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LAMPIRAN

Lampiran 1 Persamaan Diferensial Non-Eksak

Definisi 1 (Ross, 2004)

Misalkan F yaitu fungsi dari dua variabel real yang mempunyai turunan parsial pertama yang kontinu pada domain D. Turunan total dF dari fungsi F didefinisikan sebagai berikut

$$dF(x,y) = \frac{\partial F(x,y)}{\partial x}dx + \frac{\partial F(x,y)}{\partial y}dy$$

untuk setiap $(x,y) \in D$.

Definisi 2 (Ross, 2004)

$$M(x,y)dx + N(x,y)dy. \qquad (1)$$

Ekspresi (1) disebut diferensial eksak pada domain D jika terdapat fungsi F yang merupakan fungsi dari dua variabel real sedemikian sehingga

$$\frac{\partial F(x,y)}{\partial x} = M(x,y), \qquad \frac{\partial F(x,y)}{\partial y} = N(x,y)$$

untuk setiap $(x,y) \in D$.

Jika $M(x,y)dx + N(x,y)dy$ adalah diferensial eksak, maka persamaan diferensial berikut

$$M(x,y)dx + N(x,y)dy = 0$$

disebut persamaan diferensial eksak.

Teorema 1 (Ross, 2004)

Misalkan terdapat persamaan diferensial

$$M(x,y)dx + N(x,y)dy = 0 \qquad (2)$$

dengan M dan N mempunyai turunan parsial pertama yang kontinu pada semua titik (x,y) dalam domain D.

1. *Jika Persamaan (2) adalah persamaan diferensial eksak pada domain D, maka*

$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$$

untuk semua $(x,y) \in D$.

2. *Sebaliknya, jika*

$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$$

untuk semua $(x, y) \in D$*, maka Persamaan (2) adalah persamaan diferensial eksak pada domain* D*.*

Teorema 2 (Ross, 2004)

Misalkan persamaan diferensial

$$M(x,y)dx + N(x,y)dy = 0$$

memenuhi Teorema 1 dan eksak pada domain D*, maka* $F(x,y) = c$ *dengan* F *adalah fungsi sedemikian sehingga*

$$\frac{\partial F(x,y)}{\partial x} = M(x,y), \qquad \frac{\partial F(x,y)}{\partial y} = N(x,y)$$

untuk setiap $(x,y) \in D$ *dan* c *merupakan sembarang konstanta.*

Definisi 3 (Ross, 2004)

Jika persamaan diferensial

$$M(x,y)dx + N(x,y)dy = 0 \tag{3}$$

bukan persamaan diferensial eksak pada domain D*, tetapi persamaan diferensial*

$$\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0$$

merupakan persamaan diferensial eksak pada domain D*, maka* $\mu(x,y)$ *disebut faktor integrasi dari persamaan diferensial (3).*

1. Solusi umum dari $\tilde{g}_0$ pada (4.62)

Pada (4.62) terdapat persamaan diferensial berikut yang perlu diselesaikan

$$\frac{d\tilde{g}_0}{d\tilde{\tau}} = -\alpha_G^* \phi_G^* \tilde{n}_0 \tilde{g}_0 + \delta_G^* (1 - \tilde{g}_0). \tag{4}$$

Substitusi solusi dari $\tilde{n}_0$ pada (4.64) yaitu

$$\tilde{n}_0 = \frac{p_S e^{-\tilde{\tau}}}{\alpha_G^* \tilde{g}_0}$$

ke Persamaan (4), diperoleh persamaan berikut

$$\frac{d\tilde{g}_0}{d\tilde{\tau}} = -\phi_G^* p_S e^{-\tilde{\tau}} + \delta_G^* (1 - \tilde{g}_0),$$

$$\left(\phi_G^* p_S e^{-\tilde{\tau}} - \delta_G^* (1 - \tilde{g}_0)\right) d\tilde{\tau} + d\tilde{g}_0 = 0. \tag{5}$$

