DAFTAR PUSTAKA

Agustin, U., 2012. *Principles of Seismology*. Madrid: Cambridge University Press.

Bamberger, A., Enquist, B., Halpern, L. & Joly, P., 1988. Parabolic Wave Equation Approximation in Heterogeneous Media. *SIAM J. Appl. Math 48*, pp.99-128.

Berkhout, A.J., 1982. Seismic Migration: Imaging of Acoustic Energy by Wave Field Extrapolation. *ASME Journal Applied Mechanics Vol.49*, pp.682-83.

Bermudez, A., Prieto, A., Hervella-Nieto, L. & Rodrigues*, R., 2006. An Optimal Finite-Element/PML Method for The Simulation of Acoustik Wave Propagation Phenomena. *Variational Formulation in Mechanics: Theory and Aplication*.

Beylkin, G., Oristaglio, M. & Miller, D., 1985. Spatial Resolution Of Migration Algorithms. *Acoustical Imaging Vol.14*, pp.155-68.

Biondi, B. & Palacharla, G., 1996. 3-D Prestack Migration of Common-Azimuth Data. *Geophysics 61*, pp.1822-32.

Chen, L., 2020. *Finite Difference Methods for Poisson Equation*. [Online] Available at: https://www.math.uci.edu/~chenlong/226/FDM.pdf [Accessed 2 January 2021].

Claerbout, J.F., 1971. Toward a Unified Theory of Reflector Mapping. *Geophysics, 36*, pp.467-81.

Claerbout, J.F., 1985. Imaging the Earth's Interior. *Blackwell Scientific Publications*.

Collino, F. & Joly, P., 1995. Spliting of Operators, Alternate Directions, and Paraxial Approximations for the Three-Dimensional Wave Equation. *SIAM J. Sci Comput 16*, pp.1019-48.

Erlangga, Y.A., 2005. *A Robust and Efficient Iterative Method for the Numerical Solution of the Helmholtz Rquation*. PhD Thesis. Delft: Technische Universiteit Delft.

Jin, S., Wu, R.-S. & Peng, C., 1998. Prestack Depth Migration Using a Hybrid Pseudo-Screen Propagator. *68th SEG Annual Mtg*, pp.1819-22.

Kwangjin, Y., Kurt J, M. & William, S., 2004. Challenges in Reverse Time Migration. *SEG Int'l Exposition an 74th Annual Meeting*.

Mikhail, B., Maxim, D., Victor, K. & Dmitry, N., 2017. An Iterative Solver for the 3D Helmholtz Equation. *Journal of Computational Physics 345*, pp.330-34.

Mulder, W.A. & Plessix, R.E., 2004. How to Choose a subset of Frequencies in Frequency-Domain Finite-Difference Migration. *Geophys. J. Int 158*, pp.801-12.

Nagle, K.R. & Saff, E.B., 1996. *Fundamentals of Differential Equations and Boundary Value Problems*. South Florida: University of South Florida.

Operto, S., Xu, S. & Lambare, G., 2000. Can We Quantitatively Image Complex Structures with Rays. *Geophysics VOl. 65 NO. 4*, pp.1223-38.

Singer, I. & Turkel, E., 2004. A Perfectly Match Layer for The Helmholtz Equation in a Semi-Infinite Strip. *Journal of Computational Physics 201*, pp.439-65.

Spiegel, M.R., 1983. *Advanced Mathematics for Engineer and Scientists*. Jakarta: Erlangga.

Telford, W.M., Geldart, L.P. & Sheriff, R.E., 1990. *APPLIED GEOPHYSICS SECOND EDITION: Seismic Methods*. USA: Cambridge University Press.

ten Kroode, A.P.E., Smit, D.J. & Verdel, A.R., 1998. A Microlocal Analysis of Migration. *Wave Motion*, pp.149-72.

Triatmodjo, B., 2002. *Metode Numerik: Dilengkapi dengan Program Komputer*. Yogyakarta: Fakultas Teknik Universitas Gajahmada.

