

DAFTAR PUSTAKA

- Cox, Douglas; Natalia Clifton; John W. Bartok; Taryn Lascola. 2010. *Greenhouse BMPs*. Massachusetts: Massachusetts Department of Agricultural Resources.
- Klainerman, Sergiu. 2006. *Partial Differential Equation*. Princeton: Princeton University.
- Pinchover, Yehuda dan Jacob Rubinstein. 2005. *An Indtroduction to Partial Defferential Equations*. New York: Cambridge University Press.
- Kusuma, Jeffry. (2018). *Persamaan Diferensial Parsial*. Makassar: Pusat Kajian Media dan Sumber Belajar Universitas Hasanuddin.
- Hoffman, Klaus A. dan Steve T. Chiang. 2000. *Computation Fluid Dynamics For Engineers*. Wichita: A Publication of Engineering Education System.
- Causon, D. M. dan C. G. Mingham. 2010. *Introductory Finite Difference Method for PDEs*. Manchester: Manchester Metropolitan University.
- Croft, David R. dan David J. Lilley. 1977. *Heat Transfer Calculation Using Finite Difference Equation*. Essex: Applied Science Publisher Ltd.
- Drazin, P. G. dan N. Riley. 2006. *The Navier-Stokes Equation: a Classification of Flows and Exact Solution*. New York: Cambridge University Press.
- Hud, Volodymyr; Oleg Pinchuk; Petro Martyniuk; Ievgenii Gerasimov; Pavlo Volk. 2019. *Mathematical Modelling of Heat Transfer in a Greenhouse with Surface Soil Heating System*. Rivne: Scientific Review Emgineering and Environmental Sciences.
- Zaman, Fahad Sadekin; Tanmoy Saha Turja; Md. Mamun Molla. 2013. *Buoyancy Driven Natural Convection Flow In an Enclosure with Two Discrete Heating From Below*. Dhaka: Elsevier Ltd.

LAMPIRAN

1. Program MATLAB Penyebaran Panas Di Dalam *Greenhouse*

```
clc
clear all

input('Tekan enter untuk memulai program atau ctrl+c untuk
membatalkan program')

dn      =1/100;
dx      =dn; dy=dn;
dt      =0.001;
nt      =200000;
waktu  =dt*nt;
nx      =100;ny=100;nk=0;nks=100;tn=0; tn2=0;
Re      =10000;
Pr      =0.71;
docalc =1;
error   =0.001;
tahap   =0;
kn      =0;
ts      =100000;
nsave   =0;
PIT     =0;
MAXIT   =1000;
MINIT   =1;
slcv    =0;
Beta    =dx/dy;

badumtes=input('0 kalau n>1, 1 kalau n=1 \n badumtes=')

if badumtes==0
    bk    =nt;
    U     =zeros(nx,ny); V=U;
    TPxku =matfile('TPx14005.mat'); TPx=TPxku.TPx;
    TPku  =matfile('TP14005.mat'); TP=TPku.TP; TP00=TP;
    omgxku=matfile('omgx14005.mat'); omgx=omgxku.omg;
    omgku =matfile('omg14005.mat'); omg=omgku.omg;
    vorku =matfile('vor14005.mat'); vor=vorku.vor;
    for i=2:nx-1
        for j=2:ny-1
            U(i,j) =(vor(i+1,j)-vor(i-1,j))./(2*dn);
            V(i,j) =(vor(i,j-1)-vor(i,j+1))./(2*dn);
        end
    end
else
    omg   =zeros(nx,ny); U=omg;V=omg;omgx=omg;
    vor   =zeros(nx,ny);
    TP    =zeros(nx,ny); TPx=omg;
end

nTP   =0; abTP=1;

mx    =nx-2;
my    =ny-2;

a     = ones(mx,1);
```

```

b      = ones(mx,1);
c      = ones(mx,1);
d      = ones(mx,1);
Y      = ones(mx,1);

fprintf('Real time simulasi diasumiskan dihitung untuk t =
%f',waktu)
input('')

while docalc==1

%Batas atas
U(1,2:ny-1)=0; V(1,2:ny-1)=0; vor(1,2:ny-1)=0;
%Batas kanan
U(2:nx-1,end)=0; V(2:nx-1,end)=0; vor(2:nx-1,end)=0;
%Batas bawah
U(end,round(ny/4):round(ny/2))=0;
U(end,round(ny/2)+1:round(3*ny/4)-1)=0; V(end,2:ny-1)=0;
vor(end,2:ny-1)=0;
%Batas kiri
U(2:nx-1,1)=0; V(2:nx-1,1)=0; vor(2:nx-1,1)=0;

%Batas kiri-atas
U(1,1)=0; V(1,1)=0; vor(1,1)=0;
%Batas kanan-atas
U(1,end)=0; V(1,end)=0; vor(1,end)=0;
%Batas kiri-bawah
U(end,1)=0; V(end,1)=0; vor(end,1)=0;
%Batas kanan-bawah
U(end,end)=0; V(end,end)=0; vor(end,end)=0;

%Batas atas omg
omg(1,2:ny-1)=2*(vor(1,2:ny-1)-vor(2,2:ny-1))./(dn.^2);
%Batas kanan omg
omg(2:nx-1,end)=2*(vor(2:nx-1,end)-vor(2:nx-1,end-1))./(dn.^2);
%Batas bawah omg
omg(end,2:ny-1)=2*(vor(end,2:ny-1)-vor(end-1,2:ny-1))./(dn.^2);
%Batas kiri omg
omg(2:nx-1,1)=2*(vor(2:nx-1,1)-vor(2:nx-1,2))./(dn.^2);

%Batas kiri atas
omg(1,1)=(omg(1,2)+omg(2,1))./2;
%Batas kanan atas
omg(1,end)=(omg(1,end-1)+omg(2,end))./2;
%Batas kiri bawah
omg(end,1)=(omg(end-1,1)+omg(end,2))./2;
%Batas kanan bawah
omg(end,end)=(omg(end-1,end)+omg(end,end-1))./2;

%Batas atas TP
TP(1,2:ny-1)=TP(2,2:ny-1);
TP(1,round(ny/4)-2:round(ny/4)+2)=1;
TP(1,round(3*ny/4)-2:round(3*ny/4)+2)=1;
TP(1,2:round(ny/4)-3)=TP(2,2:round(ny/4)-3);
TP(1,round(ny/4)+3:round(ny/2)-3)=TP(2,round(ny/4)+3:round(ny/2)-
3);
TP(1,round(3*ny/4)+3:ny-1)=TP(2,round(3*ny/4)+3:ny-1);