Misalkan $u = e^{\tilde{\tau}}$, maka $\frac{du}{d\tilde{\tau}} = e^{\tilde{\tau}}$ atau $d\tilde{\tau} = \frac{du}{e^{\tilde{\tau}}} = \frac{du}{u}$, sehingga Persamaan (5) menjadi

$$\left(\frac{\phi_G^* p_S}{u^2} - \frac{\delta_G^* (1 - \tilde{g}_0)}{u}\right) du + d\tilde{g}_0 = 0 \tag{6}$$

Misalkan $M_1(u,\tilde{g}_0)=\left(\frac{\phi_G^* p_S}{u^2}-\frac{\delta_G^*(1-\tilde{g}_0)}{u}\right)$ dan $N_1(u,\tilde{g}_0)=1$, maka diperoleh $\frac{\partial M_1(u,\tilde{g}_0)}{\partial \tilde{g}_0}=\frac{\delta_G^*}{u}$ dan $\frac{\partial N_1(u,\tilde{g}_0)}{\partial u}=0$. Berdasarakan Teorema 1, Persamaan (6) bukan merupakan persamaan diferensial eksak sebab $\frac{\partial M_1(u,\tilde{g}_0)}{\partial \tilde{g}_0}\neq\frac{\partial N_1(u,\tilde{g}_0)}{\partial u}$. Oleh karena itu, perlu dicari terlebih dahulu faktor integrasi $\mu(u)$ sehingga Persamaan (6) menjadi persamaan diferensial eksak, yaitu

$$\frac{\partial\big(\mu(u)M_1(u,\tilde{g}_0)\big)}{\partial \tilde{g}_0}=\frac{\partial\big(\mu(u)N_1(u,\tilde{g}_0)\big)}{\partial u},$$

$$\mu(u)\frac{\partial M_1(u,\tilde{g}_0)}{\partial \tilde{g}_0}=N_1(u,\tilde{g}_0)\frac{\partial \mu(u)}{\partial u}+\mu(u)\frac{\partial N_1(u,\tilde{g}_0)}{\partial u},$$

$$\frac{\partial \mu(u)}{\partial u}=\frac{\mu(u)}{N_1(u,\tilde{g}_0)}\left(\frac{\partial M_1(u,\tilde{g}_0)}{\partial \tilde{g}_0}-\frac{\partial N_1(u,\tilde{g}_0)}{\partial u}\right),$$

$$\ln\mu(u)=\int\frac{1}{N_1(u,\tilde{g}_0)}\left(\frac{\partial M_1(u,\tilde{g}_0)}{\partial \tilde{g}_0}-\frac{\partial N_1(u,\tilde{g}_0)}{\partial u}\right)\partial u\,,$$

$$\begin{aligned}\mu(u)&=e^{\int\frac{1}{N_1(u,\tilde{g}_0)}\left(\frac{\partial M_1(u,\tilde{g}_0)}{\partial \tilde{g}_0}-\frac{\partial N_1(u,\tilde{g}_0)}{\partial u}\right)\partial u}\\&=e^{\int\frac{\delta_G^*}{u}\partial u}\\&=e^{\ln u^{\delta_G^*}+A_1}\\&=Au^{\delta_G^*}.\end{aligned}$$

Misalkan $A=1$, sehingga faktor integrasi yang digunakan yaitu $\mu(u)=u^{\delta_G^*}$. Kalikan faktor integrasi $\mu(u)$ tersebut ke Persamaan (6), diperoleh

$$u^{\delta_G^*}\left(\frac{\phi_G^* p_S}{u^2}-\frac{\delta_G^*(1-\tilde{g}_0)}{u}\right)du+u^{\delta_G^*}d\tilde{g}_0=0 \qquad (7)$$

