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LAMPIRAN

Lampiran 1 : Pembuktian Persamaan 2.39 dan 2.40

Pembuktian dari Triana,S (2017) sebagai berikut
dari persaman 2.34 dan 2.38 diperoleh persamaan sebagai berikut.

$$E\left(C_{i,n-i}\middle|C_{i,n}\right) = p_i C_{i,n}$$
$$Var\left(C_{i,n-i}\middle|C_{i,n}\right) = p_i q_i \alpha_i^2\left(C_{i,n}\right)$$

Dengan demikian, akan diperoleh bentuk $Var\left(C_{i,n-i}\right)$ $dan\ Cov\ \left(C_{i,n-1}, C_{i,n}\right)$ sebagai berikut.

$$\begin{aligned}
Var\left(C_{i,n-i}\right) &= E\left[Var\left[C_{i,n-i}|C_{i,n}\right]\right] + Var\left[E\left[C_{i,n-i}|C_{i,n}\right]\right] \\
&= p_i q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2 Var\left[C_{i,n}\right] \\
&= p_i(1-p_i)E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2 Var\left[C_{i,n}\right] \\
&= p_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] - p_i^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2 Var\left[C_{i,n}\right] \\
Var\left(C_{i,n-i}\right) &= p_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)
\end{aligned}$$

$$\begin{aligned}
Cov\left(C_{i,n-i}, C_{i,n}\right) &= E\left[Cov\left[C_{i,n-i}, C_{i,n}|C_{i,n}\right]\right] + Cov\left[E\left[C_{i,n-i}|C_{i,n}\right], E\left[C_{i,n}|C_{i.n}\right]\right] \\
&= E\left[Cov\left[C_{i,n-i}, C_{i,n}|C_{i,n}\right]\right] + Cov\left[E\left[C_{i,n-i}|C_{i,n}\right], C_{i,n}\right] \\
&= E\left[E\left(C_{i,n-i}C_{i,n}|C_{i,n}\right) - E\left(C_{i,n-i}|C_{i,n}\right)E\left(C_{i,n}|C_{i,n}\right)\right] + Cov\left[p_i C_{i,n}, C_{i,n}\right] \\
&= E\left[E\left(C_{i,n-i}C_{i,n}|C_{i,n}\right) - p_i C_{i,n} C_{i,n}\right] + p_i Cov\left[C_{i,n}, C_{i,n}\right] \\
&= E\left[E\left(C_{i,n-i}C_{i,n}|C_{i,n}\right)\right] - p_i E\left[C_{i,n}^2\right] + p_i Var\left[C_{i,n}\right] \\
&= E\left[C_{i,n-i}C_{i,n}\right] - p_i E\left[C_{i,n}^2\right] + p_i Var\left[C_{i,n}\right] \\
&= E\left[p_i C_{i,n} C_{i,n}\right] - p_i E\left[C_{i,n}^2\right] + p_i Var\left[C_{i,n}\right] \\
&= p_i E\left[C_{i,n}^2\right] - p_i E\left[C_{i,n}^2\right] + p_i Var\left[C_{i,n}\right] \\
Cov\left(C_{i,n-i}, C_{i,n}\right) &= p_i Var\left[C_{i,n}\right]
\end{aligned}$$

Subtitusikan $Var\left(C_{i,n-i}\right)$ $dan\ Cov\ \left(C_{i,n-1}, C_{i,n}\right)$ maka didapatkan nilai $Cov\ \left(C_{i,n-1}, R_i\right)$ sebagai berikut

$$\begin{aligned}Cov[C_{i,n-i},R_i] &= Cov[C_{i,n-i},C_{i,n}-C_{i,n-i}] = Cov[C_{i,n-i},C_{i,n}] - Cov[C_{i,n-i},C_{i,n-i}]\\ &= Cov[C_{i,n-i},C_{i,n}] - Var[C_{i,n-i}]\\ &= p_iVar[C_{i,n}] - p_iE[\alpha_i^2(C_{i,n})] - p_i^2(Var[C_{i,n}] - E[\alpha_i^2(C_{i,n})])\\ &= p_iVar[C_{i,n}] - p_iE[\alpha_i^2(C_{i,n})] - p_i(1-q_i)(Var[C_{i,n}] - E[\alpha_i^2(C_{i,n})])\\ &= p_iVar[C_{i,n}] - p_iE[\alpha_i^2(C_{i,n})] - (p_i - p_iq_i)(Var[C_{i,n}] - E[\alpha_i^2(C_{i,n})])\\ &= p_iVar[C_{i,n}] - p_iE[\alpha_i^2(C_{i,n})] - p_iVar[C_{i,n}] + p_iq_iVar[C_{i,n}]\\ &\quad + p_iE[\alpha_i^2(C_{i,n})] - p_iq_iE[\alpha_i^2(C_{i,n})]\end{aligned}$$