```

```

TP(1,round(ny/2)+3:round(3*ny/4)-
3)=TP(2,round(ny/2)+3:round(3*ny/4)-3);
TP(1,round(nx/2)-2:round(nx/2)+2)=1;
%Batas kanan TP
TP(2:nx-1,end)=0;
%Batas bawah TP
TP(end,2:ny-1)=TP(end-1,2:ny-1);
%Batas kiri TP
TP(2:nx-1,1)=0;

%Batas kiri atas
TP(1,1)=TP(2,1);
%Batas kanan atas
TP(1,end)=TP(2,end);
%Batas kiri bawah
TP(end,1)=TP(end-1,1);
%Batas kanan bawah
TP(end,end)=TP(end-1,end);

if kn~=0
    for i=2:nx-1
        for j=2:ny-1
            if U(i,j)>0
                epx=1;
            else
                epx=-1;
            end
            if V(i,j)>0
                epy=1;
            else
                epy=-1;
            end
            a(j-1)=-0.5*(0.5*(1+epx)*U(i,j-
1)*dt./dn+(dt/(dn.^2)));
            b(j-1)=1+(dt/(dn.^2))+0.5*epx*U(i,j)*dt./dn;
            c(j-1)=0.5*(0.5*(1-epx)*U(i,j+1)*dt/dn-(dt/(dn.^2)));
            d(j-1)=0.5*(0.5*(1+epy)*V(i-
1,j)*dt/dn+(dt/(dn.^2)))*omg(i-1,j)+(1-(dt/(dn.^2))-0.5*epy*V(i,j)*dt/dn)*omg(i,j)+0.5*(-0.5*(1-
epy)*V(i+1,j)*dt/dn+(dt/(dn.^2)))*omg(i+1,j)+dt*(TP(i,j+1)-TP(i,j-
1)).*Re/(2*(Pr*2*dn));
        end

        beta=b(1); omgx(i,2)=d(1)/beta;

        %Decomposition dan Forward substitution
        for j=3:ny-1
            gamma(j-1)=c(j-2)/beta;
            beta=b(j-1)-a(j-1)*gamma(j-1);
        if (beta==0)
            fprintf('Failed')
            pause
        end
        omgx(i,j)=(d(j-1)-a(j-1)*omgx(i,j-1))/beta;
    end

    %Backward substitution
    for k=ny-2:-1:2

