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LAMPIRAN

Lampiran 1 Pelabelan *Graceful* pada graf roda W_n dengan beberapa nilai n.

Untuk $n = 7$

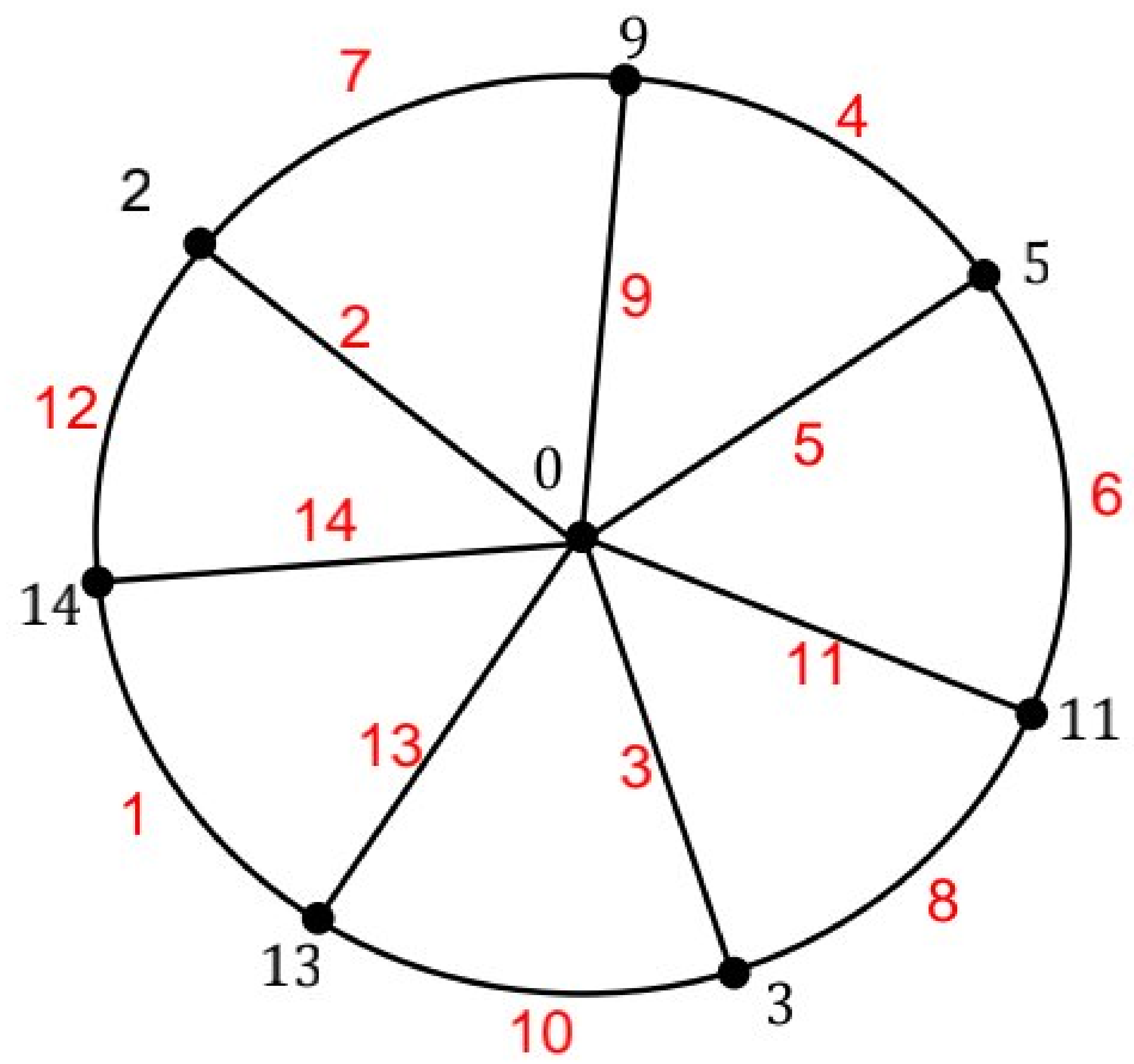
- Pelabelan titik

$f(v_0) = 0$
$f(v_1) = 2$
$f(v_2) = 9$
$f(v_3) = 5$
$f(v_4) = 11$
$f(v_5) = 3$
$f(v_6) = 13$
$f(v_7) = 14$

- Menentukan pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)| = |0 - 2| = 2$
  - $f(v_0, v_2) = |f(v_0) - f(v_2)| = |0 - 9| = 9$
  - $f(v_0, v_3) = |f(v_0) - f(v_3)| = |0 - 5| = 5$
  - $f(v_0, v_4) = |f(v_0) - f(v_4)| = |0 - 11| = 11$
  - $f(v_0, v_5) = |f(v_0) - f(v_5)| = |0 - 3| = 3$
  - $f(v_0, v_6) = |f(v_0) - f(v_6)| = |0 - 13| = 13$
  - $f(v_0, v_7) = |f(v_0) - f(v_7)| = |0 - 14| = 14$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)| = |2 - 9| = 7$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)| = |9 - 5| = 4$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)| = |5 - 11| = 6$
  - $f(v_4, v_5) = |f(v_4) - f(v_5)| = |11 - 3| = 8$
  - $f(v_5, v_6) = |f(v_5) - f(v_6)| = |3 - 13| = 10$
  - $f(v_6, v_7) = |f(v_6) - f(v_7)| = |13 - 14| = 1$
  - $f(v_7, v_1) = |f(v_7) - f(v_1)| = |14 - 2| = 12$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf roda dengan $n = 7$ merupakan *graceful*.

Dapat diilustrasikan sebagai berikut:



Untuk $n = 9$

- Pelabelan titik

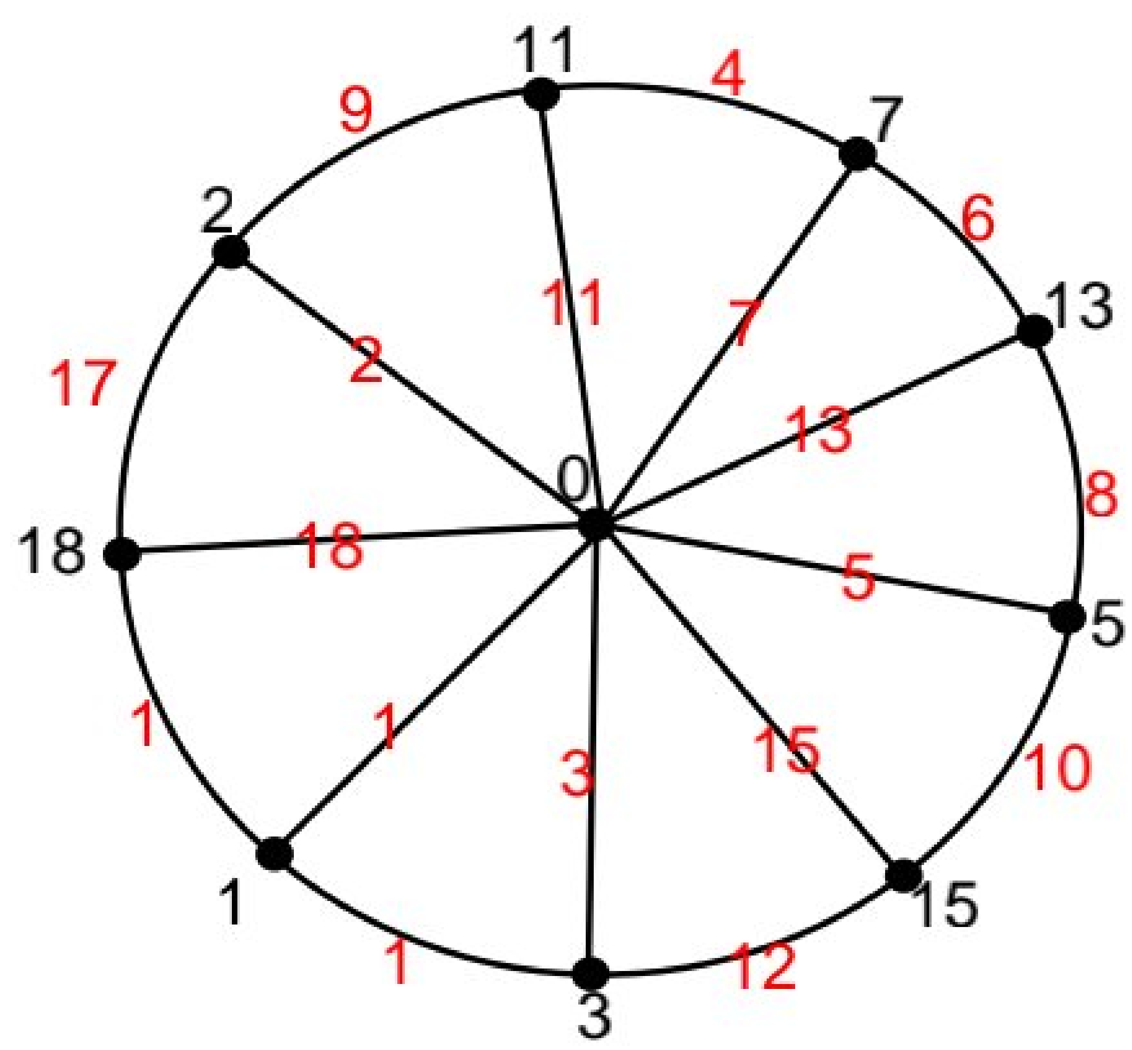
$f(v_0) = 0$
$f(v_1) = 2$
$f(v_2) = 11$
$f(v_3) = 7$
$f(v_4) = 13$
$f(v_5) = 5$
$f(v_6) = 15$
$f(v_7) = 3$
$f(v_8) = 17$
$f(v_9) = 18$

- Pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)| = |0 - 2| = 2$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)| = |0 - 11| = 11$
  - $f(v_o, v_3) = |f(v_0) - f(v_3)| = |0 - 7| = 7$
  - $f(v_o, v_4) = |f(v_0) - f(v_4)| = |0 - 13| = 13$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)| = |2 - 11| = 9$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)| = |11 - 7| = 4$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)| = |7 - 13| = 6$
  - $f(v_4, v_5) = |f(v_4) - f(v_5)| = |13 - 5| = 8$

- $f(v_o, v_5) = |f(v_0) - f(v_5)|$
  $= |0 - 5|$
  $= 5$
- $f(v_o, v_6) = |f(v_0) - f(v_6)|$
  $= |0 - 15|$
  $= 15$
- $f(v_o, v_7) = |f(v_0) - f(v_7)|$
  $= |0 - 3|$
  $= 3$
- $f(v_o, v_8) = |f(v_0) - f(v_8)|$
  $= |0 - 17|$
  $= 17$
- $f(v_o, v_9) = |f(v_0) - f(v_9)|$
  $= |0 - 18|$
  $= 18$
- $f(v_5, v_6) = |f(v_5) - f(v_6)|$
  $= |5 - 15|$
  $= 10$
- $f(v_6, v_7) = |f(v_6) - f(v_7)|$
  $= |15 - 3|$
  $= 12$
- $f(v_7, v_8) = |f(v_7) - f(v_8)|$
  $= |3 - 17|$
  $= 14$
- $f(v_8, v_9) = |f(v_8) - f(v_9)|$
  $= |17 - 18|$
  $= 1$
- $f(v_9, v_1) = |f(v_9) - f(v_1)|$
  $= |18 - 2|$
  $= 16$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf roda dengan $n = 9$ merupakan *graceful*.

Dapat diilustrasikan sebagai berikut:



Untuk $n = 12$

- Pelabelan titik

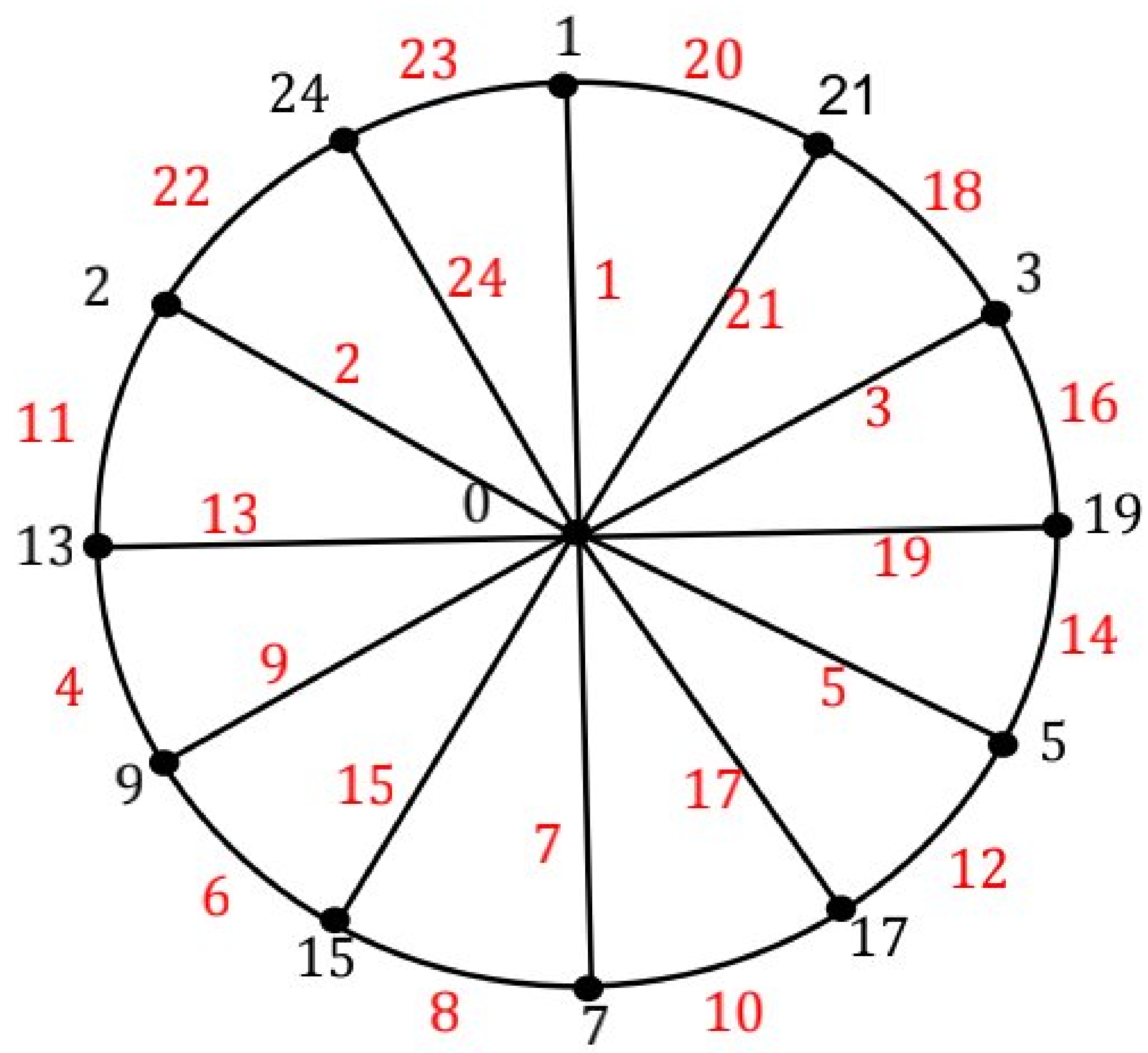
$f(v_0) = 0$
$f(v_1) = 1$
$f(v_2) = 21$
$f(v_3) = 3$
$f(v_4) = 19$
$f(v_5) = 5$
$f(v_6) = 17$
$f(v_7) = 7$
$f(v_8) = 15$
$f(v_9) = 9$
$f(v_{10}) = 13$
$f(v_{11}) = 2$
$f(v_{12}) = 24$

- Pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)| = |0 - 1| = 1$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)| = |0 - 21| = 21$
  - $f(v_o, v_3) = |f(v_0) - f(v_3)| = |0 - 3| = 3$
  - $f(v_o, v_4) = |f(v_0) - f(v_4)| = |0 - 19| = 19$
  - $f(v_o, v_5) = |f(v_0) - f(v_5)| = |0 - 5| = 5$
  - $f(v_o, v_6) = |f(v_0) - f(v_6)| = |0 - 17| = 17$
  - $f(v_o, v_7) = |f(v_0) - f(v_7)| = |0 - 7| = 7$
  - $f(v_o, v_8) = |f(v_0) - f(v_8)| = |0 - 15| = 15$
  - $f(v_o, v_9) = |f(v_0) - f(v_9)| = |0 - 9| = 9$
  - $f(v_o, v_{10}) = |f(v_0) - f(v_{10})| = |0 - 13| = 13$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)| = |1 - 21| = 20$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)| = |21 - 3| = 18$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)| = |3 - 19| = 16$
  - $f(v_4, v_5) = |f(v_4) - f(v_5)| = |19 - 5| = 14$
  - $f(v_5, v_6) = |f(v_5) - f(v_6)| = |5 - 17| = 12$
  - $f(v_6, v_7) = |f(v_6) - f(v_7)| = |17 - 7| = 10$
  - $f(v_7, v_8) = |f(v_7) - f(v_8)| = |7 - 15| = 8$
  - $f(v_8, v_9) = |f(v_8) - f(v_9)| = |15 - 9| = 6$
  - $f(v_9, v_{10}) = |f(v_9) - f(v_{10})| = |9 - 13| = 4$
  - $f(v_{10}, v_{11}) = |f(v_{10}) - f(v_{11})| = |13 - 2| = 11$