$$Cov[C_{i,n-i},R_i] = p_iq_i(Var[C_{i,n}] - E[\alpha_i^2(C_{i,n})])$$

Maka dengan mensubtitusikan $Var(C_{i,n-i})$ dan $Cov[C_{i,n-i},R_i]$ ke persamaan didapatkan

$$c_i^* = \frac{p_i}{q_i}\frac{Cov[C_{i,n-i},R_i] + p_iq_iVar[U_i^{BC}]}{Var[C_{i,n-i}] + p_i^2Var[U_i^{BC}]}$$

$$= \frac{p_i}{q_i}\frac{p_iq_i(Var[C_{i,n}]-E[\alpha_i^2(C_{i,n})]) + p_iq_iVar[U_i^{BC}]}{p_iE[\alpha_i^2(C_{i,n})] + p_i^2(Var[C_{i,n}]-E[\alpha_i^2(C_{i,n})]) + p_i^2Var[U_i^{BC}]}$$

$$= p_i\frac{Var[C_{i,n}]-E[\alpha_i^2(C_{i,n})] + Var[U_i^{BC}]}{E[\alpha_i^2(C_{i,n})] + p_i(Var[C_{i,n}]-E[\alpha_i^2(C_{i,n})]) + p_iVar[U_i^{BC}]}$$

$$= \frac{p_i(Var[C_{i,n}]-E[\alpha_i^2(C_{i,n})] + Var[U_i^{BC}])}{p_i(Var[C_{i,n}]-E[\alpha_i^2(C_{i,n})] + Var[U_i^{BC}]) + E[\alpha_i^2(C_{i,n})]}$$

$$= \frac{p_i}{p_i + \frac{E[\alpha_i^2(C_{i,n})]}{Var[C_{i,n}]-E[\alpha_i^2(C_{i,n})] + Var[U_i^{BC}]}}$$

$$c_i^* = \frac{p_i}{p_i + t_i^*}$$

Dengan $t_i^* = \frac{E[\alpha_i^2(C_{i,n})]}{Var[C_{i,n}]-E[\alpha_i^2(C_{i,n})] + Var[U_i^{BC}]}$ maka persamaan 3.39 dan 3.40 Terbukti

Lampiran 2 : Pembuktian Persamaan 3.44

Pembuktian dari Triana,S (2017) sebagi berikut:
Subtitusikan persamaan 3.41 Serta asumsi $Var\left[C_{i,n}\right] = Var\left[U_i^{BC}\right] dan\ E\left[C_{i,n}\right] = E\left[U_i^{BC}\right]\ pada\ Var\left(C_{i,n-i}\right)$ yang telah didapat sebelumnya. Sehingga akan diperoleh bentuk $Var\left(C_{i,n-i}\right)$ yang lain

$$
\begin{aligned}
Var\left(C_{i,n-i}\right) &= p_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right) \\
&= p_i \beta_i^2\left(Var\left[U_i^{BC}\right] + (E[U_i^{BC}])^2\right) \\
&+ p_i^2\left(Var\left[U_i^{BC}\right] - \beta_i^2\left(Var\left[U_i^{BC}\right] + (E[U_i^{BC}])^2\right)\right) \\
&= p_i \beta_i^2 Var\left[U_i^{BC}\right] + p_i \beta_i^2 (E[U_i^{BC}])^2 + p_i^2\, Var\left[U_i^{BC}\right] - p_i^2 \beta_i^2\, Var\left[U_i^{BC}\right] \\
&- p_i^2 \beta_i^2 (E[U_i^{BC}])^2 \\
&= Var\left[U_i^{BC}\right]\left(p_i\beta_i^2 + p_i^2 - p_i^2\beta_i^2 + \frac{(E[U_i^{BC}])^2}{Var[U_i^{BC}]}[p_i\beta_i^2 - p_i^2\beta_i^2]\right) \\
Var\left(C_{i,n-i}\right) &= Var\left[U_i^{BC}\right]\left(p_i[\beta_i^2(1-p_i) + p_i] + \frac{(E[U_i^{BC}])^2}{Var[U_i^{BC}]}[p_i\beta_i^2(1-p_i)]\right)
\end{aligned}
$$