```

```

        omgx(i,k)=omgx(i,k)-gamma(k)*omgx(i,k+1);
    end
end
for j=2:ny-1
    for i=2:nx-1
        if U(i,j)>0
            epx=1;
        else
            epx=-1;
        end
        if V(i,j)>0
            epy=1;
        else
            epy=-1;
        end
        a(i-1)=-0.5*(0.5*(1+epy)*V(i-
1,j)*dt./dn+(dt/(dn.^2)));
        b(i-1)=1+(dt/(dn.^2))+0.5*epy*V(i,j)*dt./dn;
        c(i-1)=0.5*(0.5*(1-epy)*V(i+1,j)*dt/dn-(dt/(dn.^2)));
        d(i-1)=0.5*(0.5*(1+epx)*U(i,j-
1)*dt/dn+(dt/(dn.^2)))*omgx(i,j-1)+(1-(dt/(dn.^2))-0.5*epx*U(i,j)*dt/dn)*omgx(i,j)+0.5*(-0.5*(1-
epx)*U(i,j+1)*dt/dn+(dt/(dn.^2)))*omgx(i,j+1)+dt*(TP(i,j+1)-
TP(i,j-1)).*Re/(2*(Pr*2*dn));
    end

beta=b(1); omg(2,j)=d(1)/beta;

%Decomposition dan Forward substitution
for i=3:nx-1
    gamma(i-1)=c(i-2)/beta;
    beta=b(i-1)-a(i-1)*gamma(i-1);
if (beta==0)
    fprintf('Failed')
    pause
end
omg(i,j)=(d(i-1)-a(i-1)*omg(i-1,j))/beta;
end

%Backward substitution
for k=nx-2:-1:2
    omg(k,j)=omg(k,j)-gamma(k)*omg(k+1,j);
end
end
end

if tn==1
    if kn>1
        omg1(1:nx,1:ny,tn)=omg(:,:,_);
        vor01(1:nx,1:ny,tn)=vor;
        vor11(1:nx,1:ny,tn)=vor;
        vor01(:,:,1)=0;
        vor01(:,:,end)=0;
        vor01(1,:,:)=0;
        vor01(end,:,:)=0;
        vor11(:,:,1)=0;
        vor11(:,:,end)=0;
        vor11(1,:,:)=0;
    end
end

```

```

        vor11(end,:,:)=0;
    else
        omg1(1:nx,1:ny,tn+1)=omg(:,:,);
        vor01(1:nx,1:ny,tn+1)=vor;
        vor11(1:nx,1:ny,tn+1)=vor;
    end
else
    omg1(1:nx,1:ny,tn+1)=omg(:,:,);
    vor01(1:nx,1:ny,tn+1)=vor;
    vor11(1:nx,1:ny,tn+1)=vor;
end

errach=(kn==0);
mite=0;
cp=(kn~=1).*1+(kn==1).*10;

while errach==0
    for i=2:nx-1
        for j=2:ny-1

vor11(i,j,tn+1)=0.25*vor01(i,j)+((dn.^2)*omg1(i,j,tn)+vor01(i,j+1,
tn)+vor01(i,j-1,tn+1)+((Beta).^2)*(vor01(i+1,j,tn)+vor01(i-
1,j,tn+1))).*0.75./(2*(1+(Beta).^2));
        end
    end
    errach=(max(max(max(abs(vor11(2:nx-1,2:ny-1,tn+1)-
vor01(2:nx-1,2:ny-1,tn+1))))))<=error;
    mite=mite+1;
    PFF=abs(vor11(2:nx-1,2:ny-1,tn+1)-vor01(2:nx-1,2:ny-
1,tn+1));
    vor01(:,:,:,tn+1)=vor11(:,:,:,tn+1);
    if mite>=(MAXIT*cp);
        errach=1;
    end
    if mite<MINIT
        errach=0;
    end
    if k>100 && slcv==0 && (mean(PIT)>(MAXIT)/2);

fprintf('!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!\n')
        fprintf('Slow convergence for Pressure poisson solver
(P.P) was detected!\n')
        fprintf('To increase speed of convergence you could stop
calculations and modify some\n')
        fprintf('parameters such as;\n')
        fprintf('-> decrease maximum allowed iterations P.P
("MAXIT")\n')
        fprintf('-> tolerance of error for pressure poisson
("error")\n')
        fprintf('-> delta time (dt)\n')
        fprintf('-> resolution of scenario ("MI")\n')
        fprintf('-> reynolds number (RE)\n')
        fprintf('-> velocity magnitude at boundary conditions
("velxi") and/or ("velyi")\n')
        fprintf('In the mean, time calculations will run until
manually stopped\n')

