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LAMPIRAN

Lampiran 1. Titik ekuilibrium non endemik dan endemik model penyebaran penyakit meningitis.

```
> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
```

$$\beta := \mathbf{0.2} : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$$

```
> P1 := Λ + ψ·(R1 + R2) − β·S·(Is/N) − μ·S :
> P2 := (β·τ1·S·Is/N) − (μ + ϵ + δ)·C :
> P3 := (β·(1 − τ1)·Is/N)·S − (μ + ω)·Ia :
> P4 := Ia·ω + δ·C − (μ + α + χ)·Is :
> P5 := ϵ·C + χ·τ2·Is − (μ + ψ + σ)·R1 :
> P6 := χ·(1 − τ2)·Is + σ·R1 − (μ + ψ)·R2 :
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, Ia, Is, R1, R2}) :
```

Lampiran 2. Analisis Sensitivitas menggunakan *software Maple*

```
> restart :

> with(linalg) :
>
```

$$R0 := (\Lambda\beta(\delta\mu\,\tau 1 - \mu\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\omega + \mu\omega + \omega\epsilon))/(\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon));$$

$$R0 := (\Lambda\beta(\delta\mu\,\tau 1 - \mu\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\omega + \mu\omega + \omega\epsilon))/(\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon))$$

```
> #Untuk Parameter β
> P1 := diff(R0, β)
```

$$P1 := (\Lambda(\delta\mu\,\tau 1 - \mu\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\omega + \mu\omega + \omega\epsilon))/(\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon))$$

```
> P11 := P1·β/R0
```

$$P11 := 1$$

```
> #Untuk Parameter τ1
> P2 := diff(R0, τ1)
```

$$P2 := (\Lambda\beta(\delta\mu - \mu\omega - \omega\epsilon))/(\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon))$$

```
> P22 := P2·τ1/R0
```

$$P22 := \frac{(\delta\mu - \mu\omega - \omega\epsilon)\,\tau 1}{\delta\mu\,\tau 1 - \mu\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\omega + \mu\omega + \omega\epsilon}$$

```
> #Untuk Parameter Λ
> P3 := diff(R0, Λ)
```

$$P3 := (\beta(\delta\mu\,\tau 1 - \mu\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\omega + \mu\omega + \omega\epsilon))/(\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon))$$

```
> P33 := P3·Λ/R0
```

$$P33 := 1$$

> **#Untuk Parameter χ**

> $P4 := diff(R0, \chi)$

$$P4 := -\left(\Lambda\,\beta\,(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon)\,(\delta\,\mu + \delta\,\omega + \mu^2 + \mu\,\omega + \mu\,\epsilon + \omega\,\epsilon)\right) \Big/ \left(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)^2\right)$$

> $P44 := P4 \cdot \frac{\chi}{R0}$

$$P44 := -\left(\left(\delta\,\mu + \delta\,\omega + \mu^2 + \mu\,\omega + \mu\,\epsilon + \omega\,\epsilon\right)\chi\right) \Big/ \left(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon\right)$$

>

> **#Untuk Parameter ω**

> $P5 := diff(R0, \omega)$

$$P5 := \left(\Lambda\,\beta\,(-\mu\,\tau 1 - \tau 1\,\epsilon + \delta + \mu + \epsilon)\right) \Big/ \left(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)\right) - \left(\Lambda\,\beta\,(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon)\,(\alpha\,\delta + \alpha\,\mu + \alpha\,\epsilon + \chi\,\delta + \chi\,\mu + \chi\,\epsilon + \delta\,\mu + \mu^2 + \mu\,\epsilon)\right) \Big/ \left(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)^2\right)$$