Misalkan $M_2(u,\tilde{g}_0)=u^{\delta_G^*}\left(\frac{\phi_G^* p_S}{u^2}-\frac{\delta_G^*(1-\tilde{g}_0)}{u}\right)$ dan $N_2(u,\tilde{g}_0)=u^{\delta_G^*}$, maka diperoleh $\frac{\partial M_2(u,\tilde{g}_0)}{\partial \tilde{g}_0}=\delta_G^* u^{\delta_G^*-1}$ dan $\frac{\partial N_2(u,\tilde{g}_0)}{\partial u}=\delta_G^* u^{\delta_G^*-1}$. Berdasarkan Teorema 1, Persamaan (7) merupakan persamaan diferensial eksak sebab $\frac{\partial M_2(u,\tilde{g}_0)}{\partial \tilde{g}_0}=\frac{\partial N_2(u,\tilde{g}_0)}{\partial u}$. Berdasarkan Definisi 1 dan 2, terdapat fungsi $F(u,\tilde{g}_0)$ sedemikian sehingga

$$\frac{\partial F(u,\tilde{g}_0)}{\partial u}=M_2(u,\tilde{g}_0)=u^{\delta_G^*}\left(\frac{\phi_G^* p_S}{u^2}-\frac{\delta_G^*(1-\tilde{g}_0)}{u}\right) \qquad (8)$$

dan

$$\frac{\partial F(u,\tilde{g}_0)}{\partial \tilde{g}_0} = N_2(u,\tilde{g}_0) = u^{\delta_G^*}. \tag{9}$$

Berdasarkan Persamaan (8), diperoleh

$$F(u,\tilde{g}_0) = \int M_2(u,\tilde{g}_0)\,\partial u + \gamma(\tilde{g}_0)$$

$$= \int u^{\delta_G^*}\left(\frac{\phi_G^* p_S}{u^2} - \frac{\delta_G^*(1-\tilde{g}_0)}{u}\right)\partial u + \gamma(\tilde{g}_0)$$

$$= u^{\delta_G^*}\left(\frac{\phi_G^* p_S}{(\delta_G^*-1)u} - 1 + \tilde{g}_0\right) + \gamma(\tilde{g}_0). \tag{10}$$

Selanjutnya, berdasarkan Persamaan (10), diperoleh

$$\frac{\partial F(u,\tilde{g}_0)}{\partial \tilde{g}_0} = u^{\delta_G^*} + \frac{d\gamma(\tilde{g}_0)}{d\tilde{g}_0}. \tag{11}$$

Bandingkan Persamaan (9) dengan Persamaan (11), diperoleh

$$u^{\delta_G^*} + \frac{d\gamma(\tilde{g}_0)}{d\tilde{g}_0} = u^{\delta_G^*},$$

$$\frac{d\gamma(\tilde{g}_0)}{d\tilde{g}_0} = 0,$$

$$\gamma(\tilde{g}_0) = k_1. \tag{12}$$

Substitusi Persamaan (12) ke Persamaan (10), diperoleh

$$F(u,\tilde{g}_0) = u^{\delta_G^*}\left(\frac{\phi_G^* p_S}{(\delta_G^*-1)u} - 1 + \tilde{g}_0\right) + k_1.$$

Berdasarkan Teorema 2 yaitu $F(u,\tilde{g}_0) = k_2$ maka diperoleh

$$u^{\delta_G^*}\left(\frac{\phi_G^* p_S}{(\delta_G^*-1)u} - 1 + \tilde{g}_0\right) + k_1 = k_2,$$

$$\tilde{g}_0 = -\frac{\phi_G^* p_S}{(\delta_G^*-1)u} + 1 + \frac{K_{17}}{u^{\delta_G^*}}. \tag{13}$$

Substitusi $u = e^{\tilde{\tau}}$ ke Persamaan (13), diperoleh solusi umum dari $\tilde{g}_0$ pada (4.62) yaitu

$$\tilde{g}_0 = -\frac{\phi_G^* p_S}{\delta_G^*-1}e^{-\tilde{\tau}} + 1 + K_{17}e^{-\delta_G^*\tau}.$$

2. Solusi umum dari g_0° pada (4.99)

Pada (4.99) terdapat persamaan diferensial berikut yang perlu diselesaikan

$$\frac{dg_0^\circ}{d\tau^\circ} = -\alpha_G^*\phi_G^* n_0^\circ g_0^\circ + \delta_G^*(1-g_0^\circ). \tag{14}$$

