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LAMPIRAN

Lampiran 1

```
clear all;
clc;
clf;
global a1 a2 a3 w1 w2 w3 lambda alpha eta1 eta2 miu beta omega
gamma phi rho teta epsilon delta tau qiu pii J

% Nilai parameter model
%lambda= 0.0121; miu=0.0096; alpha=0.01842; eta1=0.04;
eta2=0.05372; beta=0.036;
%gamma=0.003; omega=0.002; phi=0.0009; epsilon=0.02; rho=0.6;
teta=0.4; delta=0.03; tau=0.94; qiu=0.11;

lambda= 0.0096; miu=0.0096; alpha=0.01458; eta1=0.04;
eta2=0.05372; beta=0.036;
gamma=0.003; omega=0.002; phi=0.0009; epsilon=0.02; rho=0.6;
teta=0.4; delta=0.025; tau=0.94; qiu=0.11; pii=0.03;

%Nilai Bobot
a1=1; a2=50; a3=20; w1=1; w2=20; w3=20;
% Nilai awal state


x10=535.000;
x20=160.500;
x30=107.000;
x40=80.250;
x50=26.750;
x60=53.500;


x0=[x10;x20;x30;x40;x50;x60];
%Nilai akhir Costate (syarat transversalitas)
nx=6;
lambdaT=zeros(nx,1);
%Interval waktu
Ntime=500;
tf=50;
% tf=15;
ti=linspace(0,tf,Ntime);
%Batas kontrol
M1=0;
M2=1;
nv=3;
Lb=M1.*ones(nv,Ntime);
Ub=M2.*ones(nv,Ntime);
%Parameter Sweep
```

```
test=-1;
deltaa=0.000001;
k=0;
%tebakan awal untuk fungsi kontrol u1, u2 dan u3
u=0*ones(nv,Ntime);
%solving sistem tanpa kontrol (u1=0 u2=0 u3=0)
options = odeset('AbsTol',1e-3,'RelTol',1e-3);
xc=ode45(@(t,x) ika_state(t, x, u, ti),[0 tf],x0,options);
xc=deval(xc,ti);
%Awal Metode Sweep
x=zeros(nx,Ntime);
p=zeros(nx,Ntime);
while(test<0)
 k=k+1;
 oldx=x;
 oldp=p;
 oldu=u;
 %Forward Runge Kutta
 x=deval(ode45(@(t,x) ika_state(t, x, u, ti), [0 tf], x0),ti);
 %Backward Runge Kutta %yg digunakan nilai akhir lambdaT=0
 p=deval(ode45(@(t,p) ika_costate(t, p, x, u, ti),[tf
0],lambdaT),ti);
 %menghitung nilai u dari syarat optimal sistem
 u1=ika_kontrol(x,p); %menggunakan u dH/du=0
 %membuat u berada dalam interval yang diharapkan
 u1=ika_simplebounds(u1,Lb,Ub);
 %mengupdate nilau u dalam metode sweep menggunakan kombinasi
konveks
 u=0.5*(u1+oldu); %uji Konvergensi u yang pertama
  %menghitung nilai error
 temp1=deltaa*sum(abs(u))-sum(abs(oldu-u));
 temp2=deltaa*sum(abs(x))-sum(abs(oldx-x));
 temp3=deltaa*sum(abs(p))-sum(abs(oldp-p));
 test=min(temp1,min(temp2,temp3)); %Buku Lenhart Hal:55
 %menghitung nilai fungsi tujuan menggunakan u akhir

 J(k)=ika_objektif(x,u,ti);
  disp(['it:',num2str(k),',Test:',num2str(test)])
end
 %menghitung nilai fungsi tujuan menggunakan u optimal
 [m,n]=size(J);
 Ju=ika_objektif(x,u,ti);

XC1 = sum(xc(1,1:end-1)).*tf./Ntime;
disp (XC1)
XC2 = sum(xc(2,1:end-1)).*tf./Ntime;
disp (XC2)
XC3 = sum(xc(3,1:end-1)).*tf./Ntime;
disp (XC3)
XC4 = sum(xc(4,1:end-1)).*tf./Ntime;
disp (XC4)
XC5 = sum(xc(5,1:end-1)).*tf./Ntime;