LAMPIRAN-LAMPIRAN

Lampiran I Skrip Uji Solusi Eksak dan Numerik di Program Matlab

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Telah diselesaikan persamaan Helmholtz
%  - Delta u - k^2 u = f in [L, R] x [B, T]
%
% dengan mengaplikasikan PML dalam kordinat Kartesian
%
%  Zona PML yang dibuat didefenisikan sebagai W, maka untuk zona normal
%  dapat didefenisikan sbb;
%  [L+W, R-W] x [B+W, T-W]
%
% Dalam zona PML diselesaikan dengan persamaan di bawah
% - alpha_x^2 u_xx - alpha_y^2 u_yy - k^2 u = f
%                                u = 0 on x=L, x=R, y=B, y=T
% dimana digunakan syarat batas Dirichlet sebagai batas eksteriornya
%
% Parameter otpimal dari PML merujuk pada artikel
% A. BermÃºdez, L. Hervella-Nieto, A. Prieto, R. RodrÃ-guez, 2006,
% An optimal finite-element/pml method for the simulation of acoustic wave
% propagation phenomena,
% Variational Formulations in Mechanics: Theory and Applications
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

close all
clear all
tic
f=1;
omg = 2*pi*f;
c = 1;
k = omg/c;

L = -3;
R = 3;
T = 3;
B = -3;
W = 1.;

h = 0.01;

[x, y] = meshgrid(L:h:R, B:h:T);
m = size(x,1);
n = size(x,2);
```

```
gammax = ones(m,n); dergammax = zeros(m,n);
gammay = ones(m,n); dergammay = zeros(m,n);

gammax(x<=L+W) = 1 + 1i/omg * c./(x(x<=L+W)-L); %(-1+0.2*1i);
gammax(x>=R-W) = 1 + 1i/omg * c./(R-x(x>=R-W)); %(1-0.2*1i);
gammay(y<=B+W) = 1 + 1i/omg * c./(y(y<=B+W)-B); %(-1+0.2*1i);

dergammax(x<=L+W) = -1i*c/omg ./(x(x<=L+W)-L).^2;
dergammax(x>=R-W) =  1i*c/omg ./(R-x(x>=R-W)).^2;
dergammay(y<=B+W) = -1i*c/omg ./(y(y<=B+W)-B).^2;

dergammax(abs(dergammax) == Inf) = 0;
dergammay(abs(dergammay) == Inf) = 0;

A  = sparse(m*n, m*n);
P  = speye(m*n, m*n);
PP = speye(m*n, m*n);
f  = sparse(m*n,1);
Uex = fh(x, y, k, T);

for i=1:m
   for j=1:n
      A(n*(i-1)+j, n*(i-1)+j) = 2/gammax(i,j)^2 + 2/gammay(i,j)^2 - h^2*k^2;
      try
         A(n*(i-1)+j, n*(i-1)+j-1) = -1/gammax(i,j-1)^2 - h/2*dergammax(i,j-
1)/gammax(i,j-1)^3; %left, lower diag
      end
      try
         A(n*(i-1)+j, n*(i-1)+j+1) = -1/gammax(i,j+1).^2 +
h/2*dergammax(i,j+1)/gammax(i,j+1)^3; %right, upper diag
      end
      try
         A(n*(i-1)+j, n*(i-2)+j) = -1/gammay(i-1,j).^2 - h/2*dergammay(i-
1,j)/gammay(i-1,j)^3; %top, lower2 diag
         if i==m
            A(n*(i-1)+j, n*(i-2)+j) = 2 * A(n*(i-1)+j, n*(i-2)+j);
         end
      end
      try
         A(n*(i-1)+j, n*i+j) = -1/gammay(i+1,j).^2 +
h/2*dergammay(i+1,j)/gammay(i+1,j)^3; %bottom, upper2 diag
      end

      if i==1 || j==1 || j==n
         A(n*(i-1)+j,:) = P(n*(i-1)+j,:);
      end
```

```
    end
end

for i=(m+1)/2+2:-1:(m+1)/2-2
    for j=(n+1)/2+2:-1:(n+1)/2-2
        if i==(m+1)/2-2 || i == (m+1)/2+2 || j == (n+1)/2-2 || j == (n+1)/2+2
            A(n*(i-1)+j, :) = P(n*(i-1)+j, :);
            f(n*(i-1)+j)    = fh(x(1,i),y(j,1),k, T);
        else
            A(n*(i-1)+j, :)  = [];
            A(:, n*(i-1)+j)  = [];
            f(n*(i-1)+j)     = [];
            P(n*(i-1)+j, :)  = [];
            P(:, n*(i-1)+j)  = [];
            PP(n*(i-1)+j, :) = [];
            Uex(i,j) = 0;
        end
    end
end
toc

tic
U = A\f;
toc

lw = round(W/h,0)+1;
rw = size(x,1) - round(W/h,0);
bw = round(W/h,0)+1;
tw = size(x,2) - round(W/h,0);