```

```

fprintf('!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!\n')
    slcv=1;
end
end

PIT(size(PIT,2)+1)=mite;

if kn~=0
    if sum(sum(isnan(vor01(:,:,tn+1))))~=0 ||
sum(sum(isinf(vor01(:,:,tn+1))))~=0 % if Pressure has
infinite or nan value
        fprintf('\n Error! The solution for pressure has diverged
(going to inf or nan)\n')
        fprintf('Try modifying dt,mu,xinc, domainX,velxi/velyi and
try again!\n')
        force_exit_code=intentional_error;
    end
end

vor(:,:,)=vor11(:,:,tn+1);

for i=2:nx-1
    for j=2:ny-1
        U(i,j)=(vor(i+1,j)-vor(i-1,j))./(2*dn);
        V(i,j)=(vor(i,j-1)-vor(i,j+1))./(2*dn);
    end
end

if kn~=0
    for i=2:nx-1
        for j=2:ny-1
            if U(i,j)>0
                epx=1;
            else
                epx=-1;
            end
            if V(i,j)>0
                epy=1;
            else
                epy=-1;
            end
            a(j-1)=-0.5*(0.5*(1+epx)*U(i,j-
1)*dt./dn+(dt/(Pr*(dn.^2))));
            b(j-1)=1+(dt/(Pr*(dn.^2)))+0.5*epx*U(i,j)*dt./dn;
            c(j-1)=0.5*(0.5*(1-epx)*U(i,j+1)*dt/dn-
(dt/(Pr*(dn.^2))));
            d(j-1)=0.5*(0.5*(1+epy)*V(i-
1,j)*dt/dn+(dt/(Pr*(dn.^2)))*TP(i-1,j)+(1-(dt/(Pr*(dn.^2)))-
0.5*epy*V(i,j)*dt/dn)*TP(i,j)+0.5*(-0.5*(1-
epy)*V(i+1,j)*dt/dn+(dt/(Pr*(dn.^2)))*TP(i+1,j));
        end
        beta=b(1); TPx(i,2)=d(1)/beta;
    end
    %Decomposition dan Forward substitution

```

```

for j=3:ny-1
    gamma(j-1)=c(j-2)/beta;
    beta=b(j-1)-a(j-1)*gamma(j-1);
if (beta==0)
    fprintf('Failed')
    pause
end
TPx(i,j)=(d(j-1)-a(j-1)*TPx(i,j-1))/beta;
end

%Backward substitution
for k=ny-2:-1:2
    TPx(i,k)=TPx(i,k)-gamma(k)*TPx(i,k+1);
end

end
for j=2:ny-1
    for i=2:nx-1
        if U(i,j)>0
            epx=1;
        else
            epx=-1;
        end
        if V(i,j)>0
            epy=1;
        else
            epy=-1;
        end
        a(i-1)=-0.5*(0.5*(1+epy)*V(i-
1,j)*dt./dn+(dt/(Pr*(dn.^2))));%
        b(i-1)=1+(dt/(Pr*(dn.^2)))+0.5*epy*V(i,j)*dt./dn;
        c(i-1)=0.5*(0.5*(1-epy)*V(i+1,j)*dt/dn-
(dt/(Pr*(dn.^2))));%
        d(i-1)=0.5*(0.5*(1+epx)*U(i,j-
1)*dt/dn+(dt/(Pr*(dn.^2)))*TPx(i,j-1)+(1-(dt/(Pr*(dn.^2)))-%
0.5*epx*U(i,j)*dt/dn)*TPx(i,j)+0.5*(-0.5*(1-
epx)*U(i,j+1)*dt/dn+(dt/(Pr*(dn.^2)))*TPx(i,j+1));
    end

beta=b(1); TP(2,j)=d(1)/beta;

%Dekomposisi dan Forward substitution
for i=3:nx-1
    gamma(i-1)=c(i-2)/beta;
    beta=b(i-1)-a(i-1)*gamma(i-1);
if (beta==0)
    fprintf('Failed')
    pause
end
TP(i,j)=(d(i-1)-a(i-1)*TP(i-1,j))/beta;
end

%Backward substitution
for k=nx-2:-1:2
    TP(k,j)=TP(k,j)-gamma(k)*TP(k+1,j);
end
end
end