- $f(v_o, v_{11}) = |f(v_0) - f(v_{11})|$
  $= |0 - 2|$
  $= 2$
- $f(v_{11}, v_{12}) = |f(v_{11}) - f(v_{12})|$
  $= |2 - 24|$
  $= 22$
- $f(v_o, v_{12}) = |f(v_0) - f(v_{12})|$
  $= |0 - 24|$
  $= 24$
- $f(v_{12}, v_1) = |f(v_{12}) - f(v_1)|$
  $= |24 - 1|$
  $= 23$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf roda dengan n=12 merupakan *graceful.*

Dapat diilustrasikan sebagai berikut:



Untuk $n = 15$

- Pelabelan titik

$f(v_0) = 0$
$f(v_1) = 2$
$f(v_2) = 17$
$f(v_3) = 13$
$f(v_4) = 19$
$f(v_5) = 11$
$f(v_6) = 21$
$f(v_7) = 9$
$f(v_8) = 23$
$f(v_9) = 7$
$f(v_{10}) = 25$
$f(v_{11}) = 5$
$f(v_{12}) = 27$
$f(v_{13}) = 3$
$f(v_{14}) = 29$
$f(v_{15}) = 30$

- Menentukan pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)|$
    $= |0 - 2|$
    $= 2$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)|$
    $= |2 - 17|$
    $= 15$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)|$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)|$

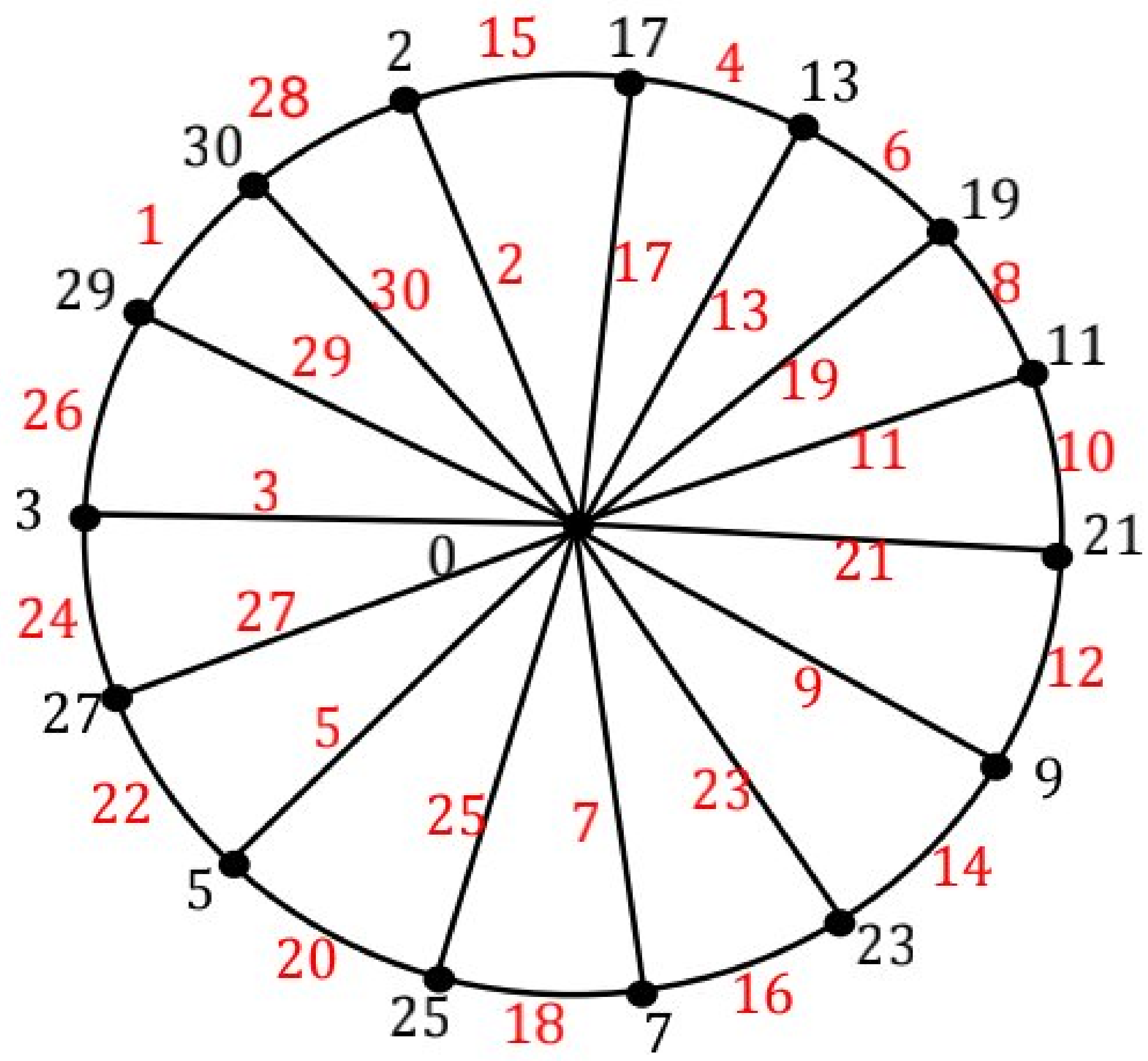
$= |0 - 17|$
$= 17$

- $f(v_o, v_3) = |f(v_0) - f(v_3)|$
  $= |0 - 13|$
  $= 13$
- $f(v_o, v_4) = |f(v_0) - f(v_4)|$
  $= |0 - 19|$
  $= 19$
- $f(v_o, v_5) = |f(v_0) - f(v_5)|$
  $= |0 - 11|$
  $= 11$
- $f(v_o, v_6) = |f(v_0) - f(v_6)|$
  $= |0 - 21|$
  $= 21$
- $f(v_o, v_7) = |f(v_0) - f(v_7)|$
  $= |0 - 9|$
  $= 9$
- $f(v_o, v_8) = |f(v_0) - f(v_8)|$
  $= |0 - 23|$
  $= 23$
- $f(v_o, v_9) = |f(v_0) - f(v_9)|$
  $= |0 - 7|$
  $= 7$
- $f(v_o, v_{10}) = |f(v_0) - f(v_{10})|$
  $= |0 - 25|$
  $= 25$
- $f(v_o, v_{11}) = |f(v_0) - f(v_{11})|$
  $= |0 - 5|$
  $= 5$
- $f(v_o, v_{12}) = |f(v_0) - f(v_{12})|$
  $= |0 - 27|$
  $= 27$
- $f(v_o, v_{13}) = |f(v_0) - f(v_{13})|$
  $= |0 - 3|$
  $= 3$
- $f(v_o, v_{14}) = |f(v_0) - f(v_{14})|$
  $= |0 - 29|$
  $= 29$
- $f(v_o, v_{15}) = |f(v_0) - f(v_{15})|$
  $= |0 - 30|$
  $= 30$

$= |17 - 13|$
$= 4$

- $f(v_3, v_4) = |f(v_3) - f(v_4)|$
  $= |13 - 19|$
  $= 6$
- $f(v_4, v_5) = |f(v_4) - f(v_5)|$
  $= |19 - 11|$
  $= 8$
- $f(v_5, v_6) = |f(v_5) - f(v_6)|$
  $= |11 - 21|$
  $= 10$
- $f(v_6, v_7) = |f(v_6) - f(v_7)|$
  $= |21 - 9|$
  $= 12$
- $f(v_7, v_8) = |f(v_7) - f(v_8)|$
  $= |9 - 23|$
  $= 14$
- $f(v_8, v_9) = |f(v_8) - f(v_9)|$
  $= |23 - 7|$
  $= 16$
- $f(v_9, v_{10}) = |f(v_9) - f(v_{10})|$
  $= |7 - 25|$
  $= 18$
- $f(v_{10}, v_{11}) = |f(v_{10}) - f(v_{11})|$
  $= |25 - 5|$
  $= 20$
- $f(v_{11}, v_{12}) = |f(v_{11}) - f(v_{12})|$
  $= |5 - 27|$
  $= 22$
- $f(v_{12}, v_{13}) = |f(v_{12}) - f(v_{13})|$
  $= |27 - 3|$
  $= 24$
- $f(v_{13}, v_{14}) = |f(v_{13}) - f(v_{14})|$
  $= |3 - 29|$
  $= 26$
- $f(v_{14}, v_{15}) = |f(v_{14}) - f(v_{15})|$
  $= 29 - 30$
  $= 1$
- $f(v_{15}, v_1) = |f(v_{15}) - f(v_1)|$
  $= |30 - 2|$
  $= 28$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf roda dengan $n = 15$ merupakan *graceful.*
Dapat diilustrasikan sebagai berikut:



Untuk $\boldsymbol{n = 16}$

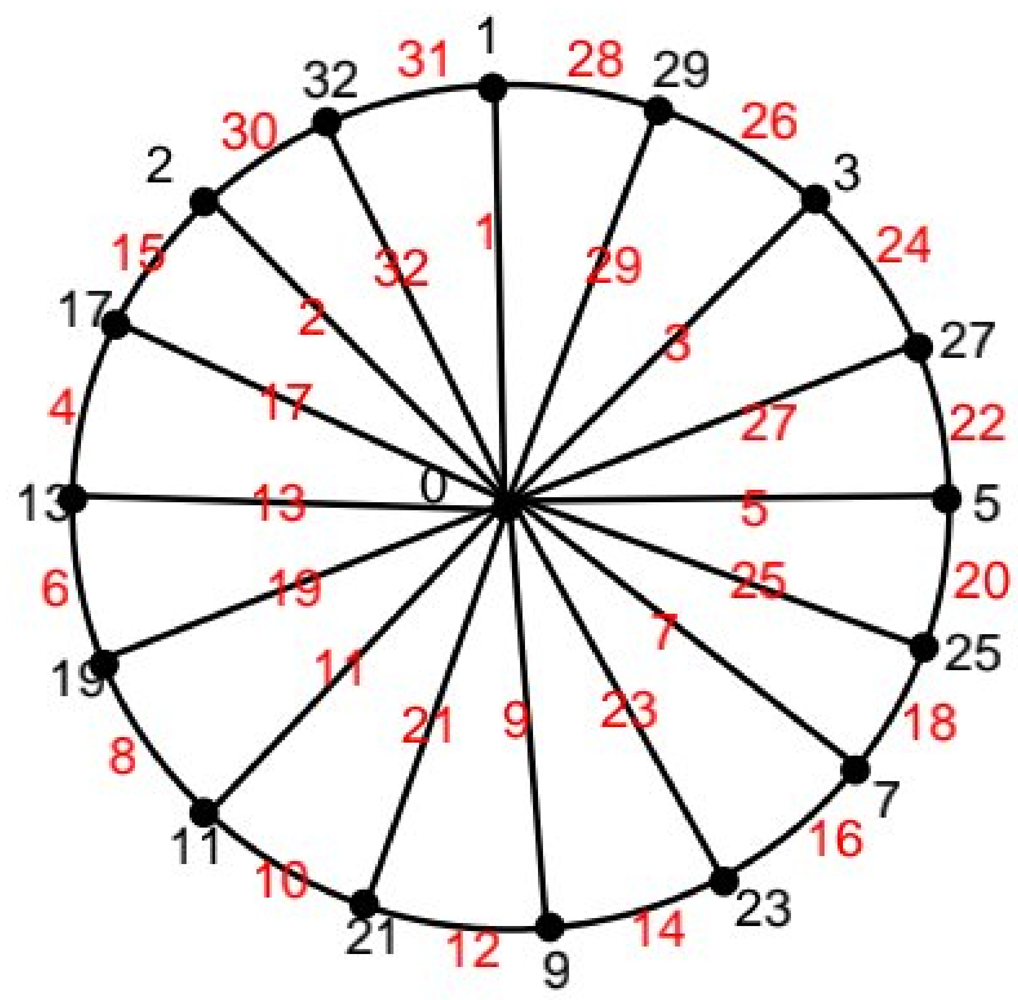
- Pelabelan titik

$f(v_0) = 0$
$f(v_1) = 1$
$f(v_2) = 29$
$f(v_3) = 3$
$f(v_4) = 27$
$f(v_5) = 5$
$f(v_6) = 25$
$f(v_7) = 7$
$f(v_8) = 23$
$f(v_9) = 9$
$f(v_{10}) = 21$
$f(v_{11}) = 11$
$f(v_{12}) = 19$
$f(v_{13}) = 13$
$f(v_{14}) = 17$
$f(v_{15}) = 2$
$f(v_{16}) = 32$

- Pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)| = |0 - 1| = 1$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)| = |0 - 29| = 29$
  - $f(v_o, v_3) = |f(v_0) - f(v_3)| = |0 - 3| = 3$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)| = |1 - 29| = 28$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)| = |29 - 3| = 26$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)| = |3 - 27| = 24$

- $f(v_o, v_4) = |f(v_0) - f(v_4)|$
  $= |0 - 27|$
  $= 27$
- $f(v_o, v_5) = |f(v_0) - f(v_5)|$
  $= |0 - 5|$
  $= 5$
- $f(v_o, v_6) = |f(v_0) - f(v_6)|$
  $= |0 - 25|$
  $= 25$
- $f(v_o, v_7) = |f(v_0) - f(v_7)|$
  $= |0 - 7|$
  $= 7$
- $f(v_o, v_8) = |f(v_0) - f(v_8)|$
  $= |0 - 23|$
  $= 23$
- $f(v_o, v_9) = |f(v_0) - f(v_9)|$
  $= |0 - 9|$
  $= 9$
- $f(v_o, v_{10}) = |f(v_0) - f(v_{10})|$
  $= |0 - 21|$
  $= 21$
- $f(v_o, v_{11}) = |f(v_0) - f(v_{11})|$
  $= |0 - 11|$
  $= 11$
- $f(v_o, v_{12}) = |f(v_0) - f(v_{12})|$
  $= |0 - 19|$
  $= 19$
- $f(v_o, v_{13}) = |f(v_0) - f(v_{13})|$
  $= |0 - 13|$
  $= 13$
- $f(v_o, v_{14}) = |f(v_0) - f(v_{14})|$
  $= |0 - 17|$
  $= 17$
- $f(v_o, v_{15}) = |f(v_0) - f(v_{15})|$
  $= |0 - 2|$
  $= 2$
- $f(v_o, v_{16}) = |f(v_0) - f(v_{15})|$
  $= |0 - 32|$
  $= 32$
- $f(v_4, v_5) = |f(v_4) - f(v_5)|$
  $= |27 - 5|$
  $= 22$
- $f(v_5, v_6) = |f(v_5) - f(v_6)|$
  $= |5 - 25|$
  $= 20$
- $f(v_6, v_7) = |f(v_6) - f(v_7)|$
  $= |25 - 7|$
  $= 18$
- $f(v_7, v_8) = |f(v_7) - f(v_8)|$
  $= |7 - 23|$
  $= 16$
- $f(v_8, v_9) = |f(v_8) - f(v_9)|$
  $= |23 - 9|$
  $= 14$
- $f(v_9, v_{10}) = |f(v_9) - f(v_{10})|$
  $= |9 - 21|$
  $= 12$
- $f(v_{1o}, v_{11}) = |f(v_{10}) - f(v_{11})|$
  $= |21 - 11|$
  $= 10$
- $f(v_{11}, v_{12}) = |f(v_{11}) - f(v_{12})|$
  $= |11 - 19|$
  $= 8$
- $f(v_{12}, v_{13}) = |f(v_{12}) - f(v_{13})|$
  $= |19 - 13|$
  $= 6$
- $f(v_{13}, v_{14}) = |f(v_{13}) - f(v_{14})|$
  $= |13 - 17|$
  $= 4$
- $f(v_{14}, v_{15}) = |f(v_{14}) - f(v_{15})|$
  $= 17 - 2$
  $= 15$
- $f(v_{15}, v_{16}) = |f(v_{15}) - f(v_1)|$
  $= |2 - 32|$
  $= 30$
- $f(v_{16}, v_1) = |f(v_{16}) - f(v_1)|$
  $= |32 - 1|$
  $= 31$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf roda dengan $n = 16$ merupakan *graceful.*
Dapat diilustrasikan sebagai berikut:



Untuk $n = 20$

- pelabelan titik

$f(v_0) = 0$
$f(v_1) = 1$
$f(v_2) = 37$
$f(v_3) = 3$
$f(v_4) = 35$
$f(v_5) = 5$
$f(v_6) = 33$
$f(v_7) = 7$
$f(v_8) = 31$
$f(v_9) = 9$
$f(v_{10}) = 29$
$f(v_{11}) = 11$
$f(v_{12}) = 27$
$f(v_{13}) = 13$
$f(v_{14}) = 25$
$f(v_{15}) = 15$
$f(v_{16}) = 23$
$f(v_{17}) = 17$
$f(v_{18}) = 21$
$f(v_{19}) = 2$
$f(v_{20}) = 40$