UU  = reshape(PP'*U, size(x)).';

err = norm(UU(lw:rw, bw:tw) - Uex(lw:rw, bw:tw))/norm(Uex(lw:rw, bw:tw))

figure
plot(x(1,:), real(UU(41,:)), x(1,:), real(Uex(41,:)))
legend('U Numerik','U Eksak')
xlabel ('x'), ylabel('Amplitude')
title('Profil 1D Solusi Eksak dan Solusi Numerik Error=0.08')

figure
plot(y(1,:), real(UU(41,:)), y(1,:), real(Uex(41,:)))
legend('U Numerik','U Eksak')
xlabel ('y'), ylabel('Amplitude')
title('Profil 1D Solusi Eksak dan Solusi Numerik Error=0.08')
```

```
figure
imagesc(real(UU))
xlabel('x-Jarak (cm)'), ylabel('y-Kedalaman')
title('Solusi Numerik Model Penjalaran pada Medium Berlapis')
colorbar

figure
imagesc(real(Uex))
xlabel('x'), ylabel('y')
title('Solusi Eksak Model Penjalaran')
colorbar

figure
surf(x,y,real(UU))
xlabel('x'), ylabel('y')
title('Solusi Numerik Model Penjalaran')

figure
surf(x,y,real(Uex))
xlabel('x'), ylabel('y')
title('Solusi Eksak Model Penjalaran')
```

Skrip fungsi (fh)

```
function res = fh(x,y,k, T)
xp = 0;
yp = 0;

res = 1i/4 * (besselh(0,k * sqrt((x-xp).^2 + (y-yp).^2)) + besselh(0,k * sqrt((x-
xp).^2 + (y-yp-2*T).^2)));

end
```

Lampiran II Skrip Model Solusi Numerik di Program Matlab

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Telah diselesaikan persamaan Helmholtz
%  - Delta u - k^2 u = f in [L, R] x [B, T]
%
% dengan mengaplikasikan PML dalam kordinat Kartesian
%
%  Zona PML yang dibuat didefenisikan sebagai W, maka untuk zona normal
%  dapat didefenisikan sbb;
%  [L+W, R-W] x [B+W, T-W]
%
% Dalam zona PML diselesaikan dengan persamaan di bawah
% - alpha_x^2 u_xx - alpha_y^2 u_yy - k^2 u = f
%                              u = 0 on x=L, x=R, y=B, y=T
% dimana digunakan syarat batas Dirichlet sebagai batas eksteriornya
%
% Parameter otpimal dari PML merujuk pada artikel
% A. BermÃºdez, L. Hervella-Nieto, A. Prieto, R. RodrÃ-guez, 2006,
% An optimal finite-element/pml method for the simulation of acoustic wave
% propagation phenomena,
% Variational Formulations in Mechanics: Theory and Applications
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

close all
clear all

%seting pewarnaan Map
set(groot,'DefaultFigureColormap',rdbuMap())

tic
%kecepatan tiap lapisan
c1=0.1; %layer 1
c2=0.2; %layer 2
c3=0.5; %layer 3

% Geometri Omega R^2 (meter)
L = -3; %domain kiri
R = 3;  %domain kanan
T = 3;  %domain atas
B = -3; %domain bawah
W = 1.;  %lebar domain PML

h = 0.02; %lebar grid
```

```
[x, y] = meshgrid(L:h:R, B:h:T);
m = size(x,1);
n = size(x,2);


%Parameter PML
gammax = ones(m,n); dergammax = zeros(m,n);
gammay = ones(m,n); dergammay = zeros(m,n);

%parameter kecepatan
c = ones(m,n);
c(y >=0.11.*x+1.33)=c1; %2
c(y <0.11.*x+1.33)=c2;  %2
c(y <-0.16.*x-1.5)=c3;   %0

%Frekuensi Source (Hz)
f=1;
omg =2*pi*f;

%parameter bilangan gelombang
k=omg./c;

%parameter Slowness
gammay(y<=B+W) = 1 + 1i./omg * c(y<=B+W)./(y(y<=B+W)-B);
gammax(x<=L+W) = 1 + 1i./omg * c(x<=L+W)./(x(x<=L+W)-L);
gammax(x>=R-W) = 1 + 1i./omg * c(x>=R-W)./(R-x(x>=R-W));


dergammax(x<=L+W) = -1i*c(x<=L+W)./omg ./(x(x<=L+W)-L).^2;
dergammax(x>=R-W) =  1i*c(x>=R-W)./omg ./(R-x(x>=R-W)).^2;
dergammay(y<=B+W) = -1i*c(y<=B+W)./omg ./(y(y<=B+W)-B).^2;


dergammax(abs(dergammax) == Inf) = 0;
dergammay(abs(dergammay) == Inf) = 0;

A  = sparse(m*n, m*n);
P  = speye(m*n, m*n);
PP = speye(m*n, m*n);
f  = sparse(m*n,1);


for i=1:m
   for j=1:n
      A(n*(i-1)+j, n*(i-1)+j) = 2/gammax(i,j)^2 + 2/gammay(i,j)^2 - h^2*k(i,j)^2;
```