Substitusi solusi dari n_0° pada (4.101) yaitu

$$n_0^\circ = \frac{p_S e^{-\mu_1-\mu_2-\tau^\circ}}{\alpha_G^* g_0^\circ}$$

ke Persamaan (14), diperoleh persamaan berikut

$$\frac{dg_0^\circ}{d\tau^\circ} = -\phi_G^* p_S e^{-\mu_1-\mu_2-\tau^\circ} + \delta_G^*(1 - g_0^\circ). \tag{15}$$

Berdasarkan Persamaan (4.94) yaitu

$$\mu_2 = \ln \frac{\phi_G^* p_S}{\delta_G^*} - \mu_1$$

maka Persamaan (15) dapat ditulis sebagai berikut

$$\frac{dg_0^\circ}{d\tau^\circ} = \delta_G^*\left(-e^{-\tau^\circ} + 1 - g_0^\circ\right),$$

$$\delta_G^*\left(e^{-\tau^\circ} - 1 + g_0^\circ\right)d\tau^\circ + dg_0^\circ = 0. \tag{16}$$

Misalkan $u = e^{\tau^\circ}$, maka $\frac{du}{d\tau^\circ} = e^{\tau^\circ}$ atau $d\tau^\circ = \frac{du}{e^{\tau^\circ}} = \frac{du}{u}$, sehingga Persamaan (16) menjadi

$$\delta_G^*\left(\frac{1}{u^2} + \frac{g_0^\circ - 1}{u}\right)du + dg_0^\circ = 0. \tag{17}$$

Misalkan $M_1(u, g_0^\circ) = \delta_G^*\left(\frac{1}{u^2} + \frac{g_0^\circ - 1}{u}\right)$ dan $N_1(u, g_0^\circ) = 1$, maka diperoleh $\frac{\partial M_1(u,g_0^\circ)}{\partial g_0^\circ} = \frac{\delta_G^*}{u}$ dan $\frac{\partial N_1(u,g_0^\circ)}{\partial u} = 0$. Berdasarkan Teorema 1, Persamaan (17) bukan merupakan persamaan diferensial eksak sebab $\frac{\partial M_1(u,g_0^\circ)}{\partial g_0^\circ} \neq \frac{\partial N_1(u,g_0^\circ)}{\partial u}$. Oleh karena itu, perlu dicari terlebih dahulu factor integrasi $\mu(u)$ sehingga Persamaan (17) menjadi persamaan diferensial eksak, yaitu

$$\frac{\partial\big(\mu(u)M_1(u,g_0^\circ)\big)}{\partial g_0^\circ} = \frac{\partial\big(\mu(u)N_1(u,g_0^\circ)\big)}{\partial u},$$

$$\mu(u)\frac{\partial M_1(u,g_0^\circ)}{\partial g_0^\circ} = N_1(u,g_0^\circ)\frac{\partial \mu(u)}{\partial u} + \mu(u)\frac{\partial N_1(u,g_0^\circ)}{\partial u},$$

$$\frac{\partial \mu(u)}{\partial u} = \frac{\mu(u)}{N_1(u,g_0^\circ)}\left(\frac{\partial M_1(u,g_0^\circ)}{\partial g_0^\circ} - \frac{\partial N_1(u,g_0^\circ)}{\partial u}\right),$$

$$\ln \mu(u) = \int \frac{1}{N_1(u,g_0^\circ)}\left(\frac{\partial M_1(u,g_0^\circ)}{\partial g_0^\circ} - \frac{\partial N_1(u,g_0^\circ)}{\partial u}\right)\partial u\,,$$

$$\begin{aligned}\mu(u) &= e^{\int \frac{1}{N_1(u,g_0^\circ)}\left(\frac{\partial M_1(u,g_0^\circ)}{\partial g_0^\circ} - \frac{\partial N_1(u,g_0^\circ)}{\partial u}\right)\partial u} \\ &= e^{\int \frac{\delta_G^*}{u}\partial u} \\ &= e^{\ln u^{\delta_G^*} + A_1} \\ &= Au^{\delta_G^*}.\end{aligned}$$