Dengan menggunakan asumsi $Var\left[C_{i,n}\right] = Var\left[U_i^{BC}\right] dan\ E\left[C_{i,n}\right] = E\left[U_i^{BC}\right]$ dan persamaan 3.41 pada persamaan 3.40 maka akan diperoleh etimasi dari t_i sebagai beriku

$$
t_i = \frac{E\left[\alpha_i^2\left(C_{i,n}\right)\right]}{Var[U_i^{BC}] + Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]}
$$

$$
= \frac{\beta_i^2\left(Var[U_i^{BC}] + \left(E[U_i^{BC}]\right)^2\right)}{Var[U_i^{BC}] + Var[U_i^{BC}] - \beta_i^2\left(Var[U_i^{BC}] + \left(E[U_i^{BC}]\right)^2\right)}
$$

$$
= \frac{\beta_i^2\left(Var[U_i^{BC}] + \left(E[U_i^{BC}]\right)^2\right)}{2Var[U_i^{BC}] - \beta_i^2\left(Var[U_i^{BC}] + \left(E[U_i^{BC}]\right)^2\right)} \frac{\frac{1}{Var\left[U_i^{BC}\right]}}{\frac{1}{Var\left[U_i^{BC}\right]}}
$$

$$
t_i = \frac{\beta_i^2\left(1 + \frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)}{2 - \beta_i^2\left(1 + \frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)}
$$

sehingga diperoleh

$$t_i\left(2-\beta_i^2\left(1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)\right)=\beta_i^2\left(1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)$$

$$2t_i-t_i\beta_i^2\left(1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)=\beta_i^2\left(1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)$$

$$(1+t_i)\beta_i^2\left(1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)=2t_i$$

$$\beta_i^2\left(1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)=\frac{2t_i}{(1+t_i)}$$

Maka dengan mensubtitusikan nilai $Var\left(C_{i,n-i}\right)$ dan $\beta_i^2\left(1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)$ yang telah didapat ke dalam persamaan 3.43 maka didapatkan

$$\begin{aligned}
Var[R_i^B] &= c_i^2\left(\frac{q_i}{p_i}\right)^2 Var\left[C_{i,n-i}\right]+(1-c_i)^2q_i^2Var\left[U_i^{BC}\right]\\
&= c_i^2\left(\frac{q_i}{p_i}\right)^2 Var\left[U_i^{BC}\right]\left(p_i[\beta_i^2(1-p_i)+p_i]+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}[p_i\beta_i^2(1-p_i)]\right)\\
&\qquad +(1-c_i)^2Var[R_i^{BF}]\\
&= Var[R_i^{BF}]\left(c_i^2\left(\frac{1}{p_i}[\beta_i^2(1-p_i)+p_i]+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\left[\frac{1}{p_i}\beta_i^2(1-p_i)\right]\right)\right.\\
&\qquad \left.+(1-c_i)^2\right)\\
&= Var[R_i^{BF}]\left(c_i^2\left(\beta_i^2\frac{(1-p_i)}{p}+1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\left[\beta_i^2\frac{(1-p_i)}{p_i}\right]\right)+(1-c_i)^2\right)\\
&= Var[R_i^{BF}]\left(c_i^2\left(1+\beta_i^2\frac{(1-p_i)}{p}\left(1+\frac{\left(E\left[U_i^{BC}\right]\right)^2}{Var\left[U_i^{BC}\right]}\right)\right)+(1-c_i)^2\right)\\
&= Var[R_i^{BF}]\left(c_i^2\left(1+\frac{(1-p_i)}{p_i}\left(\frac{2t_i}{(1+t_i)}\right)\right)+c_i^2-2c_i+1\right)\\
&= Var[R_i^{BF}]\left(c_i^2+2c_i^2\left(\frac{1-p_i}{p_i}\right)\left(\frac{t_i}{1+t_i}\right)+c_i^2-2c_i+1\right)\\
&= Var[R_i^{BF}]\left(2c_i^2+2c_i^2\left(\frac{1-p_i}{p_i}\right)\left(\frac{t_i}{1+t_i}\right)-2c_i+1\right)\\
&= Var[R_i^{BF}]\left(2c_i^2\left[1+\left(\frac{1-p_i}{p_i}\right)\left(\frac{t_i}{1+t_i}\right)-2c_i+1\right]\right)
\end{aligned}$$