```

```

kn=kn+1;
disp(kn)

if tn>= ts
    nsave=nsave+1;
    tn=0;
    clc
    tahap=tahap+1;
    disp(tahap)
end

%determine if calcuations are finished
if kn~=0
    if kn>=nt %if maximum allowed number of timesteps is reached
        docalc=0; %set parameter to finish calculations (while loop
does not continue)
        if tn~=0
            end
        fprintf('selesai')
        else
            tn=1;
        end
    else
        tn=tn+1;
    end
    fprintf('\n tn nih')
    disp(tn)
    bk=kn;
end

save(['TP' num2str(bk) '.mat'], 'TP');
save(['TPx' num2str(bk) '.mat'], 'TPx');
save(['omg' num2str(bk) '.mat'], 'omg');
save(['omgx' num2str(bk) '.mat'], 'omgx');
save(['vor' num2str(bk) '.mat'], 'vor');

fprintf('selesai')

```

2. Nondimensionalisasi Model Persamaan Navier-Stokes dan Persamaan Energi

Persamaan Navier-Stokes

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \zeta \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \zeta \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g \beta (T - T_c).$$

- Nondimensionalisasi

Misalkan:

$$X = \frac{x}{L}, Y = \frac{y}{L}, \tau = \frac{t\alpha}{L^2}, U = \frac{uL}{\alpha},$$

$$V = \frac{vL}{\alpha}, \Psi = \frac{\psi}{\alpha}, Re = \frac{g\beta\Delta TK}{\alpha\zeta}, Pr = \frac{\alpha}{\zeta}.$$

Maka,

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\alpha}{L} \frac{\partial U}{\partial \tau} \frac{\partial \tau}{\partial t} = \frac{\alpha^2}{L^3} \frac{\partial U}{\partial \tau}, & \frac{\partial v}{\partial t} &= \frac{\alpha}{L} \frac{\partial V}{\partial \tau} \frac{\partial \tau}{\partial t} = \frac{\alpha^2}{L^3} \frac{\partial V}{\partial \tau}, \\ u \frac{\partial u}{\partial x} &= \frac{\alpha}{L} U \frac{\partial u}{\partial x} = \frac{\alpha^2}{L^3} U \frac{\partial U}{\partial X}, & u \frac{\partial v}{\partial x} &= \frac{\alpha}{L} U \frac{\partial v}{\partial x} = \frac{\alpha^2}{L^3} U \frac{\partial V}{\partial X}, \\ v \frac{\partial u}{\partial y} &= \frac{\alpha}{L} V \frac{\partial u}{\partial y} = \frac{\alpha^2}{L^3} V \frac{\partial U}{\partial Y}, & v \frac{\partial v}{\partial y} &= \frac{\alpha}{L} V \frac{\partial v}{\partial y} = \frac{\alpha^2}{L^3} V \frac{\partial V}{\partial Y} \\ -\frac{1}{\rho} \frac{\partial p}{\partial x} &= -\frac{1}{\rho} \frac{L^2}{\rho \alpha^2} \frac{\partial P}{\partial X} \frac{\partial X}{\partial x} = -\frac{L}{\rho^2 \alpha^2} \frac{\partial P}{\partial X}, \\ -\frac{1}{\rho} \frac{\partial p}{\partial y} &= -\frac{1}{\rho} \frac{L^2}{\rho \alpha^2} \frac{\partial P}{\partial Y} \frac{\partial Y}{\partial y} = -\frac{L}{\rho^2 \alpha^2} \frac{\partial P}{\partial Y}, \\ \frac{\partial^2 u}{\partial x^2} &= \frac{\partial}{\partial x} \frac{\partial u}{\partial x} = \frac{\partial}{\partial X} \frac{\partial U}{\partial x} \left(\frac{\alpha}{L^2} \frac{\partial U}{\partial X} \right) = \frac{\alpha}{L^3} \frac{\partial^2 U}{\partial X^2}, \\ \frac{\partial^2 v}{\partial x^2} &= \frac{\partial}{\partial x} \frac{\partial v}{\partial x} = \frac{\partial}{\partial X} \frac{\partial V}{\partial x} \left(\frac{\alpha}{L^2} \frac{\partial V}{\partial X} \right) = \frac{\alpha}{L^3} \frac{\partial^2 V}{\partial X^2}, \\ \frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial y} \frac{\partial u}{\partial y} = \frac{\partial}{\partial Y} \frac{\partial U}{\partial y} \left(\frac{\alpha}{L^2} \frac{\partial U}{\partial Y} \right) = \frac{\alpha}{L^3} \frac{\partial^2 U}{\partial Y^2}, \\ \frac{\partial^2 v}{\partial y^2} &= \frac{\partial}{\partial y} \frac{\partial v}{\partial y} = \frac{\partial}{\partial Y} \frac{\partial V}{\partial y} \left(\frac{\alpha}{L^2} \frac{\partial V}{\partial Y} \right) = \frac{\alpha}{L^3} \frac{\partial^2 V}{\partial Y^2}, \end{aligned}$$