> $P55 := P5 \cdot \frac{\omega}{R0}$

$$P55 := \left(\left((\Lambda \beta (-\mu\, \tau 1 - \tau 1\, \epsilon + \delta + \mu + \epsilon)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) \right) - (\Lambda \beta (\delta \mu\, \tau 1 - \mu \omega\, \tau 1 - \omega\, \tau 1\, \epsilon + \delta \omega + \mu \omega + \omega \epsilon) (\alpha \delta + \alpha \mu + \alpha \epsilon + \chi \delta + \chi \mu + \chi \epsilon + \delta \mu + \mu^2 + \mu \epsilon)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2)) \right) \omega \mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon) \Big/ (\Lambda \beta (\delta \mu\, \tau 1 - \mu \omega\, \tau 1 - \omega\, \tau 1\, \epsilon + \delta \omega + \mu \omega + \omega \epsilon))$$

> **#Untuk Parameter** δ

> $P6 := diff(R0, \delta)$

$$P6 := (\Lambda \beta (\mu\, \tau 1 + \omega)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2)$$

> $P66 := P6 \cdot \frac{\delta}{R0}$

$$P66 := \left(\left((\Lambda \beta (\mu\, \tau 1 + \omega)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) \right) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2) \right) \delta \mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon) \Big/ (\Lambda \beta (\delta \mu\, \tau 1 - \mu \omega\, \tau 1 - \omega\, \tau 1\, \epsilon + \delta \omega + \mu \omega + \omega \epsilon))$$

> **#Untuk Parameter** ω

> $P7 := diff(R0, \omega)$

$$P7 := (\Lambda\,\beta\,(-\mu\,\tau 1 - \tau 1\,\epsilon + \delta + \mu + \epsilon))\Big/(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)) - (\Lambda\,\beta\,(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon)\,(\alpha\,\delta + \alpha\,\mu + \alpha\,\epsilon + \chi\,\delta + \chi\,\mu + \chi\,\epsilon + \delta\,\mu + \mu^2 + \mu\,\epsilon))\Big/(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)^2)$$

> $P77 := P7 \cdot \dfrac{\omega}{R0}$

$$P77 := \left(\left((\Lambda\,\beta\,(-\mu\,\tau 1 - \tau 1\,\epsilon + \delta + \mu + \epsilon))\Big/(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)) - (\Lambda\,\beta\,(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon)\,(\alpha\,\delta + \alpha\,\mu + \alpha\,\epsilon + \chi\,\delta + \chi\,\mu + \chi\,\epsilon + \delta\,\mu + \mu^2 + \mu\,\epsilon))\Big/(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)^2)\right)\omega\,\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)\Big)\Big/(\Lambda\,\beta\,(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon))$$

> ***#Untuk Parameter*** ϵ

> $P8 := diff(\mathrm{R0}, \epsilon)$

$$P8 := (\Lambda\,\beta\,(-\omega\,\tau 1 + \omega))\Big/(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)) - (\Lambda\,\beta\,(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon)\,(\alpha\,\mu + \alpha\,\omega + \chi\,\mu + \chi\,\omega + \mu^2 + \mu\,\omega))\Big/(\mu\,(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon)^2)$$

> $P88 := P8 \cdot \dfrac{\epsilon}{R0}$

$$P88 := \left(\left((\Lambda \beta (-\omega \tau 1 + \omega)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \right.\right.$$

$$\left. + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) \right.$$

$$\left. - \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2) \right) \epsilon \mu (\alpha \delta \mu$$

$$+ \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon$$

$$+ \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) \Big/ (\Lambda \beta (\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega$$

$$+ \mu \omega + \omega \epsilon))$$

> **#*Untuk Parameter*** μ

> $P9 := diff(R0, \mu)$

$$P9 := (\Lambda \beta (\delta \tau 1 - \omega \tau 1 + \omega)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu$$

$$+ \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon))$$

$$- (\Lambda \beta (\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) \Big/ (\mu^2 (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2$$

$$+ \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega$$

$$+ \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) - (\Lambda \beta (\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega$$

$$+ \omega \epsilon) (\alpha \delta + 2 \alpha \mu + \alpha \omega + \alpha \epsilon + \chi \delta + 2 \chi \mu + \chi \omega + \chi \epsilon + 2 \delta \mu + \delta \omega + 3 \mu^2$$

$$+ 2 \mu \omega + 2 \mu \epsilon + \omega \epsilon)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon$$