$$= Var[R_i^{BF}]\left(2\left(\frac{p_i}{p_i+t_i}\right)^2\left[1+\left(\frac{1-p_i}{p_i}\right)\left(\frac{t_i}{1+t_i}\right)\right]-2\frac{p_i}{p_i+t_i}+1\right)$$
$$= Var[R_i^{BF}]\left(\frac{2p_i^2}{(p_i+t_i)^2}+\frac{2p_i(1-p_i)}{(p_i+t_i)^2}\left(\frac{t_i}{1+t_i}\right)-\frac{2p_i}{p_i+t_i}+1\right)$$
$$= Var[R_i^{BF}]\left(\frac{2p_i^2(1+t_i)+2p_it_i(1-p_i)-2p_i(p_i+t_i)(1+t_i)}{(p_i+t_i)^2(1+t_i)}+1\right)$$
$$= Var[R_i^{BF}]\left(\frac{2p_i^2+2p_i^2t_i+2p_it_i-2p_i^2t_i-2p_i^2-2p_i^2t_i-2p_it_i-2p_it_i^2}{(p_i+t_i)^2(1+t_i)}+1\right)$$
$$= Var[R_i^{BF}]\left(\frac{-2p_i^2t_i-2p_it_i^2}{(p_i+t_i)^2(1+t_i)}+1\right)$$
$$Var[R_i^{BF}]\left(1-\frac{(2p_it_i(p_i+t_i))}{(p_i+t_i)^2(1+t_i)}\right)$$
$$Var[R_i^B] = Var[R_i^{BF}]\left(1-\frac{2p_it_i}{(p_i+t_i)(1+t_i)}\right)$$

Apabila $Var[R_i^B]$ diatas diturunkan terhadap t_i, maka diperoleh t_i yang meminmumkan variansi sebagai berikut

$$\frac{\partial}{\partial t_i}Var[R_i^B] = \frac{\partial}{\partial t_i}\left[Var[R_i^{BF}]\left(1-\frac{2p_it_i}{(p_i+t_i)(1+t_i)}\right)\right]$$

$$0 = \frac{\partial}{\partial t_i}\left[Var[R_i^{BF}]\left(1-\frac{2p_it_i}{(p_i+t_i)(1+t_i)}\right)\right]$$

$$0 = \frac{\partial}{\partial t_i}\left[\frac{2p_it_i}{(p_i+p_it_i+t_i+t_i^2)}\right]$$

$$0 = \frac{2p_i\left(p_i+p_it_i^*+t_i^*+t_i^{*2}\right)-(p_i+1+2t_i^*)(2p_it_i^*)}{\left(p_i+p_it_i^*+t_i^*+t_i^{*2}\right)^2}$$

$$0 = 2p_i^2+2p_i^2t_i^*+2p_it_i^*+2p_it_i^{*2}-2p_i^2t_i^*-2p_it_i^*-4p_it_i^{*2}$$

$$0 = 2p_i^2-2p_it_i^{*2}$$

$$t_i^{*2} = \frac{2p_i^2}{2p_i}$$

$$t_i^* = \sqrt{p_i}$$

Karena $p_i \neq 0$ untuk semua i dengan demikian t_i yang meminmumkan variansi $Var[R_i^B]$ adalah $t_i^* = \sqrt{p_i}$. dengan demikian persamaan 3.44 terbukti.

Lampiran 3 : pembuktian persamaan 3.46

Pembuktian dari Triana,S (2017) sebagi berikut :
Sebelum membuktikan persamaan 3.46 terlebih dulu akan dicari nilai $Var[R_i]$ menggunakan persamaan $Var\left(C_{i,n-i}\right)$ dan $Cov\left(C_{i,n-i}, C_{i,n}\right)$ yang telah didapat pada lampiran 1 sebagi berikut

$$Var\left(C_{i,n-i}\right) \quad = p_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)$$

