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Another Antimagic Conjecture

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Abstract

An antimagic labeling of a graph G is a bijection $f : E(G) \rightarrow \{1, . . . , |E(G)|\}$ such that the weights $w(x)=\sum_{y\sim x}f(y)$ distinguish all vertices. A well-known conjecture of Harts field and Ringel (1990) is that every connected graph other than K2 admits an antimagic labeling. For a set of distances D, a D-antimagic labeling of a graph G is a bijection $f : V(G) \rightarrow \{1, . . . , |V(G)|\}$ such that the weight $\omega(x) = \sum_{y\in ND(x)}f(y)$ is distinct for each vertex x, where $ND(x) = \{y \in V(G)|d(x,y) \in D\}$ is the D-neighbourhood set of a vertex x. If ND(x) = r, for every vertex x in G, a graph G is said to be (D, r)-regular. In this paper, we conjecture that a graph admits a D-antimagic labeling if and only if it does not contain two vertices having the same D-neighbourhood set. We also provide evidence that the conjecture is true. We present computational results that, for D = {1}, all graphs of order up to 8 concur with the conjecture. We prove that the set of (D, r)-regular D-antimagic graphs is closed under union. We provide examples of disjoint union of symmetric (D, r)-regular that are D-antimagic and examples of disjoint union of non-symmetric non-(D, r)-regular graphs that are D-antimagic. Furthermore, lastly, we show that it is possible to obtain a D-antimagic graph from a previously known distance antimagic graph.

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