$$+ \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon$$

$$+ \mu \omega \epsilon)^2)$$

> $P99 := P9 \cdot \frac{\mu}{R0}$

$$P99 := \left(\left((\Lambda \beta (\delta \tau 1 - \omega \tau 1 + \omega)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) \right. \right.$$

$$- (\Lambda \beta (\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) \Big/ (\mu^2 (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) - (\Lambda \beta (\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon) (\alpha \delta + 2 \alpha \mu + \alpha \omega + \alpha \epsilon + \chi \delta + 2 \chi \mu + \chi \omega + \chi \epsilon + 2 \delta \mu + \delta \omega + 3 \mu^2 + 2 \mu \omega + 2 \mu \epsilon + \omega \epsilon)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2)) \mu^2 (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) \Big/ (\Lambda \beta (\delta \mu \tau 1$$

$$- \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon))$$

> **#Untuk Parameter** α

> $P10 := diff(R0, \alpha)$

$$P10 := -(\Lambda \beta (\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon) (\delta \mu + \delta \omega + \mu^2 + \mu \omega + \mu \epsilon + \omega \epsilon)) \Big/ (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2)$$

> $P110 := P10 \cdot \frac{\alpha}{R0}$

$$P110 := -((\delta \mu + \delta \omega + \mu^2 + \mu \omega + \mu \epsilon + \omega \epsilon) \alpha) \Big/ (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)$$

Lampiran 3. Penentuan persamaan bifurkasi

```
> restart : with(plots) :
>
```

$$\omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$$

```
> restart;
> a := μ + σ + ψ :
> b := μ + ψ :
> d := μ + ε + δ :
> f := μ + ω :
> g := μ + α + χ :
> h := τ1·β :
```

$$> C := \frac{\left(h \cdot a \cdot b \cdot \Lambda \cdot X + h \cdot \psi \cdot X^2 \cdot \left(\tau 2 \cdot b \cdot \chi + (1 - \tau 2) \cdot a \cdot \chi + \sigma \cdot \tau 2 \cdot \chi\right)\right)}{a \cdot b \cdot d \cdot \mu + X \cdot \left(a \cdot b \cdot d \cdot \beta - b \cdot h \cdot \psi \cdot \epsilon - h \cdot \psi \cdot \sigma \cdot \epsilon\right)} :$$

$$> S := \frac{C \cdot d}{h \cdot X} :$$

$$> Ia := \frac{\beta \cdot (1 - \tau 1) \cdot S \cdot X}{f} :$$

$$> K := \omega \cdot Ia + \delta \cdot C - (\mu + \alpha + \chi) \cdot X = 0 :$$

$$> J := numer(lhs(K)) \cdot denom(rhs(K)) = numer(rhs(K)) \cdot denom(lhs(K)) :$$

$$> K := lhs\left(\frac{J}{X}\right) :$$

```
> K1 := lhs(J) :
> implicitplot(K1 = 0, β = 0..1, X = 0..0.1) :
> y := coeff(K1, X) :
> n := coeff(K1, X, 0) :
> u := coeff(K1, X^2) :
>
```

$$R0 := \left(\Lambda \beta \left(\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon\right)\right) \Big/ \left(\mu \left(\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon\right)\right) :$$

```
> M1 := solve(R0 - 1 = 0, β) :
```

Lampiran 4. Sintax simulasi bifurkasi dengan nilai parameter β

$$(\boldsymbol{\beta = 0.2})$$

```
> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
```

$$\beta := \mathbf{0.2} : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$$

```
> P1 := Λ + ψ·(R1 + R2) − β·S·(Is/N) − μ·S :
> P2 := (β·τ1·S·Is/N) − (μ + ϵ + δ)·C :
> P3 := (β·(1 − τ1)·Is/N)·S − (μ + ω)·Ia :
> P4 := Ia·ω + δ·C − (μ + α + χ)·Is :
> P5 := ϵ·C + χ·τ2·Is − (μ + ψ + σ)·R1 :
> P6 := χ·(1 − τ2)·Is + σ·R1 − (μ + ψ)·R2 :
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, Ia, Is, R1, R2}) :
>
```