$$Cov\left(C_{i,n-i}, C_{i,n}\right) = p_i Var\left[C_{i,n}\right]$$

Maka nilai $Var[R_i]$ adalah

$$\begin{aligned}
Var[R_i] &= Var\left[C_{i,n} - C_{i,n-i}\right] \\
&= Var\left[C_{i,n}\right] - 2Cov\left[C_{i,n} - C_{i,n-i}\right] + Var\left[C_{i,n-i}\right] \\
&= Var\left[C_{i,n}\right] - 2\left(p_i Var\left[C_{I,n-i}\right]\right) + p_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2\left(Var\left[C_{i,n}\right] - \right. \\
&\quad \left. E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right) \\
&= Var\left[C_{i,n}\right] - 2p_i Var\left[C_{I,n}\right] + p_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2 Var\left[C_{i,n}\right] - p_i^2\, E\left[\alpha_i^2\left(C_{i,n}\right)\right] \\
&= Var\left[C_{i,n}\right](1 - 2p_i + p_i^2) + p_i E\left[\alpha_i^2\left(C_{i,n}\right)\right](1 - p_i) \\
&= Var\left[C_{i,n}\right](1 - p_i)^2 + p_i q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] \\
&= q_i^2 Var\left[C_{i,n}\right] + q_i(1 - q_i) E\left[\alpha_i^2\left(C_{i,n}\right)\right] \\
&= q_i^2 Var\left[C_{i,n}\right] + (q_i - q_i^2) E\left[\alpha_i^2\left(C_{i,n}\right)\right] \\
Var[R_i] &= q_i\, E\left[\alpha_i^2\left(C_{i,n}\right)\right] + q_i^2\left(Var\left[C_{I,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)
\end{aligned}$$

a. MSE Chain Ladder

Selanjutnya dengan asumsi $E[R_{CL}] = E[R_i]$ diperoleh mean square eror metode *Chain Ladder* sebagai berikut

$$\begin{aligned}
mse\left(R_i^{CL}\right) &= E[(R^{CL} - R_i)^2] \\
&= Var[R^{CL} - R_i] \\
&= Var\left[R_i^{CL}\right] - 2\, Cov\left[R_i^{CL}, R_i\right] + Var\,[R_i] \\
&= \left(\frac{q_i}{p_i}\right)^2 Var\left[C_{i,n-i}\right] - 2\frac{q_i}{p_i} Cov\left[C_{i,n-i}, R_i\right] + Var\,[R_i] \\
&= \left(\frac{q_i}{p_i}\right)^2 \left(p_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + p_i^2\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)\right) - \\
&\quad 2\frac{q_i}{p_i}\left(p_i, q_i\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)\right) + q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + \\
&\quad q_i^2\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)
\end{aligned}$$

$$= \frac{q_i^2}{p_i}\ E\left[\alpha_i^2\left(C_{i,n}\right)\right] + q_i^2\ Var\ \left[C_{i,n}\right] - q_i^2\ E\left[\alpha_i^2\left(C_{i,n}\right)\right] - 2q_i^2 Var\left[C_{i,n}\right] + \ 2q_i^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right] + q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + q_i^2 Var\ \left[C_{i,n}\right] - q_i^2 E(\alpha_i^2\left(C_{i,n}\right)$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\ \left(\frac{q_i^2}{p_i} + q_i\right)$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left(\frac{q_i^2 + q_i p_i}{p_i}\right)$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left(\frac{q_i^2 + q_i(1-p_i)}{p_i}\right)$$

$$mse\ \left(R_i^{CL}\right) = E\left[\alpha_i^2\left(C_{i,n}\right)\right]\frac{q_i}{p_i}$$

b. MSE Bornhuetter Ferguson

Dari asumsi sebelumnya, diperoleh bahwa $E[R_I^{BF}] = q_i E\left[U_i^{BC}\right] = q_i E\left[C_{i,n}\right] = E\left[C_{i,n} - P_i C_{i,n}^{CL}\right] = E\left[C_{i,n} - C_{i,n-i}\right] = E[R_i]$. Asumsikan bahwa $Cov[R_i^{BF}, R_i] = q_i Cov\left[U_i^{BC}, R_i\right] = 0$. Dengan demikian, diperoleh *mean squared eror* dari metode Bornhuetter Ferguson sebagai beriku

$$mse\ (R_i^{BF}) = E[(R^{BF} - R_i)^2]$$

$$= Var[R_i^{BF} - R_i]$$

$$= Var[R_i^{BF}] + Var[R_i]$$

$$= q_i^2 Var\left[U_i^{BC}\right] + q_i^2\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right) + q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right]$$