disp (XC5)
XC6 = sum(xc(6,1:end-1)).*tf./Ntime;
disp (XC6)
```

```
XK1 = sum(x(1,1:end-1)).*tf./Ntime;
disp (XK1)
XK2 = sum(x(2,1:end-1)).*tf./Ntime;
disp (XK2)
XK3 = sum(x(3,1:end-1)).*tf./Ntime;
disp (XK3)
XK4 = sum(x(4,1:end-1)).*tf./Ntime;
disp (XK4)
XK5 = sum(x(5,1:end-1)).*tf./Ntime;
disp (XK5)
XK6 = sum(x(6,1:end-1)).*tf./Ntime;
disp (XK6)


%Jint=trapz(ti,J);
%menghitung nilai fungsi tujuan menggunakan u optimal
figure (1)
subplot (3,2,1)
plot(ti,xc(1,:),'R--',ti, x(1,:),'b-','LineWidth',2)
  %plot(ti, x(1,:),'black-','LineWidth',2)
 xlabel('Waktu (Tahun)')
 ylabel('S(t)')
legend('S Tanpa Kontrol','S dengan Kontrol')
  axis('tight')
 grid on
 hold on

%,u_2(t),u_3(t)
subplot (3,2,2)
plot(ti,xc(2,:),'R--',ti, x(2,:),'b-','LineWidth',2)
%plot(ti, x(2,:),'black-','LineWidth',2)
 xlabel('Waktu (Tahun)')
 ylabel('E(t)')
legend('E Tanpa Kontrol','E dengan Kontrol')
 axis('tight')
 grid on
 hold on


subplot (3,2,3)
plot(ti,xc(3,:),'R--',ti, x(3,:),'b-','LineWidth',2)
%plot(ti, x(3,:),'black-','LineWidth',2)
 xlabel('Waktu (Tahun)')
 ylabel('A(t)')
legend('A Tanpa Kontrol','A dengan Kontrol')
 axis('tight')
 grid on
 hold on


subplot (3,2,4)
plot(ti,xc(4,:),'R--',ti, x(4,:),'b-','LineWidth',2)
% plot(ti, x(4,:),'black-','LineWidth',2)
 xlabel('Waktu (Tahun)')
 ylabel('C(t)')
```

```
legend('C Tanpa Kontrol','C dengan Kontrol')
 axis('tight')
 grid on
 hold on


subplot (3,2,5)
plot(ti,xc(5,:),'R--',ti, x(5,:),'b-','LineWidth',2)
 % plot(ti, x(5,:),'black-','LineWidth',2)
 xlabel('Waktu (Tahun)')
 ylabel('L(t)')
legend('L Tanpa Kontrol','L dengan Kontrol')
 axis('tight')
 grid on
 hold on

subplot (3,2,6)
plot(ti,xc(6,:),'R--',ti, x(6,:),'b-','LineWidth',2)
 %plot(ti, x(6,:),'black-','LineWidth',2)
 xlabel('Waktu (Tahun)')
 ylabel('R(t)')
legend('R Tanpa Kontrol','R dengan Kontrol')
 axis('tight')
 grid on
 hold on



figure (2)
subplot(221)
plot(ti,u(1,:),'b-',ti,u(2,:),'g-',ti,u(3,:),'r-','LineWidth',2)
legend('kontrol u_1 (Vaksinasi)','kontrol u_2(Kampanye)','kontrol
u_3 (Pengobatan)')
xlabel('Waktu ')
ylabel('u_1(t), u_2(t), u_3(t)')
title('Fungsi Kontrol')
axis('tight')
grid on

subplot(222)
plot(ti,u(2,:),'g-','LineWidth',2)
legend('kontrol u_2(Kampanye)')
xlabel('Waktu ')
ylabel('u_2(t)')
title('Fungsi Kontrol')
axis('tight')
grid on

subplot(223)
plot(ti,u(3,:),'r-','LineWidth',2)
legend('kontrol u_3(Pengobatan)')
xlabel('Waktu ')
ylabel('u_3(t)')
title('Fungsi Kontrol')
axis('tight')
```

```
grid on

subplot(224)
plot(ti,u(1,:),'b-','LineWidth',2)
legend('kontrol u_1(Vaksinasi)')
xlabel('Waktu ')
ylabel('u_1(t)')
title('Fungsi Kontrol')