$$R0 := (\Lambda\beta(\delta\mu\tau 1 - \mu\omega\tau 1 - \omega\tau 1\epsilon + \delta\omega + \mu\omega + \omega\epsilon)) / (\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)) :$$

```
> with(linalg) : with(VectorCalculus) :
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);
  -0.0000200000000000000, -0.500020000000000, -0.650020000000000,
      -0.986197492134063, -0.337733651369526, -0.126128856496411

> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);
  -1.15973255775372, 0.00158087934783999, -0.00157849745389581, -0.290319408284038,
      -0.500027083163507, -0.650020000000000
```

$$(\boldsymbol{\beta = 0.3})$$

```
> restart :

> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
```

>

$\beta := \mathbf{0.3} : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N} \right) - \mu \cdot S :$

> $P2 := \left(\frac{\beta \cdot \tau 1 \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$

> $P3 := \left(\frac{\beta \cdot (1 - \tau 1) \cdot Is}{N} \right) \cdot S - (\mu + \omega) \cdot Ia :$

> $P4 := Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$

> $P5 := \epsilon \cdot C + \chi \cdot \tau 2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$

> $P6 := \chi \cdot (1 - \tau 2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$

> $T := solve(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, \{S, C, Ia, Is, R1, R2\}) :$

>

$R0 := (\Lambda \beta (\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$

> $with(linalg) : with(VectorCalculus) :$

> $T1 := T[1] : T2 := T[2] :$

> $jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :$

> $jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);$

$-0.0000200000000000000, -0.500020000000000, -0.650020000000000, -1.06865294393117, -0.308608607267027, -0.0727984488018024$

> $jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);$

$-1.15973438367225, 0.00119060401712791, -0.00119781283223920, -0.290321024853564, -0.500024048088934, -0.650020000000000$

$(\boldsymbol{\beta = 0.4})$

> $restart :$

> $with(DEtools) :$

> $with(linalg) : with(VectorCalculus) :$

>

$\beta := \mathbf{0.4} : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N} \right) - \mu \cdot S :$

> $P2 := \left(\frac{\beta \cdot \tau 1 \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$

> $P3 := \left(\frac{\beta\cdot(1-\tau 1)\cdot Is}{N}\right)\cdot S-(\mu+\omega)\cdot Ia$:

> $P4 := Ia\cdot\omega+\delta\cdot C-(\mu+\alpha+\chi)\cdot Is$:

> $P5 := \epsilon\cdot C+\chi\cdot\tau 2\cdot Is-(\mu+\psi+\sigma)\cdot R1$:

> $P6 := \chi\cdot(1-\tau 2)\cdot Is+\sigma\cdot R1-(\mu+\psi)\cdot R2$:

> $T := solve(\{P1=0,P2=0,P3=0,P4=0,P5=0,P6=0\},\{S,C,Ia,Is,R1,R2\})$:

>

$$R0 := (\Lambda\beta(\delta\mu\,\tau 1-\mu\omega\,\tau 1-\omega\,\tau 1\,\epsilon+\delta\omega+\mu\omega+\omega\epsilon))\Big/(\mu(\alpha\delta\mu+\alpha\delta\omega+\alpha\mu^2+\alpha\mu\omega+\alpha\mu\epsilon+\alpha\omega\epsilon+\chi\delta\mu+\chi\delta\omega+\chi\mu^2+\chi\mu\omega+\chi\mu\epsilon+\chi\omega\epsilon+\delta\mu^2+\delta\mu\omega+\mu^3+\mu^2\omega+\mu^2\epsilon+\mu\omega\epsilon))$$:

> $with(linalg)$: $with(VectorCalculus)$:

> $T1 := T[1]$: $T2 := T[2]$:

> $jac := Jacobian([P1,P2,P3,P4,P5,P6],[S,C,Ia,Is,R1,R2])$:

> $jac1 := subs(T1,evalm(jac))$: $eigenvalues(jac1)$;

$$-0.0000200000000000000,\ -0.500020000000000,\ -0.650020000000000,$$
$$-1.13841518316760,\ -0.0179680903767637,\ -0.293676726455640$$