$$= q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + q_i^2\left(Var\left[U_i^{BC}\right] + Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right] q_i \left(1 + \frac{q_i\left(Var\left[U_i^{BC}\right] + Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)}{E\left[\alpha_i^2\left(C_{i,n}\right)\right]}\right)$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right] q_i \left(1 + \frac{q_i}{\frac{E\left[\alpha_i^2\left(C_{i,n}\right)\right]}{Var\left[U_i^{BC}\right] + Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]}}\right)$$

$$mse\ (R_i^{BF}) = E\left[\alpha_i^2\left(C_{i,n}\right)\right] q_i \left(1 + \frac{q_i}{t_i}\right)$$

c. MSE Benktander

$$mse\left(R_i^{GB}\right) = E\left[\left(c_i\left(R_i^{CL} - R_i\right) + (1 - c_i)(R_i^{BF} - R_i)\right)^2\right]$$

$$= c_i^2 mse\left(R_i^{CL}\right) + 2c_i(1 - c_i)E\left[\left(R_i^{CL} - R_i\right)(R_i^{BF} - R_i)\right] + (1 - c_i)^2 mse\ (R_i^{BF})$$

$$= c_i^2 E\left[\alpha_i^2\left(C_{i,n}\right)\frac{q_i}{p_i}\right] + 2c_i(1 - c_i)Cov\left[R_i^{CL} - R_i, R_i^{BF} - R_i\right] + (1 - c_i)^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right]q_i\left(1 + \frac{q_i}{t_i}\right)$$

$$= c_i^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right]\frac{q_i}{p_i} + 2c_i(1 - c_i)\left(Var[R_i] - Cov\left[R_i^{CL}, R_i\right]\right) + (1 - c_i)^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right]q_i\left(1 + \frac{q_i}{t_i}\right)$$

$$= c_i^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right]\frac{q_i}{p_i} + 2c_i(1 - c_i)\left(q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + q_i^2\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right) - \frac{q_i}{p_i}Cov\left[C_{i,n-i}, R_i\right]\right) + (1 - c_i)^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right]q_i\left(1 + \frac{q_i}{t_i}\right)$$

$$= c_i^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right]\frac{q_i}{p_i} + 2c_i(1 - c_i)\left(q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right] + q_i^2\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right) - \frac{q_i}{p_i}q_i p_i\left(Var\left[C_{i,n}\right] - E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right)\right) + (1 - c_i)^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right]q_i\left(1 + \frac{q_i}{t_i}\right)$$

$$= c_i^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right] + 2c_i(1 - c_i)\left(q_i E\left[\alpha_i^2\left(C_{i,n}\right)\right]\right) + (1 - c_i)^2 E\left[\alpha_i^2\left(C_{i,n}\right)\right]q_i\left(1 + \frac{q_i}{t_i}\right)$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left[c_i^2\frac{q_i}{p_i} + 2c_i(1 - c_i)(q_i) + (1 - c_i)^2 q_i\left(1 + \frac{q_i}{t_i}\right)\right]$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left[c_i^2\frac{q_i}{p_i} + 2c_i q_i - 2c_i^2 q_i + (1 - c_i)^2 q_i + (1 - c_i)^2\frac{q_i^2}{t_i}\right]$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left[c_i^2\frac{q_i}{p_i} + 2c_i q_i - 2c_i^2 q_i + q_i - 2c_i q_i + c_i^2 q_i + (1 - c_i)^2\frac{q_i^2}{t_i}\right]$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left[c_i^2\left(\frac{q_i}{p_i} - q_i\right) + q_i + (1 - c_i)^2\frac{q_i^2}{t_i}\right]$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left[c_i^2\left(\frac{q_i - q_i(1-q_i)}{p_i}\right) + q_i + (1 - c_i)^2\frac{q_i^2}{t_i}\right]$$

$$= E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left[c_i^2\left(\frac{q_i^2}{p_i}\right) + q_i + (1 - c_i)^2\frac{q_i^2}{t_i}\right]$$

$$mse\left(R_i^{GB}\right) = E\left[\alpha_i^2\left(C_{i,n}\right)\right]\left(\frac{c_i^2}{p_i} + \frac{1}{q_i} + \frac{(1-c_i)^2}{t_i}\right)q_i^2$$

Dengan demikian persamaan 3.46 terbukti.