Misalkan $A = 1$, sehingga faktor integrasi yang digunakan yaitu $\mu(u) = u^{\delta_G^*}$. Kalikan faktor integrasi $\mu(u)$ tersebut ke Persamaan (17), diperoleh

$$u^{\delta_G^*}\delta_G^*\left(\frac{1}{u^2} + \frac{g_0^\circ - 1}{u}\right)du + u^{\delta_G^*}dg_0^\circ = 0. \qquad (18)$$

Misalkan $M_2(u,g_0^\circ) = u^{\delta_G^*}\delta_G^*\left(\frac{1}{u^2} + \frac{g_0^\circ - 1}{u}\right)$ dan $N_2(u,g_0^\circ) = u^{\delta_G^*}$, maka diperoleh $\frac{\partial M_2(u,g_0^\circ)}{\partial g_0^\circ} = \delta_G^* u^{\delta_G^* - 1}$ dan $\frac{\partial N_2(u,g_0^\circ)}{\partial u} = \delta_G^* u^{\delta_G^* - 1}$. Berdasarkan Teorema 1, Persamaan (18) merupakan persamaan diferensial eksak sebab $\frac{\partial M_2(u,g_0^\circ)}{\partial g_0^\circ} = \frac{\partial N_2(u,g_0^\circ)}{\partial u}$. Berdasarkan Definisi 1 dan 2, terdapat fungsi $F(u,g_0^\circ)$ sedemikian sehingga

$$\frac{\partial F(u,g_0^\circ)}{\partial u} = M_2(u,g_0^\circ) = u^{\delta_G^*}\delta_G^*\left(\frac{1}{u^2} + \frac{g_0^\circ - 1}{u}\right) \qquad (19)$$

dan

$$\frac{\partial F(u,g_0^\circ)}{\partial g_0^\circ} = N_2(u,g_0^\circ) = u^{\delta_G^*}. \qquad (20)$$

Berdasarkan Persamaan (19), diperoleh

$$F(u, g_0^\circ) = \int M_2(u, g_0^\circ)\, \partial u + \gamma(g_0^\circ)$$

$$= \int u^{\delta_G^*} \delta_G^* \left(\frac{1}{u^2} + \frac{g_0^\circ - 1}{u} \right) \partial u + \gamma(g_0^\circ)$$

$$= u^{\delta_G^*} \left(\frac{\delta_G^*}{(\delta_G^* - 1)u} + g_0^\circ - 1 \right) + \gamma(g_0^\circ). \quad (21)$$

Selanjutnya, berdasarkan Persamaan (21), diperoleh

$$\frac{\partial F(u, g_0^\circ)}{\partial g_0^\circ} = u^{\delta_G^*} + \frac{d\gamma(g_0^\circ)}{dg_0^\circ}. \quad (22)$$

Bandingkan Persamaan (20) dengan Persamaan (22), diperoleh

$$u^{\delta_G^*} + \frac{d\gamma(g_0^\circ)}{dg_0^\circ} = u^{\delta_G^*},$$

$$\frac{d\gamma(g_0^\circ)}{dg_0^\circ} = 0,$$

$$\gamma(g_0^\circ) = k_1. \quad (23)$$

Substitusi Persamaan (23) ke Persamaan (21), diperoleh

$$F(u, g_0^\circ) = u^{\delta_G^*} \left(\frac{\delta_G^*}{(\delta_G^* - 1)u} + g_0^\circ - 1 \right) + k_1.$$

Berdasarkan Teorema 2 yaitu $F(u, g_0^\circ) = k_2$ maka diperoleh

$$u^{\delta_G^*} \left(\frac{\delta_G^*}{(\delta_G^* - 1)u} + g_0^\circ - 1 \right) + k_1 = k_2,$$

$$g_0^\circ = -\frac{\delta_G^*}{(\delta_G^* - 1)u} + 1 + \frac{K_{22}}{u^{\delta_G^*}}. \quad (24)$$