- pelabelan sisi

  - $f(v_o, v_1) = |f(v_0) - f(v_1)|$
    $= |0 - 1|$
    $= 1$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)|$
    $= |0 - 37|$
    $= 37$
  - $f(v_o, v_3) = |f(v_0) - f(v_3)|$
    $= |0 - 3|$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)|$
    $= |1 - 37|$
    $= 36$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)|$
    $= |37 - 3|$
    $= 34$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)|$
    $= |3 - 35|$

$= 3$

- $f(v_o, v_4) = |f(v_0) - f(v_4)|$
  $= |0 - 35|$
  $= 35$
- $f(v_o, v_5) = |f(v_0) - f(v_5)|$
  $= |0 - 5|$
  $= 5$
- $f(v_o, v_6) = |f(v_0) - f(v_6)|$
  $= |0 - 33|$
  $= 33$
- $f(v_o, v_7) = |f(v_0) - f(v_7)|$
  $= |0 - 7|$
  $= 7$
- $f(v_o, v_8) = |f(v_0) - f(v_8)|$
  $= |0 - 31|$
  $= 31$
- $f(v_o, v_9) = |f(v_0) - f(v_9)|$
  $= |0 - 9|$
  $= 9$
- $f(v_o, v_{10}) = |f(v_0) - f(v_{10})|$
  $= |0 - 29|$
  $= 29$
- $f(v_o, v_{11}) = |f(v_0) - f(v_{11})|$
  $= |0 - 11|$
  $= 11$
- $f(v_o, v_{12}) = |f(v_0) - f(v_{12})|$
  $= |0 - 27|$
  $= 27$
- $f(v_o, v_{13}) = |f(v_0) - f(v_{13})|$
  $= |0 - 13|$
  $= 13$
- $f(v_o, v_{14}) = |f(v_0) - f(v_{14})|$
  $= |0 - 25|$
  $= 25$
- $f(v_o, v_{15}) = |f(v_0) - f(v_{15})|$
  $= |0 - 15|$
  $= 15$
- $f(v_o, v_{16}) = |f(v_0) - f(v_{16})|$
  $= |0 - 23|$
  $= 23$
- $f(v_o, v_{17}) = |f(v_0) - f(v_{17})|$
  $= |0 - 17|$
  $= 17$

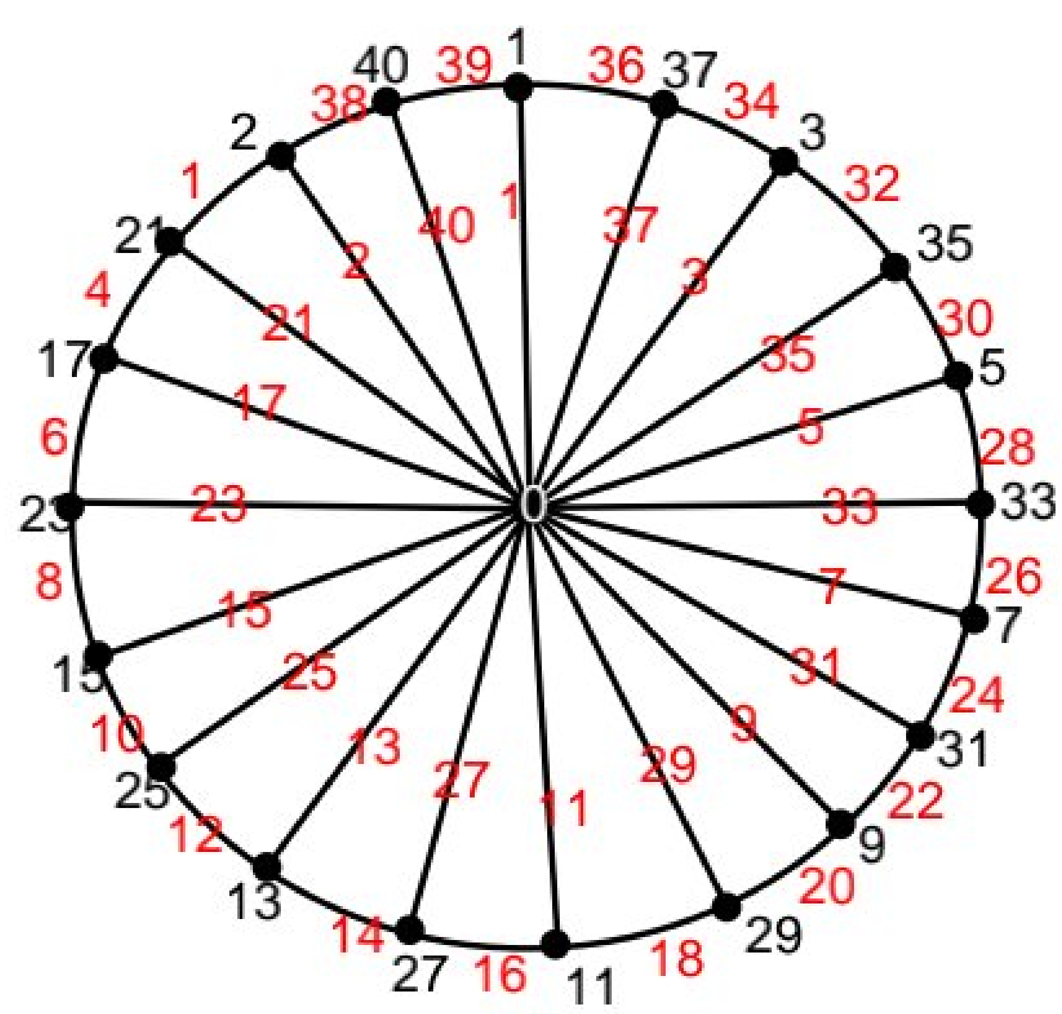
$= 32$

- $f(v_4, v_5) = |f(v_4) - f(v_5)|$
  $= |35 - 5|$
  $= 30$
- $f(v_5, v_6) = |f(v_5) - f(v_6)|$
  $= |5 - 33|$
  $= 28$
- $f(v_6, v_7) = |f(v_6) - f(v_7)|$
  $= |33 - 7|$
  $= 26$
- $f(v_7, v_8) = |f(v_7) - f(v_8)|$
  $= |7 - 31|$
  $= 24$
- $f(v_8, v_9) = |f(v_8) - f(v_9)|$
  $= |31 - 9|$
  $= 22$
- $f(v_9, v_{10}) = |f(v_9) - f(v_{10})|$
  $= |9 - 29|$
  $= 20$
- $f(v_{1o}, v_{11}) = |f(v_{10}) - f(v_{11})|$
  $= |29 - 11|$
  $= 18$
- $f(v_{11}, v_{12}) = |f(v_{11}) - f(v_{12})|$
  $= |11 - 27|$
  $= 16$
- $f(v_{12}, v_{13}) = |f(v_{12}) - f(v_{13})|$
  $= |27 - 13|$
  $= 14$
- $f(v_{13}, v_{14}) = |f(v_{13}) - f(v_{14})|$
  $= |13 - 25|$
  $= 12$
- $f(v_{14}, v_{15}) = |f(v_{14}) - f(v_{15})|$
  $= |25 - 15|$
  $= 10$
- $f(v_{15}, v_{16}) = |f(v_{15}) - f(v_{16})|$
  $= |15 - 23|$
  $= 8$
- $f(v_{16}, v_{17}) = |f(v_{16}) - f(v_{17})|$
  $= |23 - 17|$
  $= 6$
- $f(v_{17}, v_{18}) = |f(v_{17}) - f(v_{18})|$
  $= |17 - 21|$
  $= 4$

- $f(v_o, v_{18}) = |f(v_0) - f(v_{18})|$
  $= |0 - 21|$
  $= 21$
- $f(v_o, v_{19}) = |f(v_0) - f(v_{19})|$
  $= |0 - 2|$
  $= 2$
- $f(v_o, v_{20}) = |f(v_0) - f(v_{20})|$
  $= |0 - 40|$
  $= 40$
- $f(v_{18}, v_{19}) = |f(v_{18}) - f(v_{19})|$
  $= |21 - 2|$
  $= 19$
- $f(v_{19}, v_{20}) = |f(v_{19}) - f(v_{20})|$
  $= |2 - 40|$
  $= 38$
- $f(v_{20}, v_1) = |f(v_{20}) - f(v_1)|$
  $= |40 - 1|$
  $= 39$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf roda dengan $n = 20$ merupakan *graceful.*

Dapat diilustrasikan sebagai berikut:



Lampiran 2 pelabelan *graceful* pada graf kipas F_n dengan beberapa nilai n.