> $jac2 := subs(T2,evalm(jac))$: $eigenvalues(jac2)$;

$$-1.15973620964772,\ 0.000589011651680524,\ -0.000605811119774765,$$
$$-0.290322641542702,\ -0.500021012893889,\ -0.650020000000000$$

$(\beta = 0.45)$

> $restart$:

> $with(DEtools)$:

> $with(linalg)$: $with(VectorCalculus)$:

>

$$\beta := 0.45 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$$

> $P1 := \Lambda+\psi\cdot(R1+R2)-\beta\cdot S\cdot\left(\frac{Is}{N}\right)-\mu\cdot S$:

> $P2 := \left(\frac{\beta\cdot\tau 1\cdot S\cdot Is}{N}\right)-(\mu+\epsilon+\delta)\cdot C$:

> $P3 := \left(\frac{\beta\cdot(1-\tau 1)\cdot Is}{N}\right)\cdot S-(\mu+\omega)\cdot Ia$:

> $P4 := Ia\cdot\omega+\delta\cdot C-(\mu+\alpha+\chi)\cdot Is$:

> $P5 := \epsilon\cdot C+\chi\cdot\tau 2\cdot Is-(\mu+\psi+\sigma)\cdot R1$:

> $P6 := \chi\cdot(1-\tau 2)\cdot Is+\sigma\cdot R1-(\mu+\psi)\cdot R2$:

> $T := solve(\{P1=0,P2=0,P3=0,P4=0,P5=0,P6=0\},\{S,C,Ia,Is,R1,R2\})$:

>

$$R0 := \left(\Lambda\,\beta\left(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon\right)\right) \Big/ \left(\mu\left(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon\right)\right) :$$

> $with(linalg) : with(VectorCalculus) :$
> $T1 := T[1] : T2 := T[2] :$
> $jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :$
> $jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);$

$$-0.0000200000000000000,\ -0.500020000000000,\ -0.650020000000000,\ -1.17006043191717,\ 0.00885703944765298,\ -0.288856607530478$$

> $jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);$

$$-1.15973712258246,\ -0.0000107974369014347 + 0.000421542677292278\,I,\ -0.0000107974369014347 - 0.000421542677292278\,I,\ -0.290323449906240,\ -0.500019495251188,\ -0.650020000000000$$

$(\beta = 0.5)$

> $restart :$

> $with(DEtools) :$
> $with(linalg) : with(VectorCalculus) :$
>

$$\beta := 0.5 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N}\right) - \mu \cdot S :$
> $P2 := \left(\frac{\beta \cdot \tau 1 \cdot S \cdot Is}{N}\right) - (\mu + \epsilon + \delta) \cdot C :$
> $P3 := \left(\frac{\beta \cdot (1 - \tau 1) \cdot Is}{N}\right) \cdot S - (\mu + \omega) \cdot Ia :$
> $P4 := Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$
> $P5 := \epsilon \cdot C + \chi \cdot \tau 2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$
> $P6 := \chi \cdot (1 - \tau 2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$
> $T := solve(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, \{S, C, Ia, Is, R1, R2\}) :$
>

$$R0 := \left(\Lambda\,\beta\left(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon\right)\right) \Big/ \left(\mu\left(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon\right)\right) :$$

> $with(linalg) : with(VectorCalculus) :$

```
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);
  -0.0000200000000000000, -0.500020000000000, -0.650020000000000,
      -1.20002000000000, 0.0350581059358213, -0.285098105935821

> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);
  -1.15973803553262, -0.0000131951216027000 + 0.000843969637210361 I,
      -0.0000131951216027000 - 0.000843969637210361 I, -0.290324258320772,
      -0.500017977578366, -0.650020000000000
```