axis('tight')
grid on



function J=ika_objektif(x,u,ti)
global a1 a2 a3 w1 w2 w3
x2=x(2,:);
x3=x(3,:);
x4=x(4,:);
x5=x(5,:);
x6=x(6,:);
u1=u(1,:);
u2=u(2,:);
u3=u(3,:);
obj=a1.*x3+a2.*x4+a3.*x5+(w1./2).*u1.^2 +(w2./2).*u2.^2
+(w3./2).*u3.^2;
J=trapz(ti,obj);


function u = ika_kontrol(x,p)
global w1 w2 w3 qiu lambda tau
%theta

p1 = p(1,:);
p2 = p(2,:);
p3 = p(3,:);
p4 = p(4,:);
p5 = p(5,:);
p6 = p(6,:);

x1 = x(1,:);
x2 = x(2,:);
x3 = x(3,:);
x4 = x(4,:);
x5 = x(5,:);
x6 = x(6,:);

%fungsi kontrol:
u1=(lambda.*tau.*(((1-qiu.*x4).*p1)+(qiu.*x4.*p4)+p1-p6))./w1;
u2=(((p3-p6).*x3)+((p4-p6).*x4))./w2;
u3=((p5-p6).*x5)./w3;

u=[u1;u2;u3];
end
```

```
function dx=ika_state(t, x, u, ti) %ti adalah inputan %t,x,u
adalah variabel
global  lambda alpha eta1 eta2 miu beta omega gamma phi rho
epsilon delta teta tau qiu pii
x1=x(1);
x2=x(2);
x3=x(3);
x4=x(4);
x5=x(5);
x6=x(6);
u1=u(1,:);
u1=interp1(ti,u1',t);
u2=u(2,:);
u2=interp1(ti,u2',t);
u3=u(3,:);
u3=interp1(ti,u3',t);

dx=zeros(6,1);

dx(1)=(lambda.*(1-(u1.*tau).*(1-qiu.*x4)))-
(alpha.*(x3+(eta1.*x4)+(eta2.*x5)).*x1)-(miu+u1).*x1;
dx(2)=(alpha.*(x3+(eta1.*x4)+(eta2.*x5)).*x1)-(beta+miu).*x2;
dx(3)=(beta.*x2)-(omega+gamma+miu+u2).*x3;
dx(4)=(omega.*x3)-(phi+miu+delta+u2).*x4+(lambda.*(1-
(u1.*tau)).*qiu.*x4);
dx(5)=(phi.*x4)+(teta.*delta.*x4)+(rho.*gamma.*x3)-
(miu+epsilon+pii+u3).*x5;
dx(6)=((1-rho).*gamma.*x3)+((1-teta).*delta.*x4)-
(miu.*x6)+(pii.*x5)+(lambda.*tau.*u1)+(u1.*x1)+(u2.*x3)+(u2.*x4)+(
u3.*x5);
end
```

```
function dp=ika_costate(t, p, x, u, ti)
global a1 a2 a3 w1 w2 w3 lambda alpha eta1 eta2 miu beta omega
gamma phi rho teta epsilon delta tau qiu pii
%theta
x = interp1(ti,x',t);

x1 = x(1);
x2 = x(2);
x3 = x(3);
x4 = x(4);
x5 = x(5);
x6 = x(6);
u1 = u(1,:);
u2 = u(2,:);
u3 = u(3,:);
u1 = interp1(ti,u1',t);
u2 = interp1(ti,u2',t);
u3 = interp1(ti,u3',t);
```

```
p1=p(1,:);
p2=p(2,:);
p3=p(3,:);
p4=p(4,:);
p5=p(5,:);
p6=p(6,:);
dp=zeros(6,1);
dp(1)=((p1-p2).*(alpha.*(x3+(eta1.*x4)+(eta2.*x5))))+(p1.*miu);
dp(2)=((p2-p3).*beta)+(p2.*miu);
dp(3)=((p1-p2).*alpha.*x1)+((p3-p6).*u2)+((p6-
p5).*rho.*gamma)+((p3-p6).*gamma)+(p3-p4.*omega)+(p3.*miu)-a1;
dp(4)=((p1-p2).*alpha.*eta1.*x1)+((p4-
p1).*lambda.*qiu.*tau.*u1)+((p1-p4).*lambda.*qiu)+((p4-
p6).*u2)+((p4-p6).*delta)+((p4-p5).*phi)+((p6-
p5).*teta.*delta)+(p4.*miu)-a2;
dp(5)=((p1-p2).*alpha.*eta2.*x1)+((p5-p6).*u3)+((p5-
p6).*pii)+(p5.*(miu+epsilon))-a3;
dp(6)=(p6.*miu);
end



function s=ika_simplebounds(s,Lb,Ub)


% untuk batas bawah
 ns_tmp=s;
 I=ns_tmp<Lb;
 ns_tmp(I)=Lb(I);

 % untuk batas atas
 J=ns_tmp>Ub;
 ns_tmp(J)=Ub(J);

 % Update u
 s=ns_tmp;
```