Untuk $n = 11$

- Pelabelan titik

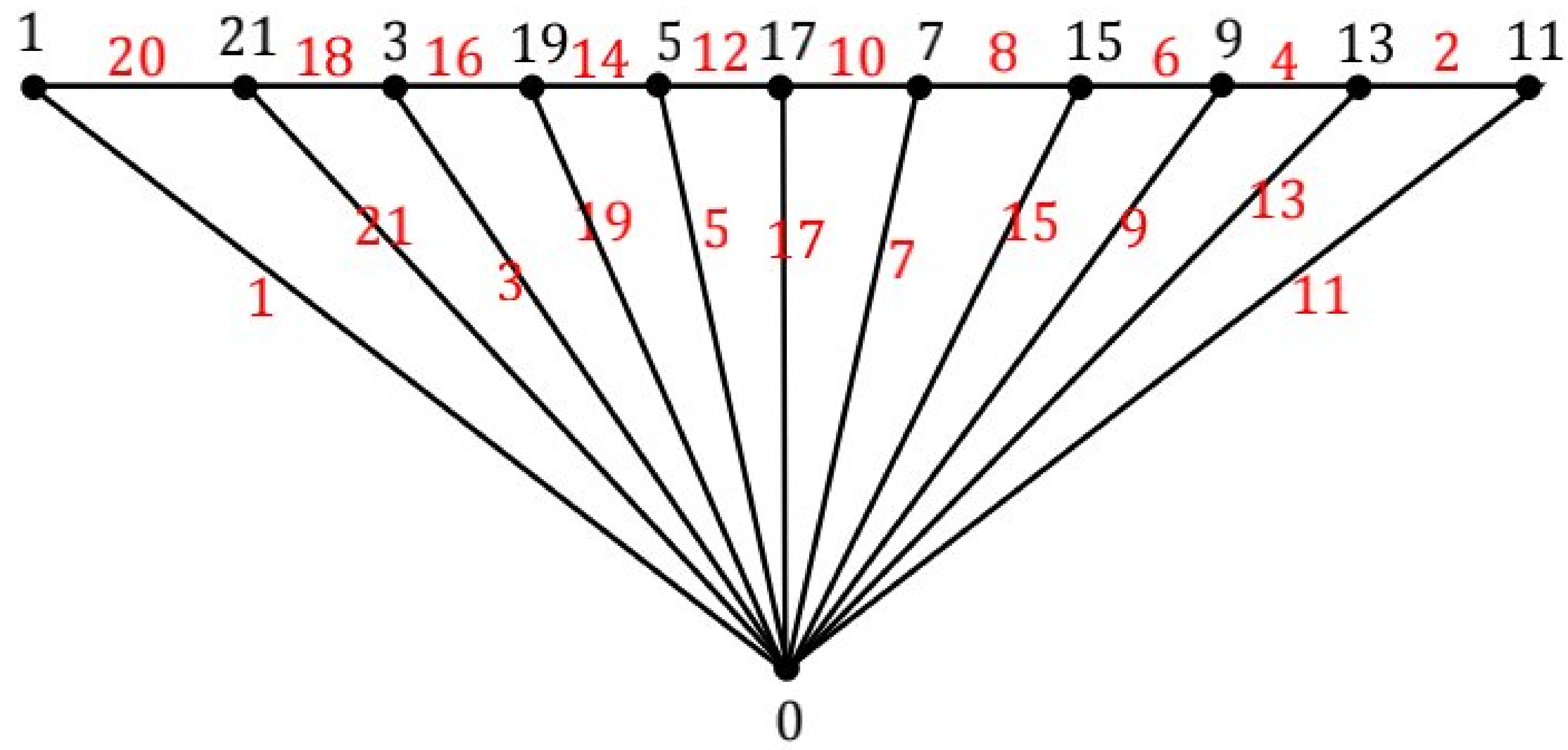
$f(v_0) = 0$ $\quad f(v_6) = 17$
$f(v_1) = 1$ $\quad f(v_7) = 7$
$f(v_2) = 21$ $\quad f(v_8) = 15$
$f(v_3) = 3$ $\quad f(v_9) = 9$
$f(v_4) = 19$ $\quad f(v_{10}) = 13$
$f(v_5) = 5$ $\quad f(v_{11}) = 11$

- Menentukan pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)| = |0 - 1| = 1$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)| = |1 - 21| = 20$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)| = |0 - 21| = 21$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)| = |21 - 3| = 18$
  - $f(v_o, v_3) = |f(v_0) - f(v_3)| = |0 - 3| = 3$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)| = |3 - 19| = 16$
  - $f(v_o, v_4) = |f(v_0) - f(v_4)| = |0 - 19| = 19$
  - $f(v_4, v_5) = |f(v_4) - f(v_5)| = |19 - 5| = 14$
  - $f(v_o, v_5) = |f(v_0) - f(v_5)| = |0 - 5| = 5$
  - $f(v_5, v_6) = |f(v_5) - f(v_6)| = |5 - 17| = 12$
  - $f(v_o, v_6) = |f(v_0) - f(v_6)| = |0 - 17| = 17$
  - $f(v_6, v_7) = |f(v_6) - f(v_7)| = |17 - 7| = 10$
  - $f(v_o, v_7) = |f(v_0) - f(v_7)| = |0 - 7| = 7$
  - $f(v_7, v_8) = |f(v_7) - f(v_8)| = |7 - 15| = 8$
  - $f(v_o, v_8) = |f(v_0) - f(v_8)| = |0 - 15| = 15$
  - $f(v_8, v_9) = |f(v_8) - f(v_9)| = |15 - 9| = 6$
  - $f(v_o, v_9) = |f(v_0) - f(v_9)| = |0 - 9| = 9$
  - $f(v_9, v_{10}) = |f(v_9) - f(v_{10})| = |9 - 13| = 4$
  - $f(v_o, v_{10}) = |f(v_0) - f(v_{10})| = |0 - 13| = 13$
  - $f(v_{1o}, v_{11}) = |f(v_{10}) - f(v_{11})| = |13 - 11| = 2$

- $f(v_o, v_{11}) = |f(v_0) - f(v_{11})|$
  $= |0 - 11|$
  $= 11$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf kipas dengan $n = 11$ merupakan *graceful.*

Dapat diilustrasikan sebagai berikut:



Untuk $n = 14$

- Pelabelan titik

$f(v_0) = 0$
$f(v_1) = 1$
$f(v_2) = 27$
$f(v_3) = 3$
$f(v_4) = 25$
$f(v_5) = 5$
$f(v_6) = 23$
$f(v_7) = 7$
$f(v_8) = 21$
$f(v_9) = 9$
$f(v_{10}) = 19$
$f(v_{11}) = 11$
$f(v_{12}) = 17$
$f(v_{13}) = 13$
$f(v_{14}) = 15$

- Pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)|$
    $= |0 - 1|$
    $= 1$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)|$
    $= |0 - 27|$
    $= 27$
  - $f(v_o, v_3) = |f(v_0) - f(v_3)|$
    $= |0 - 3|$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)|$
    $= |1 - 27|$
    $= 26$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)|$
    $= |27 - 3|$
    $= 24$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)|$
    $= |3 - 25|$