Substitusi $u = e^{\tau^\circ}$ ke Persamaan (24), diperoleh solusi umum dari g_0° pada (4.99) yaitu

$$g_0^\circ = -\frac{\delta_G^*}{(\delta_G^* - 1)} e^{-\tau^\circ} + 1 + K_{22} e^{-\delta_G^* \tau^\circ}. \quad (25)$$

Berdasarkan Persamaan (4.94) diperoleh

$$\delta_G^* = \phi_G^* p_S e^{-\mu_1 - \mu_2}$$

sehingga Persamaan (25) dapat ditulis sebagai berikut

$$g_0^\circ = -\frac{\phi_G^* p_S e^{-\mu_1 - \mu_2 - \tau^\circ}}{(\delta_G^* - 1)} + 1 + K_{22} e^{-\delta_G^* \tau^\circ}.$$

Lampiran 2 Nilai Parameter pada *Non-Dimensionalisation* Model untuk $\varepsilon = 0{,}008$

Tabel 1 Deskripsi parameter baru pada non-dimensionalisation model untuk $\varepsilon = 0{,}008$.

Parameter	**Nilai**
δ_G	0,669
δ_S	0,669
δ_{PSH}	36,789
α_S	1,002
α_G	3676,254
ϕ_S	87,17
ϕ_G	168,122
v_{F1}	0,0047
v_{R1}	0,002
k_{P1}	$1{,}706 \,.\, 10^{-9}$
k_{N1}	$9{,}167 \,.\, 10^{-10}$
v_{F2}	0,0033
v_{R2}	0,003
k_{P2}	$4{,}185 \,.\, 10^{-9}$
k_{N2}	$3{,}6901 \,.\, 10^{-9}$

Lampiran 3 Metode Newton-Rhapson

Metode Newton-Rhapson dapat digunakan untuk memperoleh solusi dari $f(x) = 0$. Langkah pertama yang dilakukan dalam metode tersebut yaitu menentukan nilai tebakan awal x_0. Berikutnya, lakukan iterasi dengan formula berikut

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0, 1, 2, 3, ...$$

sampai mencapai toleransi eror atau jumlah iterasi yang ditetapkan (Sauer, 2012; Burden & Faires, 2011). Persamaan (4.115) yaitu

$$1 + \varepsilon \frac{\phi_G p_S}{\delta_G - 1} \left(e^{-\delta_G \mu_1} - e^{-\mu_1}\right) = 0$$

tidak dapat diselesaikan secara langsung untuk menemukan nilai dari μ_1. Tetapi dengan nilai parameter pada Tabel 4.6, nilai μ_1 dapat ditentukan menggunakan metode Newton-Rhapson. Nilai tebakan awal yang digunakan yaitu ${\mu_1}_0 = 0$. Berdasarkan Tabel 2 dapat dilihat bahwa nilai dari μ_1 mulai dari iterasi 3 dan seterusnya memiliki nilai yang sama pada 6 digit pertama sehingga diperoleh $\mu_1 \approx 0{,}22404$.

Tabel 2 Nilai μ_1 pada setiap iterasi dengan menggunakan metode Newton-Rhapson

i	$\boldsymbol{\mu_{1_i}}$	$\mathbf{f(\mu_{1_i})}$	$\mathbf{f'(\mu_{1_i})}$	$\boldsymbol{\mu_{1_{i+1}}}$
0	0	1	-5,3799	0,18588
1	0,18588	0,1435472	-3,8944	0,22274
2	0,22274	0,0047287	-3,6399	0,22404
3	0,22404	5,685E-06	-3,6311	0,22404
4	0,22404	8,248E-12	-3,6311	0,22404
5	0,22404	0	-3,6311	0,22404
6	0,22404	0	-3,6311	0,22404
7	0.22404	0	-3.6311	0.22404
8	0.22404	0	-3.6311	0.22404
9	0.22404	0	-3.6311	0.22404
10	0.22404	0	-3.6311	0.22404

Lampiran 4 Sintaks Matlab

Berikut adalah sintaks matlab yang digunakan dalam tugas akhir ini
https://rb.gy/8lkix3