$$\rho g \beta (T - T_c) = \rho g \beta (\Theta(T_H - T_c) + T_c - T_c) = \rho g \beta (T_H - T_c) \Theta.$$

Diperoleh,

$$\begin{aligned} \frac{\alpha^2}{L^3} \frac{\partial U}{\partial \tau} + \frac{\alpha^2}{L^3} U \frac{\partial U}{\partial X} + \frac{\alpha^2}{L^3} V \frac{\partial U}{\partial Y} &= -\frac{L}{\rho^2 \alpha^2} \frac{\partial P}{\partial X} + \zeta \left(\frac{\alpha}{L^3} \frac{\partial^2 U}{\partial X^2} + \frac{\alpha}{L^3} \frac{\partial^2 U}{\partial Y^2} \right), \\ \frac{\alpha^2}{L^3} \frac{\partial V}{\partial \tau} + \frac{\alpha^2}{L^3} U \frac{\partial V}{\partial X} + \frac{\alpha^2}{L^3} V \frac{\partial V}{\partial Y} \\ &= -\frac{L}{\rho^2 \alpha^2} \frac{\partial P}{\partial Y} + \zeta \left(\frac{\alpha}{L^3} \frac{\partial^2 V}{\partial X^2} + \frac{\alpha}{L^3} \frac{\partial^2 V}{\partial Y^2} \right) + \rho g \beta (T_H - T_C) \Theta. \end{aligned}$$

- Formulasi Persamaan Vorticity-Stream

Turunkan secara persamaan U terhadap Y , dan persamaan V terhadap X .

$$\begin{aligned} \frac{\partial}{\partial Y} \left(\frac{\alpha^2}{L^3} \frac{\partial U}{\partial \tau} + \frac{\alpha^2}{L^3} U \frac{\partial U}{\partial X} + \frac{\alpha^2}{L^3} V \frac{\partial U}{\partial Y} \right. \\ \left. = -\frac{L}{\rho^2 \alpha^2} \frac{\partial P}{\partial X} + \zeta \left(\frac{\alpha}{L^3} \frac{\partial^2 U}{\partial X^2} + \frac{\alpha}{L^3} \frac{\partial^2 U}{\partial Y^2} \right) \right), \\ \frac{\partial}{\partial X} \left(\frac{\alpha^2}{L^3} \frac{\partial V}{\partial \tau} + \frac{\alpha^2}{L^3} U \frac{\partial V}{\partial X} + \frac{\alpha^2}{L^3} V \frac{\partial V}{\partial Y} \right. \\ \left. = -\frac{L}{\rho^2 \alpha^2} \frac{\partial P}{\partial Y} + \zeta \left(\frac{\alpha}{L^3} \frac{\partial^2 V}{\partial X^2} + \frac{\alpha}{L^3} \frac{\partial^2 V}{\partial Y^2} \right) + \rho g \beta (T_H - T_C) \Theta \right). \end{aligned}$$

Diperoleh,

$$\begin{aligned} \frac{\alpha^2}{L^3} \frac{\partial^2 U}{\partial Y \partial \tau} + \frac{\alpha^2}{L^3} \frac{\partial U}{\partial Y} \frac{\partial U}{\partial X} + \frac{\alpha^2}{L^3} U \frac{\partial^2 U}{\partial Y \partial X} + \frac{\alpha^2}{L^3} \frac{\partial V}{\partial Y} \frac{\partial U}{\partial Y} + \frac{\alpha^2}{L^3} V \frac{\partial^2 U}{\partial Y^2} \\ = -\frac{L}{\rho^2 \alpha^2} \frac{\partial^2 P}{\partial Y \partial X} + \zeta \left(\frac{\alpha}{L^3} \frac{\partial^3 U}{\partial Y \partial X^2} + \frac{\alpha}{L^3} \frac{\partial^3 U}{\partial Y^3} \right), \\ \frac{\alpha^2}{L^3} \frac{\partial^2 V}{\partial X \partial \tau} + \frac{\alpha^2}{L^3} \frac{\partial U}{\partial X} \frac{\partial V}{\partial X} + \frac{\alpha^2}{L^3} U \frac{\partial^2 V}{\partial X^2} + \frac{\alpha^2}{L^3} \frac{\partial V}{\partial X} \frac{\partial V}{\partial Y} + \frac{\alpha^2}{L^3} V \frac{\partial^2 V}{\partial X \partial Y} \\ = -\frac{L}{\rho^2 \alpha^2} \frac{\partial^2 P}{\partial Y \partial X} + \zeta \left(\frac{\alpha}{L^3} \frac{\partial^3 V}{\partial X^3} + \frac{\alpha}{L^3} \frac{\partial^3 V}{\partial X \partial Y^2} \right) + \rho g \beta (T_H - T_C) \frac{\partial \Theta}{\partial X}. \end{aligned}$$

Kurangkan persamaan nondimensional U dengan persamaan nondimensional V .

$$\begin{aligned}
& \frac{\alpha^2}{L^3} \frac{\partial}{\partial \tau} \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) + \frac{\alpha^2}{L^3} \frac{\partial U}{\partial X} \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) + \frac{\alpha^2}{L^3} U \frac{\partial}{\partial X} \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) \\
& + \frac{\alpha^2}{L^3} \frac{\partial V}{\partial Y} \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) + \frac{\alpha^2}{L^3} V \frac{\partial}{\partial Y} \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) \\
& = - \frac{L}{\rho^2 \alpha^2} \frac{\partial^2 P}{\partial Y \partial X} + \frac{L}{\rho^2 \alpha^2} \frac{\partial^2 P}{\partial Y \partial X} \\
& + \zeta \left(\frac{\alpha}{L^3} \frac{\partial^2}{\partial X^2} \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) + \frac{\alpha}{L^3} \frac{\partial^2}{\partial Y^2} \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) \right) \\
& - \rho g \beta (T_H - T_C) \frac{\partial \Theta}{\partial X},
\end{aligned}$$

dimana,

$$\begin{aligned}
-\Omega &= \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}, \\
U &= \frac{\partial \Psi}{\partial Y}, V = -\frac{\partial \Psi}{\partial X},
\end{aligned}$$

substitusi U dan V ke persamaan $-\Omega$.

$$\begin{aligned}
& \frac{\alpha^2}{L^3} \frac{\partial \Omega}{\partial \tau} + \frac{\alpha^2}{L^3} \left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) \Omega + \frac{\alpha^2}{L^3} U \frac{\partial \Omega}{\partial X} + \frac{\alpha^2}{L^3} V \frac{\partial \Omega}{\partial Y} \\
& = \zeta \left(\frac{\alpha}{L^3} \frac{\partial^2 \Omega}{\partial X^2} + \frac{\alpha}{L^3} \frac{\partial^2 \Omega}{\partial Y^2} \right) + \rho g \beta (T_H - T_C) \frac{\partial \Theta}{\partial X}, \\
-\Omega &= \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2}.
\end{aligned}$$

dimana,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$

diperoleh persamaan nonkonservatif Vorticity-Stream,

$$\begin{aligned}
& \frac{\alpha^2}{L^3} \frac{\partial \Omega}{\partial \tau} + \frac{\alpha^2}{L^3} U \frac{\partial \Omega}{\partial X} + \frac{\alpha^2}{L^3} V \frac{\partial \Omega}{\partial Y} = \frac{\zeta \alpha}{L^3} \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) + \rho g \beta (T_H - T_C) \frac{\partial \Theta}{\partial X}, \\
-\Omega &= \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2},
\end{aligned}$$

atau

$$\begin{aligned}
& \frac{\partial \Omega}{\partial \tau} + U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} = \frac{1}{Pr} \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) + \frac{Re}{Pr} \frac{\partial \Theta}{\partial X}, \\
-\Omega &= \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2},
\end{aligned}$$

dimana,

$$Pr = \frac{\alpha}{\zeta}, Re = \frac{\rho g \beta (T_H - T_C) L^3}{\alpha \zeta}.$$

Atau dalam bentuk konservatif persamaan Vorticity-Stream dituliskan seperti berikut.

$$\begin{aligned} \frac{\partial \Omega}{\partial \tau} + \frac{\partial(U\Omega)}{\partial X} + \frac{\partial(V\Omega)}{\partial Y} &= \frac{1}{Pr} \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) + \frac{Re}{Pr} \frac{\partial \Theta}{\partial X}, \\ -\Omega &= \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2}, \end{aligned}$$

dimana,

$$Pr = \frac{\alpha}{\zeta}, Re = \frac{\rho g \beta (T_H - T_C) L^3}{\alpha \zeta}.$$

Nondimensionalisasi Persamaan Energi.

$$\rho C \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \rho C \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right),$$

atau

$$\begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \\ \alpha &= \frac{\lambda}{\rho C}. \end{aligned}$$

- Pembentukan Persamaan Nondimensional Temperatur (Θ)

Misalkan,

$$\Theta = \frac{T - T_C}{T_H - T_C},$$

maka,

$$\begin{aligned} \frac{\partial T}{\partial t} &= (T_H - T_C) \frac{\partial \Theta}{\partial \tau} \frac{\partial \tau}{\partial t} = \frac{\alpha(T_H - T_C)}{L^2} \frac{\partial \Theta}{\partial \tau}, \\ u \frac{\partial T}{\partial x} &= \frac{\alpha(T_H - T_C)}{L} U \frac{\partial \Theta}{\partial X} \frac{\partial X}{\partial x} = \frac{\alpha(T_H - T_C)}{L^2} U \frac{\partial \Theta}{\partial X}, \\ v \frac{\partial T}{\partial y} &= \frac{\alpha(T_H - T_C)}{L} V \frac{\partial \Theta}{\partial Y} \frac{\partial Y}{\partial y} = \frac{\alpha(T_H - T_C)}{L^2} V \frac{\partial \Theta}{\partial Y}, \\ \frac{\partial^2 T}{\partial x^2} &= \frac{\partial}{\partial X} \frac{\partial X}{\partial x} \left(\frac{(T_H - T_C)}{L} \frac{\partial \Theta}{\partial X} \right) = \frac{(T_H - T_C)}{L^2} \frac{\partial^2 \Theta}{\partial X^2}, \end{aligned}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial Y} \frac{\partial Y}{\partial y} \left(\frac{(T_H - T_C)}{L} \frac{\partial \Theta}{\partial Y} \right) = \frac{(T_H - T_C)}{L^2} \frac{\partial^2 \Theta}{\partial Y^2}.$$

Diperoleh,

$$\begin{aligned} & \frac{\alpha(T_H - T_C)}{L^2} \frac{\partial \Theta}{\partial \tau} + \frac{\alpha(T_H - T_C)}{L^2} U \frac{\partial \Theta}{\partial X} + \frac{\alpha(T_H - T_C)}{L^2} V \frac{\partial \Theta}{\partial Y} \\ &= \frac{\alpha(T_H - T_C)}{L^2} \left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right), \end{aligned}$$

maka,

$$\frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right),$$

Atau dalam bentuk konservatif persamaan Temperatur dituliskan sebagai berikut.

$$\frac{\partial \Theta}{\partial \tau} + \frac{\partial(U\Theta)}{\partial X} + \frac{\partial(V\Theta)}{\partial Y} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2}.$$