$= 3$

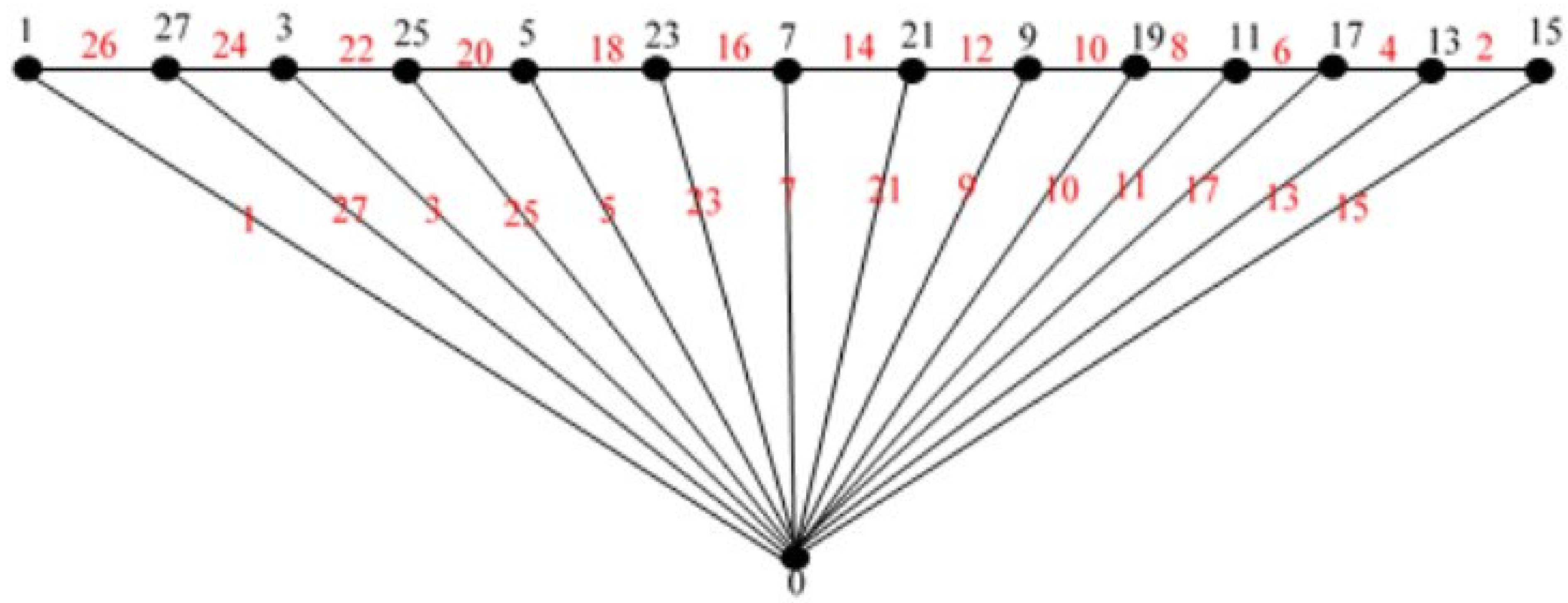
- $f(v_o, v_4) = |f(v_0) - f(v_4)|$
  $= |0 - 25|$
  $= 25$
- $f(v_o, v_5) = |f(v_0) - f(v_5)|$
  $= |0 - 5|$
  $= 5$
- $f(v_o, v_6) = |f(v_0) - f(v_6)|$
  $= |0 - 23|$
  $= 23$
- $f(v_o, v_7) = |f(v_0) - f(v_7)|$
  $= |0 - 7|$
  $= 7$
- $f(v_o, v_8) = |f(v_0) - f(v_8)|$
  $= |0 - 21|$
  $= 21$
- $f(v_o, v_9) = |f(v_0) - f(v_9)|$
  $= |0 - 9|$
  $= 9$
- $f(v_o, v_{10}) = |f(v_0) - f(v_{10})|$
  $= |0 - 19|$
  $= 19$
- $f(v_o, v_{11}) = |f(v_0) - f(v_{11})|$
  $= |0 - 11|$
  $= 11$
- $f(v_o, v_{12}) = |f(v_0) - f(v_{12})|$
  $= |0 - 17|$
  $= 17$
- $f(v_o, v_{13}) = |f(v_0) - f(v_{13})|$
  $= |0 - 13|$
  $= 13$
- $f(v_o, v_{14}) = |f(v_0) - f(v_{14})|$
  $= |0 - 15|$
  $= 15$

$= 22$

- $f(v_4, v_5) = |f(v_4) - f(v_5)|$
  $= |25 - 5|$
  $= 20$
- $f(v_5, v_6) = |f(v_5) - f(v_6)|$
  $= |5 - 18|$
  $= 18$
- $f(v_6, v_7) = |f(v_6) - f(v_7)|$
  $= |23 - 7|$
  $= 16$
- $f(v_7, v_8) = |f(v_7) - f(v_8)|$
  $= |7 - 21|$
  $= 14$
- $f(v_8, v_9) = |f(v_8) - f(v_9)|$
  $= |21 - 9|$
  $= 12$
- $f(v_9, v_{10}) = |f(v_9) - f(v_{10})|$
  $= |9 - 19|$
  $= 10$
- $f(v_{1o}, v_{11}) = |f(v_{10}) - f(v_{11})|$
  $= |19 - 11|$
  $= 8$
- $f(v_{11}, v_{12}) = |f(v_{11}) - f(v_{12})|$
  $= |11 - 17|$
  $= 6$
- $f(v_{12}, v_{13}) = |f(v_{12}) - f(v_{13})|$
  $= |17 - 13|$
  $= 14$
- $f(v_{13}, v_{14}) = |f(v_{13}) - f(v_{14})|$
  $= |13 - 15|$
  $= 2$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf kipas dengan $n = 14$ merupakan *graceful.*

Dapat diilustrasikan sebagai berikut:



<u>Untuk $n = 15$</u>

- Pelabelan titik

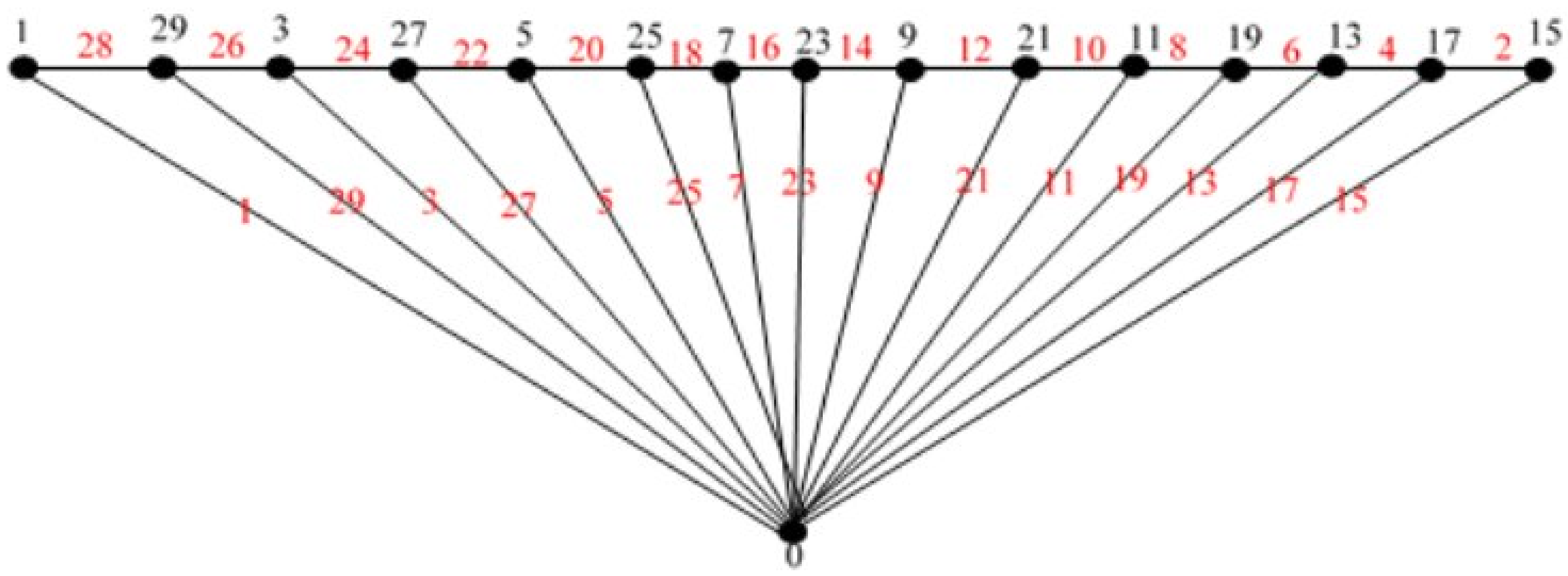
$f(v_0) = 0$
$f(v_1) = 1$
$f(v_2) = 29$
$f(v_3) = 3$
$f(v_4) = 27$
$f(v_5) = 5$
$f(v_6) = 25$
$f(v_7) = 7$
$f(v_8) = 23$
$f(v_9) = 8$
$f(v_{10}) = 21$
$f(v_{11}) = 11$
$f(v_{12}) = 19$
$f(v_{13}) = 13$
$f(v_{14}) = 17$
$f(v_{15}) = 15$

- Pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)| = |0 - 1| = 1$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)| = |0 - 29| = 29$
  - $f(v_o, v_3) = |f(v_0) - f(v_3)| = |0 - 3| = 3$
  - $f(v_o, v_4) = |f(v_0) - f(v_4)| = |0 - 27| = 27$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)| = |1 - 29| = 28$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)| = |29 - 3| = 26$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)| = |3 - 27| = 24$
  - $f(v_4, v_5) = |f(v_4) - f(v_5)| = |27 - 5| = 22$