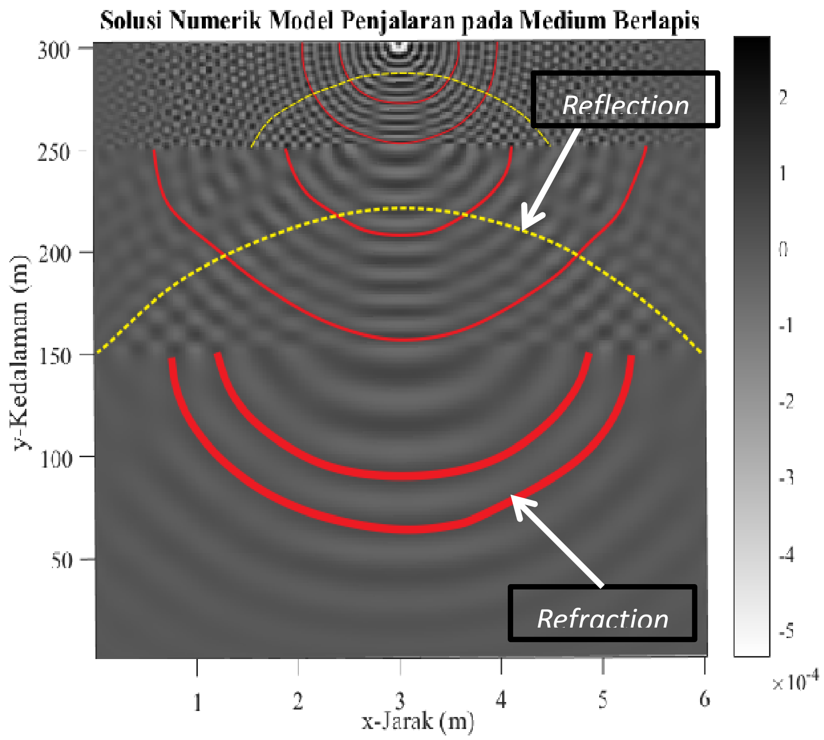
```
        try
           A(n*(i-1)+j, n*(i-1)+j-1) = -1/gammax(i,j-1)^2 - h/2*dergammax(i,j-1)/gammax(i,j-1)^3; %left, lower diag
        end
        try
           A(n*(i-1)+j, n*(i-1)+j+1) = -1/gammax(i,j+1).^2 + h/2*dergammax(i,j+1)/gammax(i,j+1)^3; %right, upper diag
        end
        try
           A(n*(i-1)+j, n*(i-2)+j) = -1/gammay(i-1,j).^2 - h/2*dergammay(i-1,j)/gammay(i-1,j)^3; %top, lower2 diag
           if i==m
              A(n*(i-1)+j, n*(i-2)+j) = 2 * A(n*(i-1)+j, n*(i-2)+j);
           end
        end
        try
           A(n*(i-1)+j, n*i+j) = -1/gammay(i+1,j).^2 + h/2*dergammay(i+1,j)/gammay(i+1,j)^3; %bottom, upper2 diag
        end

        if i==1 || j==1 || j==n
           A(n*(i-1)+j,:) = P(n*(i-1)+j,:); %untuk batas dirichlet
        end

    end
end

S = (x).^2+(y).^2;
r =1;
S(S>r^2)=0;
S(S>0)=1;
f = h^2*reshape(S,[(n)^2,1]);
toc

tic
U = A\f;
toc

lw = round(W/h,0)+1;
rw = size(x,1) - round(W/h,0);
bw = round(W/h,0)+1;
tw = size(x,2) - round(W/h,0);

UU  = reshape(PP'*U, size(x)).';

figure
```

```
imagesc(real(UU))
xlabel('x-Jarak (m)'), ylabel('y-Kedalaman (m)')
title('Solusi Numerik Model Penjalaran pada Medium Berlapis')
%colormap(flipud(gray(256)))
colorbar

figure
imagesc(c)
xlabel('x'), ylabel('y')
title('Kecepatan tiap lapisan')
%%
%Plot gamma (x,y) dan dergamma (x,y)
real_gammax =real(gammax); imag_gammax =imag(gammax);
real_dergammax =real(dergammax); imag_dergammax =imag(dergammax);

real_gammay =real(gammay); imag_gammay =imag(gammay);
real_dergammay =real(dergammay); imag_dergammay =imag(dergammay);


figure
plot(x, real_gammax,'b-',x,imag_gammax,'g--')
legend('bilangan Real','bilangan imajiner')
xlabel ('jarak (m)'), ylabel('gammax')
title('Parameter PML (Gamma x)')

figure
plot(y, real_gammay,y,imag_gammay)
legend('bilangan Real','bilangan imajiner')
xlabel ('jarak (m)'), ylabel('gammay')
title('Parameter PML (Gamma y)')
```

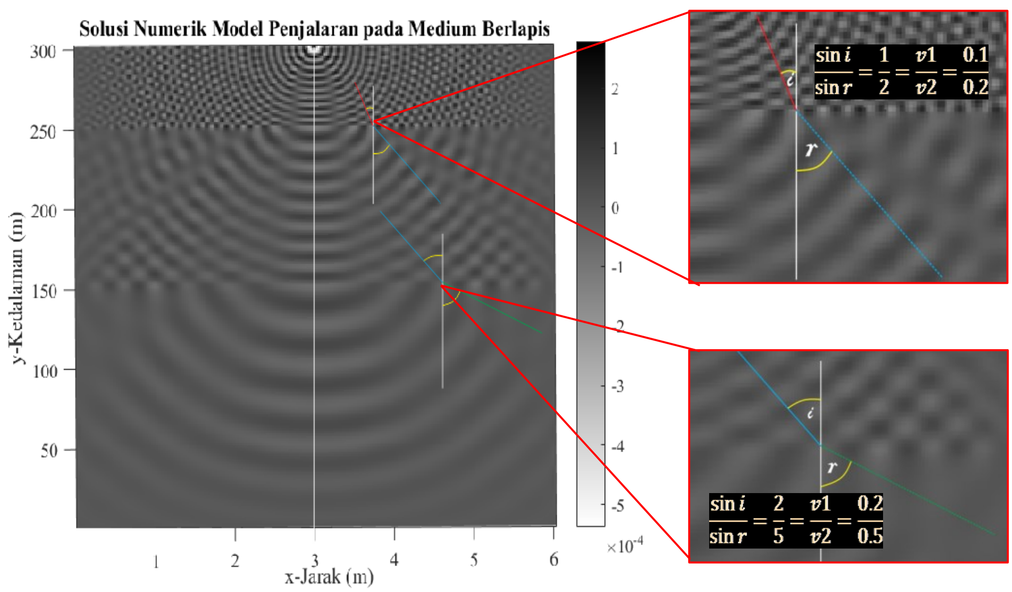
Lampiran III Hasil Model *Picking* Muka Gelombang dan Gelombang Ortogonal

1. Model 1

Muka gelombang (*wave fronts*)

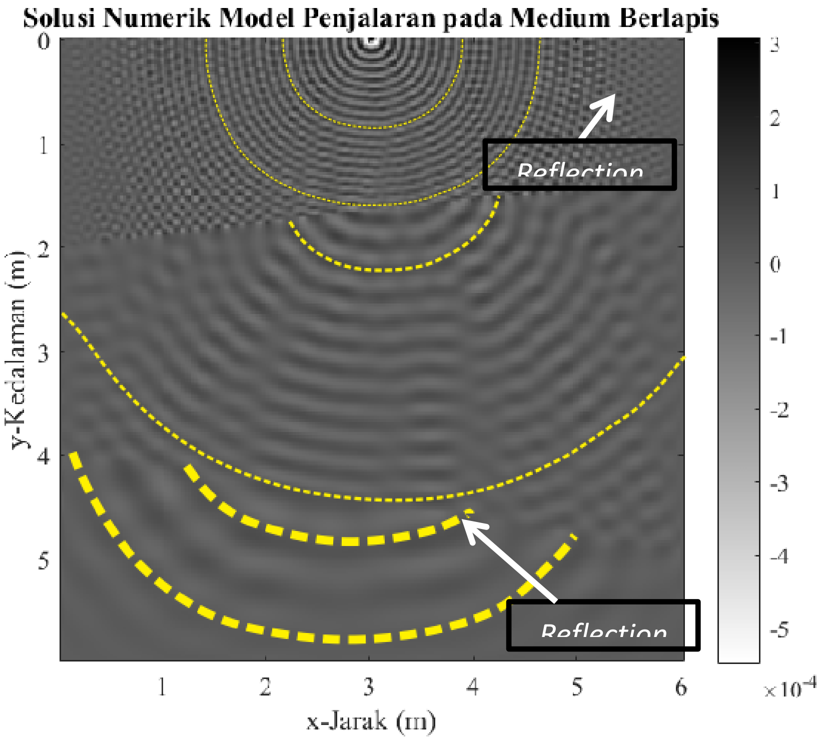


Gelombang orthogonal

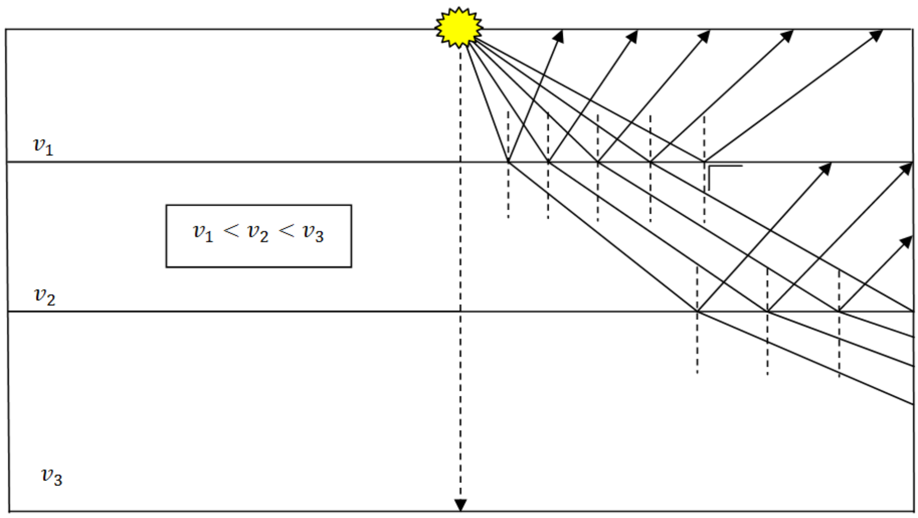


2. Model 2

Muka gelombang (*wave fronts*)

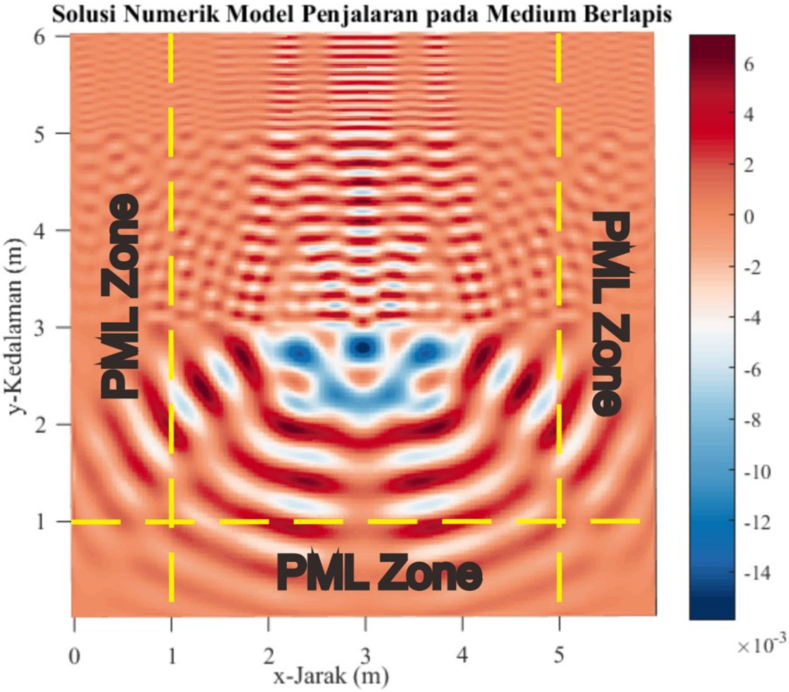


Skema orthogonal penjalaran gelombang seismik.



3. Model 3

Medium sama dengan model 1 sumber gelombang berada di tengah



4. Model 4

Medium sama dengan model 2 sumber gelombang berada di tengah