$(\beta = 0.6)$

```
> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
```

$$\beta := 0.6 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$$

```
> P1 := Λ + ψ·(R1 + R2) - β·S·(Is/N) - μ·S :
> P2 := (β·τ1·S·Is/N) - (μ + ε + δ)·C :
> P3 := (β·(1 - τ1)·Is/N)·S - (μ + ω)·Ia :
> P4 := Ia·ω + δ·C - (μ + α + χ)·Is :
> P5 := ε·C + χ·τ2·Is - (μ + ψ + σ)·R1 :
> P6 := χ·(1 - τ2)·Is + σ·R1 - (μ + ψ)·R2 :
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, Ia, Is, R1, R2}) :
>
```

$$R0 := \left(\Lambda\,\beta\left(\delta\,\mu\,\tau 1 - \mu\,\omega\,\tau 1 - \omega\,\tau 1\,\epsilon + \delta\,\omega + \mu\,\omega + \omega\,\epsilon\right)\right) \Big/ \left(\mu\left(\alpha\,\delta\,\mu + \alpha\,\delta\,\omega + \alpha\,\mu^2 + \alpha\,\mu\,\omega + \alpha\,\mu\,\epsilon + \alpha\,\omega\,\epsilon + \chi\,\delta\,\mu + \chi\,\delta\,\omega + \chi\,\mu^2 + \chi\,\mu\,\omega + \chi\,\mu\,\epsilon + \chi\,\omega\,\epsilon + \delta\,\mu^2 + \delta\,\mu\,\omega + \mu^3 + \mu^2\,\omega + \mu^2\,\epsilon + \mu\,\omega\,\epsilon\right)\right) :$$

```
> with(linalg) : with(VectorCalculus) :
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);
  -0.0000200000000000000, -0.500020000000000, -0.650020000000000,
      -1.25580772802631, 0.0854087632049604, -0.279661035178649

> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);
```

$-1.15973986153156,\ -0.0000179904632942017 + 0.00133469905857443\,I,$
$-0.0000179904632942017 - 0.00133469905857443\,I,\ -0.290325875197000,$
$-0.500014942142357,\ -0.650020000000000$

$(\beta = 0.7)$

```
> restart :

> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
β := 0.7 : ω := 0.2 : μ := 0.00002 : α := 0.5 : χ := 0.15 : δ := 0.3 : ψ := 0.5 : Λ
     := 0.00005 : ϵ := 0.3 : N := 1 : τ2 := 0.05 : τ1 := 0.8 : σ := 0.15 :


> P1 := Λ + ψ·(R1 + R2) − β·S·(Is/N) − μ·S :
> P2 := (β·τ1·S·Is/N) − (μ + ϵ + δ)·C :
> P3 := (β·(1 − τ1)·Is/N)·S − (μ + ω)·Ia :
> P4 := Ia·ω + δ·C − (μ + α + χ)·Is :
> P5 := ϵ·C + χ·τ2·Is − (μ + ψ + σ)·R1 :
> P6 := χ·(1 − τ2)·Is + σ·R1 − (μ + ψ)·R2 :
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, Ia, Is, R1, R2}) :
>
R0 := (Λ β (δ μ τ1 − μ ω τ1 − ω τ1 ϵ + δ ω + μ ω + ω ϵ)) / (μ (α δ μ + α δ ω + α μ^2
     + α μ ω + α μ ϵ + α ω ϵ + χ δ μ + χ δ ω + χ μ^2 + χ μ ω + χ μ ϵ + χ ω ϵ + δ μ^2 + δ μ ω
     + μ^3 + μ^2 ω + μ^2 ϵ + μ ω ϵ)) :

> with(linalg) : with(VectorCalculus) :
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);
-0.0000200000000000000, -0.500020000000000, -0.650020000000000,
     -1.30717463530161, 0.133062036791486, -0.275947401489881

> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);
-1.15974168758745, -0.0000227857769430146 + 0.00168834537848090 I,
     -0.0000227857769430146 − 0.00168834537848090 I, -0.290327492192871,
     -0.500011906585849, -0.650020000000000
```