- $f(v_o, v_5) = |f(v_0) - f(v_5)|$
  $= |0 - 5|$
  $= 5$
- $f(v_o, v_6) = |f(v_0) - f(v_6)|$
  $= |0 - 27|$
  $= 27$
- $f(v_o, v_7) = |f(v_0) - f(v_7)|$
  $= |0 - 7|$
  $= 9$
- $f(v_o, v_8) = |f(v_0) - f(v_8)|$
  $= |0 - 23|$
  $= 23$
- $f(v_o, v_9) = |f(v_0) - f(v_9)|$
  $= |0 - 9|$
  $= 9$
- $f(v_o, v_{10}) = |f(v_0) - f(v_{10})|$
  $= |0 - 21|$
  $= 21$
- $f(v_o, v_{11}) = |f(v_0) - f(v_{11})|$
  $= |0 - 11|$
  $= 11$
- $f(v_o, v_{12}) = |f(v_0) - f(v_{12})|$
  $= |0 - 19|$
  $= 19$
- $f(v_o, v_{13}) = |f(v_0) - f(v_{13})|$
  $= |0 - 13|$
  $= 13$
- $f(v_o, v_{14}) = |f(v_0) - f(v_{14})|$
  $= |0 - 17|$
  $= 17$
- $f(v_o, v_{15}) = |f(v_0) - f(v_{15})|$
  $= |0 - 15|$
  $= 15$
- $f(v_5, v_6) = |f(v_5) - f(v_6)|$
  $= |5 - 25|$
  $= 20$
- $f(v_6, v_7) = |f(v_6) - f(v_7)|$
  $= |25 - 7|$
  $= 18$
- $f(v_7, v_8) = |f(v_7) - f(v_8)|$
  $= |7 - 23|$
  $= 16$
- $f(v_8, v_9) = |f(v_8) - f(v_9)|$
  $= |23 - 9|$
  $= 14$
- $f(v_9, v_{10}) = |f(v_9) - f(v_{10})|$
  $= |9 - 21|$
  $= 12$
- $f(v_{10}, v_{11}) = |f(v_{10}) - f(v_{11})|$
  $= |21 - 11|$
  $= 10$
- $f(v_{11}, v_{12}) = |f(v_{11}) - f(v_{12})|$
  $= |11 - 19|$
  $= 8$
- $f(v_{12}, v_{13}) = |f(v_{12}) - f(v_{13})|$
  $= |19 - 13|$
  $= 6$
- $f(v_{13}, v_{14}) = |f(v_{13}) - f(v_{14})|$
  $= |13 - 17|$
  $= 4$
- $f(v_{14}, v_{15}) = |f(v_{14}) - f(v_{15})|$
  $= 17 - 17$
  $= 2$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa pelabelan *graceful* pada graf kipas dengan $n = 15$ merupakan *graceful.*

Dapat diilustrasikan sebagai berikut:



Untuk $n = 20$

- Pelabelan titik

$f(v_0) = 0$
$f(v_1) = 1$
$f(v_2) = 39$
$f(v_3) = 3$
$f(v_4) = 37$
$f(v_5) = 5$
$f(v_6) = 35$
$f(v_7) = 7$
$f(v_8) = 33$
$f(v_9) = 7$
$f(v_{10}) = 31$
$f(v_{11}) = 11$
$f(v_{12}) = 29$
$f(v_{13}) = 13$
$f(v_{14}) = 27$
$f(v_{15}) = 15$
$f(v_{16}) = 25$
$f(v_{17}) = 17$
$f(v_{18}) = 23$
$f(v_{19}) = 19$
$f(v_{20}) = 21$

- Menentukan pelabelan sisi
  - $f(v_o, v_1) = |f(v_0) - f(v_1)| = |0 - 1| = 1$
  - $f(v_o, v_2) = |f(v_0) - f(v_2)| = |0 - 39| = 39$
  - $f(v_o, v_3) = |f(v_0) - f(v_3)| = |0 - 3| = 3$
  - $f(v_o, v_4) = |f(v_0) - f(v_4)| = |0 - 37| = 37$
  - $f(v_1, v_2) = |f(v_1) - f(v_2)| = |1 - 39| = 38$
  - $f(v_2, v_3) = |f(v_2) - f(v_3)| = |39 - 3| = 36$
  - $f(v_3, v_4) = |f(v_3) - f(v_4)| = |3 - 37| = 34$
  - $f(v_4, v_5) = |f(v_4) - f(v_5)| = |37 - 5| = 32$

- $f(v_o, v_5) = |f(v_0) - f(v_5)|$
  $= |0 - 5|$
  $= 5$
- $f(v_o, v_6) = |f(v_0) - f(v_6)|$
  $= |0 - 35|$
  $= 35$
- $f(v_o, v_7) = |f(v_0) - f(v_7)|$
  $= |0 - 7|$
  $= 7$
- $f(v_o, v_8) = |f(v_0) - f(v_8)|$
  $= |0 - 33|$
  $= 33$
- $f(v_o, v_9) = |f(v_0) - f(v_9)|$
  $= |0 - 9|$
  $= 9$
- $f(v_o, v_{10}) = |f(v_0) - f(v_{10})|$
  $= |0 - 31|$
  $= 31$
- $f(v_o, v_{11}) = |f(v_0) - f(v_{11})|$
  $= |0 - 11|$
  $= 11$
- $f(v_o, v_{12}) = |f(v_0) - f(v_{12})|$
  $= |0 - 29|$
  $= 29$
- $f(v_o, v_{13}) = |f(v_0) - f(v_{13})|$
  $= |0 - 13|$
  $= 13$
- $f(v_o, v_{14}) = |f(v_0) - f(v_{14})|$
  $= |0 - 27|$
  $= 27$
- $f(v_o, v_{15}) = |f(v_0) - f(v_{15})|$
  $= |0 - 15|$
  $= 15$
- $f(v_o, v_{16}) = |f(v_0) - f(v_{16})|$
  $= |0 - 25|$
  $= 25$
- $f(v_o, v_{17}) = |f(v_0) - f(v_{17})|$
  $= |0 - 17|$
  $= 17$
- $f(v_o, v_{18}) = |f(v_0) - f(v_{18})|$
  $= |0 - 23|$
  $= 23$
- $f(v_o, v_{19}) = |f(v_0) - f(v_{19})|$
- $f(v_5, v_6) = |f(v_5) - f(v_6)|$
  $= |5 - 35|$
  $= 30$
- $f(v_6, v_7) = |f(v_6) - f(v_7)|$
  $= |35 - 7|$
  $= 28$
- $f(v_7, v_8) = |f(v_7) - f(v_8)|$
  $= |7 - 33|$
  $= 26$
- $f(v_8, v_9) = |f(v_8) - f(v_9)|$
  $= |33 - 9|$
  $= 24$
- $f(v_9, v_{10}) = |f(v_9) - f(v_{10})|$
  $= |9 - 31|$
  $= 22$
- $f(v_{1o}, v_{11}) = |f(v_{10}) - f(v_{11})|$
  $= |31 - 11|$
  $= 20$
- $f(v_{11}, v_{12}) = |f(v_{11}) - f(v_{12})|$
  $= |11 - 29|$
  $= 18$
- $f(v_{12}, v_{13}) = |f(v_{12}) - f(v_{13})|$
  $= |29 - 13|$
  $= 16$
- $f(v_{13}, v_{14}) = |f(v_{13}) - f(v_{14})|$
  $= |13 - 27|$
  $= 14$
- $f(v_{14}, v_{15}) = |f(v_{14}) - f(v_{15})|$
  $= |27 - 15|$
  $= 12$
- $f(v_{15}, v_{16}) = |f(v_{15}) - f(v_{16})|$
  $= |15 - 25|$
  $= 10$
- $f(v_{16}, v_{17}) = |f(v_{16}) - f(v_{17})|$
  $= |25 - 17|$
  $= 8$
- $f(v_{17}, v_{18}) = |f(v_{17}) - f(v_{18})|$
  $= |17 - 23|$
  $= 6$
- $f(v_{18}, v_{19}) = |f(v_{18}) - f(v_{19})|$
  $= |23 - 19|$
  $= 4$
- $f(v_{19}, v_{20}) = |f(v_{19}) - f(v_{20})|$

$= |0 - 19|$
$= 19$

$= |19 - 21|$
$= 2$

- $f(v_o, v_{20}) = |f(v_0) - f(v_{20})|$
$= |0 - 21|$
$= 21$

Telah dibuktikan bahwa hasil pelabelan titik tidak ada yang sama dan pelabelan sisi juga tidak ada yang sama maka dapat disimpulkan bahwa graf kipas dengan n=20 merupakan *graceful.*

Dapat diilustrasikan sebagai berikut: