
p6).*eta1+p2.*u2+p3.*theta.*u2+p2.*miu-a1;
dp(3)=(p3-p4).*beta+(p3-p5).*tau+(p3-p6).*eta2+(p3-
p5).*u3+p3.*(delta+miu)-a2;
dp(4)=(p4-p5).*alpha+(p4-p6).*eta3+(p4-p5).*u3+p4.*miu-a3;
dp(5)=(p5-p1).*epsilon+p5.*miu;
dp(6)=p6.*miub+(((p1-p3).*nu.*kappa.*x1+(p3-
p2).*rho.*nu.*kappa.*x1+(p3-p1).*u1.*nu.*kappa.*x1+(p2-
p3).*u1.*rho.*nu.*kappa.*x1)./(kappa+x6).^2);

end
```

_fungsi objektif

```
function J=syam_objektif2(x,u,ti)
global a1 a2 a3 w1 w2 w3
x2=x(2,:);
x3=x(3,:);
x4=x(4,:);

u1=u(1,:);
u2=u(2,:);
u3=u(3,:);
obj=a1.*x2+a2.*x3+a3.*x4+(1./2).*(w1.*u1.^2
+w2.*u2.^2+w3.*u3.^2);
J=trapz(ti,obj);
```

_fungsi Kontrol

```
function u = syam_kontrol2(x,p)
global w1 w2 w3 nu kappa tau rho theta beta
```

```
p1 = p(1,:);
p2 = p(2,:);
p3 = p(3,:);
p4 = p(4,:);
p5 = p(5,:);

x1 = x(1,:);
x2 = x(2,:);
x3 = x(3,:);
x4 = x(4,:);
x6 = x(6,:);
u1 =zeros(1,10000);
%u1=((p3-p1).*nu.*x6.*x1+(p2-
p3).*rho.*nu.*x6.*x1)./((kappa+x6).*w1);
u2=((p3.*theta+p2).*x2)./w2;
%u2=zeros(1,10000);
u3=zeros(1,10000);
%u3=((p4-p5).*x4+(p3-p5).*x3)./w3;
u=[u1;u2;u3];
end
```

_Simplebound

```
function s=syam_f_simplebounds2(s,Lb,Ub)

% untuk batas bawah
 ns_tmp=s;
 Z=ns_tmp<Lb;
 ns_tmp(Z)=Lb(Z);

 % untuk batas atas
 J=ns_tmp>Ub;
 ns_tmp(J)=Ub(J);

 % Update u
 s=ns_tmp;
```

Lampiran 3 Simulasi Maple

> restart
> with(linalg) :
> with(DEtools) :
> $\Lambda := 100; \nu := 0.9; \kappa := 50000; \mu := 0.0247; \delta := 0.052; \phi := 0.0003; \rho := 0.3; \eta_1 := 0.9; \eta_2 := 0.8; \eta_3 := 0.01; u_1 := 0; u_2 := 0; u_3 := 0; \varepsilon := 0.000904; \beta := 0.04; \alpha := 0.04; \tau := 0.0657; \sigma := 0.001; \theta := 0.2$

> $P1 := \Lambda + \varepsilon \cdot R - \left((1 - u_1) \cdot \frac{\nu \cdot B}{\kappa + B} \cdot S \right) - \mu \cdot S$

$$P1 := 100 + 0.000904\, R - \frac{0.9\, B\, S}{50000 + B} - 0.0247\, S \quad (2)$$

> $P2 := (1 - u_1) \cdot \frac{\rho \cdot \nu \cdot B}{\kappa + B} \cdot S - (\phi + u_2 + \theta + \eta_1 + \mu) \cdot I_c$

$$P2 := \frac{0.27\, B\, S}{50000 + B} - 1.1250\, I_c \quad (3)$$

> $P3 := (1 - u_1) \cdot \frac{(1 - \rho) \cdot \nu \cdot B}{\kappa + B} \cdot S + (1 - u_2) \cdot \theta \cdot I_c - (\tau + u_3 + \beta + \eta_2 + \delta + \mu) \cdot I$

$$P3 := \frac{0.63\, B\, S}{50000 + B} + 0.2\, I_c - 0.9824\, I \quad (4)$$

> $P4 := \beta \cdot I - (\alpha + u_3 + \eta_3 + \mu) \cdot C$

$$P4 := 0.04\, I - 0.0747\, C \quad (5)$$

> $P5 := \phi \cdot I_c + (\tau + u_3) \cdot I + (\alpha + u_3) \cdot C - (\varepsilon + \mu) \cdot R$

$$P5 := 0.0003\, I_c + 0.0657\, I + 0.04\, C - 0.025604\, R \quad (6)$$

> $P6 := \eta_1 \cdot I_c + \eta_2 \cdot I + \eta_3 \cdot C - \sigma \cdot B$

$$P6 := -0.001\, B + 0.01\, C + 0.8\, I + 0.9\, I_c \quad (7)$$

>
>

T=TITIK KESETIMBANGAN

> $T := solve(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, [S, I_c, I, C, R, B])$

$$T := [[S = 4048.582996, I_c = 0., I = 0., C = 0., R = 0., B = 0.], [S = 171.4404444, I_c = 25.59789562, I = 73.60958380, C = 39.41610913, R = 250.7605604, B = 82319.93419]] \quad (8)$$

[T1 TITIK KESETIMBANGAN NON ENDEMIK] DAN {T2 TITIK KESETIMBANGAN ENDEMIK}

>

> $T1 := T[1]; T2 := T[2]$

$$T1 := [S = 4048.582996, I_c = 0., I = 0., C = 0., R = 0., B = 0.]$$

$$T2 := [S = 171.4404444, I_c = 25.59789562, I = 73.60958380, C = 39.41610913, R = 250.7605604, B = 82319.93419] \quad (9)$$

MATRIKS JACOBIAN

> $JCOB := jacobian(vector([P1, P2, P3, P4, P5, P6]), [S, I_c, I, C, R, B]);$

$$JCOB := \begin{bmatrix} -\frac{0.9\, B}{50000 + B} - 0.0247 & 0 & 0 & 0 & 0.000904 & -\frac{0.9\, S}{50000 + B} + \frac{0.9\, B\, S}{(50000 + B)^2} \\ \frac{0.27\, B}{50000 + B} & -1.1250 & 0 & 0 & 0 & \frac{0.27\, S}{50000 + B} - \frac{0.27\, B\, S}{(50000 + B)^2} \\ \frac{0.63\, B}{50000 + B} & 0.2 & -0.9824 & 0 & 0 & \frac{0.63\, S}{50000 + B} - \frac{0.63\, B\, S}{(50000 + B)^2} \\ 0 & 0 & 0.04 & -0.0747 & 0 & 0 \\ 0 & 0.0003 & 0.0657 & 0.04 & -0.025604 & 0 \\ 0 & 0.9 & 0.8 & 0.01 & 0 & -0.001 \end{bmatrix} \quad (10)$$

>

KESTABILAN NON ENDEMIK

> $JCOB1 := subs(T1, evalm(JCOB));$

$$JCOB1 := \begin{bmatrix} -0.0247 & 0 & 0 & 0 & 0.000904 & -0.07287449393 \\ 0. & -1.1250 & 0 & 0 & 0 & 0.02186234818 \\ 0. & 0.2 & -0.9824 & 0 & 0 & 0.05101214575 \\ 0 & 0 & 0.04 & -0.0747 & 0 & 0 \\ 0 & 0.0003 & 0.0657 & 0.04 & -0.025604 & 0 \\ 0 & 0.9 & 0.8 & 0.01 & 0 & -0.001 \end{bmatrix} \quad (11)$$

$$JCOB1 := \begin{bmatrix} -0.0247 & 0 & 0 & 0 & 0.000904 & -0.07287449393 \\ 0. & -1.1250 & 0 & 0 & 0 & 0.02186234818 \\ 0. & 0.2 & -0.9824 & 0 & 0 & 0.05101214575 \\ 0 & 0 & 0.04 & -0.0747 & 0 & 0 \\ 0 & 0.0003 & 0.0657 & 0.04 & -0.025604 & 0 \\ 0 & 0.9 & 0.8 & 0.01 & 0 & -0.001 \end{bmatrix} \quad (12)$$

> Pol1 := LinearAlgebra:-CharacteristicPolynomial(JCOB1, λ)

$$Pol1 := \left(-0.005076928445 + 0.01054364577\lambda + 1.204319050\lambda^2 + 2.1831\lambda^3 + \lambda^4\right)(-0.025604 - \lambda)(-0.0247 - \lambda) \quad (13)$$

> solve({ Pol1 = 0 })

$\{\lambda = -0.02560400000\}, \{\lambda = -0.02470000000\}, \{\lambda = 0.05786578585\}, \{\lambda = -0.07487217447\}, \{\lambda = -1.048750657\}, \{\lambda = -1.117342955\}$ **(14)**

>

KESTABILAN ENDEMIK

> JCOB2 := subs(T2, evalm(JCOB));

$$JCOB2 := \begin{bmatrix} -0.5846151875 & 0 & 0 & 0 & 0.000904 & -0.0004406311997 \\ 0.1679745562 & -1.1250 & 0 & 0 & 0 & 0.0001321893599 \\ 0.3919406312 & 0.2 & -0.9824 & 0 & 0 & 0.0003084418399 \\ 0 & 0 & 0.04 & -0.0747 & 0 & 0 \\ 0 & 0.0003 & 0.0657 & 0.04 & -0.025604 & 0 \\ 0 & 0.9 & 0.8 & 0.01 & 0 & -0.001 \end{bmatrix} \quad (15)$$

>

$$JCOB2 := \begin{bmatrix} -0.5846151875 & 0 & 0 & 0 & 0.000904 & -0.0004406311997 \\ 0.1679745562 & -1.1250 & 0 & 0 & 0 & 0.0001321893599 \\ 0.3919406312 & 0.2 & -0.9824 & 0 & 0 & 0.0003084418399 \\ 0 & 0 & 0.04 & -0.0747 & 0 & 0 \\ 0 & 0.0003 & 0.0657 & 0.04 & -0.025604 & 0 \\ 0 & 0.9 & 0.8 & 0.01 & 0 & -0.001 \end{bmatrix} \quad (16)$$

> Pol2 := LinearAlgebra:-CharacteristicPolynomial(JCOB2, λ)

$$Pol2 := 1.20981202700321 \times 10^{-6} + 0.00130037566433794\lambda + 2.79331918714284\lambda^5 + 1.00000000169257\lambda^6 + 0.887822185338357\lambda^3 + 2.61157715130989\lambda^4 + 0.0700797169150085\lambda^2 \quad (17)$$

>

> solve({Pol2 = 0})

$\{\lambda = -0.0009816433311\}, \{\lambda = -0.02553853788\}, \{\lambda = -0.07472491114\}, \{\lambda = -0.5838325866\}, \{\lambda = -0.9832717554\}, \{\lambda = -1.124969748\}$ **(18)**