$(\beta = 0.88)$

```
> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
```

$$\beta := \mathbf{0.88} : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$$

$$> P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N} \right) - \mu \cdot S :$$

$$> P2 := \left(\frac{\beta \cdot \tau 1 \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$$

$$> P3 := \left(\frac{\beta \cdot (1 - \tau 1) \cdot Is}{N} \right) \cdot S - (\mu + \omega) \cdot Ia :$$

$$> P4 := Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$$

$$> P5 := \epsilon \cdot C + \chi \cdot \tau 2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$$

$$> P6 := \chi \cdot (1 - \tau 2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$$

$$> T := solve(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, \{S, C, Ia, Is, R1, R2\}) :$$

>

$$R0 := \left(\Lambda \beta \left(\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon\right)\right) \Big/ \left(\mu \left(\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon\right)\right) :$$

```
> with(linalg) : with(VectorCalculus) :
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);
```

$$-0.0000200000000000000,\ -0.500020000000000,\ -0.650020000000000,\ -1.39122830884031,\ 0.212773346931893,\ -0.271605038091584$$

```
> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);
```

$$-1.15974497434063,\ -0.0000314174380126211 + 0.00218511963365444\,I,\ -0.0000314174380126211 - 0.00218511963365444\,I,\ -0.290330403043565,\ -0.500006442280444,\ -0.650020000000000$$

Lampiran 5. Sintax simulasi untuk grafik

```
> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
> β := 0.3 :
```

$$\beta := 0.3 :$$

$$\omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.2 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$$

```
> P1 := Λ + ψ·(R + U) − β·S·(Q/N) − μ·S :
> P2 := (β·τ1·S·Q/N) − ((μ + ε + δ)·C) :
> P3 := (β·(1 − τ1)·Q/N)·S − (μ + ω)·P :
> P4 := P·ω + δ·C − (μ + χ)·Q :
> P5 := ε·C + χ·τ2·Q − ((μ + ψ + σ)·R) :
> P6 := (χ·(1 − τ2)·Q) + (σ·R) − ((μ + ψ)·U) :
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, P, Q, R, U}) :
>
R0 := (Λ β (δ μ τ1 − μ ω τ1 − ω τ1 ε + δ ω + μ ω + ω ε)) / (μ (α δ μ + α δ ω + α μ^2 + α μ ω + α μ ε + α ω ε + χ δ μ + χ δ ω + χ μ^2 + χ μ ω + χ μ ε + χ ω ε + δ μ^2 + δ μ ω + μ^3 + μ^2 ω + μ^2 ε + μ ω ε)) :
> with(linalg) : with(VectorCalculus) :
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, P, Q, R, U]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1) :
> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2) :

> T1 := (d/dt)S(t) = Λ + ψ·(R + U) − β·S·(Q/N) − μ·S :
> T21 := (d/dt)C(t) = (β·τ1·S·Q/N) − ((μ + ε + δ)·C) :
> T31 := (d/dt)P(t) = (β·(1 − τ1)·Q/N)·S − (μ + ω)·P :
> T4 := (d/dt)Q(t) = P·ω + δ·C − (μ + χ)·Q :
> T5 := (d/dt)R(t) = ε·C + χ·τ2·Q − ((μ + ψ + σ)·R) :
> T6 := (d/dt)U(t) = (χ·(1 − τ2)·Q) + (σ·R) − ((μ + ψ)·U) :
```

>

$$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$$

>

$$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0..30, ivs, linecolor = [red, blue], arrows = medium, scene = [t, S(t)], method = rosenbrock, stepsize = 0.5, title = "Susceptible", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$$

>

$$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$$

>

$$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0..35, ivs, linecolor = [red, blue], arrows = medium, scene = [t, C(t)], method = rosenbrock, stepsize = 0.5, title = "Carrier", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$$

>

$$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$$

>

$$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0..35, ivs, linecolor = [red, blue], arrows = medium, scene = [t, P(t)], method = rosenbrock, stepsize = 0.5, title = "Infected without Symptoms", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$$

>

$$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$$

>

$$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0..35, ivs, linecolor = [red, blue], arrows = medium, scene = [t, Q(t)], method = rosenbrock, stepsize = 0.5, title = "Infected with symptoms", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$$

>

$$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$$

>

$$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0..35, ivs, linecolor = [red, blue], arrows = medium, scene = [t, R(t)], method = rosenbrock, stepsize = 0.5, title = "Recovery without Disability", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$$

>

$$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$$

```
>
DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0 ..35, ivs, linecolor
    = [red, blue], arrows = medium, scene = [t, U(t)], method = rosenbrock, stepsize = 0.5,
    title = "Recovery with Disability", labels = [Bulan(t), Populasi], titlefont = ["ARIAL",
    15], labelfont = ["HELVETICA", 10]) :
```

Lampiran 6. Penjabaran untuk Untuk Memperoleh Persamaan (4.37)

$$\omega {i_A}^* + \delta c^* - (\mu + \alpha + \chi){i_s}^* = 0$$

$$\Leftrightarrow \omega \left(\frac{abdj\Lambda {i_s}^* + d\psi j({i_s}^*)^2 \left[b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2 \right]}{abdf\mu + f{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} \right) +$$

$$\delta \left(\frac{ab\Lambda\beta\tau_1 {i_s}^* + \psi\beta\tau_1 ({i_s}^*)^2 \left[b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2 \right]}{abd\mu + {i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} \right) - (\mu + \alpha + \chi){i_s}^* = 0$$

$$\Leftrightarrow \frac{abdj\omega\Lambda {i_s}^* + d\omega\psi j({i_s}^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abdf\mu + f{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} +$$

$$\frac{ab\Lambda\delta\beta\tau_1 {i_s}^* + \psi\delta\beta\tau_1 ({i_s}^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abd\mu + {i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} - (\mu + \alpha + \chi){i_s}^* = 0$$

$$\Leftrightarrow \frac{abdj\omega\Lambda {i_s}^* + d\omega\psi j({i_s}^*)^2 \left[b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2 \right]}{abdf\mu + f{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} +$$

$$\frac{abf\Lambda\delta\beta\tau_1 {I_s}^* + f\psi\delta\beta\tau_1 ({I_s}^*)^2 \left[b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2 \right]}{abdf\mu + f{I_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} -$$

$$\frac{abdf\mu + f{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)}{abdf\mu + f{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} (\mu + \alpha + \chi){i_s}^* = 0$$

misalakan $g = (\mu + \alpha + \chi)$, maka diperoleh:

$$\Leftrightarrow \frac{abdj\omega\Lambda {i_s}^* + d\omega\psi j({i_s}^*)^2 \left[b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2 \right]}{abdf\mu + f{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} +$$

$$\frac{abf\Lambda\delta\beta\tau_1 {i_s}^* + f\psi\delta\beta\tau_1 ({i_s}^*)^2 \left[b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2 \right]}{abdf\mu + f{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} -$$

$$\frac{abdfh\mu + fh{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)}{abdf\mu + f{i_s}^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} {i_s}^* = 0$$

$$\Leftrightarrow abdj\omega\Lambda {i_s}^* + d\omega\psi j({i_s}^*)^2 \left[b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2 \right] + abf\Lambda\delta\beta\tau_1 {i_s}^* +$$

$$f\psi\delta\beta\tau_1 ({i_s}^*)^2 \left[b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2 \right] - (abdfg\mu + fg{i_s}^* (abd\beta -$$

$$\psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)) {i_s}^* = 0$$

$$\Leftrightarrow (bd\omega\psi j\chi\tau_2 + ad\omega\psi j\chi(1-\tau_2) + d\omega\psi j\sigma\chi\tau_2 + bf\psi\delta\beta\tau_1\chi\tau_2$$
$$+ af\psi\delta\beta\tau_1\chi(1-\tau_2) + f\psi\delta\beta\tau_1\sigma\chi\tau_2 + fg\psi b\epsilon\beta\tau_1$$
$$+ fg\psi\sigma\epsilon\beta\tau_1 - abdfg\beta) [{i_s}^*]^2$$
$$+ (abdj\omega\Lambda + abf\Lambda\delta\beta\tau_1 - abdfg\mu) {i_s}^* = 0 \qquad (4.37)$$