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Lampiran 1. Titik Kesetimbangan Model tanpa Migrasi pada Pemangsa Menggunakan Maple

> restart : with(linalg) : with(DEtools) :

> $f1 := \left(\frac{r\cdot x\cdot(x-\eta)}{x+\theta}\cdot\left(1-\frac{x}{k}\right)\right)-\frac{\varepsilon\cdot x\cdot y}{1+x}$:

> $f2 := \frac{\varepsilon\cdot y\cdot x}{1+x}-\mu\cdot y$:

> $T := solve(\{f1, f2\}, \{x, y\})$

$$T := \{x=0, y=0\}, \left\{x=-\frac{\mu}{\mu-\varepsilon}, y=\frac{r\left(\eta k\mu^2-2\eta k\mu\varepsilon+\eta k\varepsilon^2+\eta\mu^2-\eta\mu\varepsilon+k\mu^2-k\mu\varepsilon+\mu^2\right)}{k\left(\mu^3\theta-3\mu^2\theta\varepsilon+3\mu\theta\varepsilon^2-\theta\varepsilon^3-\mu^3+2\mu^2\varepsilon-\mu\varepsilon^2\right)}\right\}, \{x=k, y=0\}, \{x=\eta, y=0\}$$

> with(VectorCalculus) : with(LinearAlgebra) :

> $Jac := Jacobian([f1,\ f2]\ , [x, y])$

$$Jac := \left[\left[\frac{r(x-\eta)\left(1-\frac{x}{k}\right)}{x+\theta}+\frac{rx\left(1-\frac{x}{k}\right)}{x+\theta}-\frac{rx(x-\eta)\left(1-\frac{x}{k}\right)}{(x+\theta)^2}-\frac{rx(x-\eta)}{(x+\theta)k}-\frac{\varepsilon y}{1+x}+\frac{\varepsilon x y}{(1+x)^2}, -\frac{\varepsilon x}{1+x}\right], \left[\frac{\varepsilon y}{1+x}-\frac{\varepsilon x y}{(1+x)^2}, \frac{\varepsilon x}{1+x}-\mu\right]\right]$$

Lampiran 2. Titik Kesetimbangan Model dengan Migrasi pada Pemangsa Menggunakan Maple

> restart : with(linalg) : with(DEtools) :

> $dx1 := \frac{x_1\cdot(x_1-\eta_1)}{x_1+\theta_1}\cdot\left(1-\frac{x_1}{k_1}\right)-\frac{\varepsilon_1\cdot x_1\cdot y_1}{1+x_1}$:

> $dy1 := \frac{\varepsilon_1\cdot x_1\cdot y_1}{1+x_1}-\mu_1\cdot y_1+\rho_1\cdot\left(\frac{\varepsilon_1\cdot x_1\cdot y_1}{1+x_1}\cdot y_2-\frac{\varepsilon_2\cdot x_2\cdot y_2}{1+x_2}\cdot y_1\right)$:

> $dx2 := \frac{r\cdot x_2\cdot(x_2-\eta_2)}{x_2+\theta_2}\cdot\left(1-\frac{x_2}{k_2}\right)-\frac{\varepsilon_2\cdot x_2\cdot y_2}{1+x_2}$:

> $dy2 := \frac{\varepsilon_2\cdot x_2\cdot y_2}{1+x_2}-\mu_2\cdot y_2+\rho_2\cdot\left(\frac{\varepsilon_2\cdot x_2\cdot y_2}{1+x_2}\cdot y_1-\frac{\varepsilon_1\cdot x_1\cdot y_1}{1+x_1}\cdot y_2\right)$:

> $T := solve([dx1, dy1, dx2, dy2], [x_1, y_1, x_2, y_2])$:

> with(VectorCalculus) : with(LinearAlgebra) :

> $Jac := Jacobian([dx1, dy1, dx2, dy2], [x_1, y_1, x_2, y_2])$

$$Jac := \left[\left[\frac{(x_1-\eta_1)\left(1-\frac{x_1}{k_1}\right)}{x_1+\theta_1} + \frac{x_1\left(1-\frac{x_1}{k_1}\right)}{x_1+\theta_1} - \frac{x_1(x_1-\eta_1)\left(1-\frac{x_1}{k_1}\right)}{(x_1+\theta_1)^2} - \frac{x_1(x_1-\eta_1)}{(x_1+\theta_1)k_1} - \frac{\varepsilon_1 y_1}{1+x_1} + \frac{\varepsilon_1 x_1 y_1}{(1+x_1)^2}, -\frac{\varepsilon_1 x_1}{1+x_1}, 0, 0\right],\right.$$

$$\left[\frac{\varepsilon_1 y_1}{1+x_1} - \frac{\varepsilon_1 x_1 y_1}{(1+x_1)^2} + \rho_1\left(\frac{\varepsilon_1 y_1 y_2}{1+x_1} - \frac{\varepsilon_1 x_1 y_1 y_2}{(1+x_1)^2}\right), \frac{\varepsilon_1 x_1}{1+x_1} - \mu_1 + \rho_1\left(\frac{\varepsilon_1 x_1 y_2}{1+x_1} - \frac{\varepsilon_2 x_2 y_2}{1+x_2}\right), \rho_1\left(-\frac{\varepsilon_2 y_2 y_1}{1+x_2} + \frac{\varepsilon_2 x_2 y_2 y_1}{(1+x_2)^2}\right), \rho_1\left(\frac{\varepsilon_1 x_1 y_1}{1+x_1} - \frac{\varepsilon_2 x_2 y_1}{1+x_2}\right)\right],$$

$$\left[0, 0, \frac{r(x_2-\eta_2)\left(1-\frac{x_2}{k_2}\right)}{x_2+\theta_2} + \frac{r x_2\left(1-\frac{x_2}{k_2}\right)}{x_2+\theta_2} - \frac{r x_2(x_2-\eta_2)\left(1-\frac{x_2}{k_2}\right)}{(x_2+\theta_2)^2} - \frac{r x_2(x_2-\eta_2)}{(x_2+\theta_2)k_2} - \frac{\varepsilon_2 y_2}{1+x_2} + \frac{\varepsilon_2 x_2 y_2}{(1+x_2)^2}, -\frac{\varepsilon_2 x_2}{1+x_2}\right],$$

$$\left.\left[\rho_2\left(-\frac{\varepsilon_1 y_1 y_2}{1+x_1} + \frac{\varepsilon_1 x_1 y_1 y_2}{(1+x_1)^2}\right), \rho_2\left(\frac{\varepsilon_2 x_2 y_2}{1+x_2} - \frac{\varepsilon_1 x_1 y_2}{1+x_1}\right), \frac{\varepsilon_2 y_2}{1+x_2} - \frac{\varepsilon_2 x_2 y_2}{(1+x_2)^2} + \rho_2\left(\frac{\varepsilon_2 y_2 y_1}{1+x_2} - \frac{\varepsilon_2 x_2 y_2 y_1}{(1+x_2)^2}\right), \frac{\varepsilon_2 x_2}{1+x_2} - \mu_2 + \rho_2\left(\frac{\varepsilon_2 x_2 y_1}{1+x_2} - \frac{\varepsilon_1 x_1 y_1}{1+x_1}\right)\right]\right]$$

Lampiran 3. Simulasi Numerik Model tanpa Migrasi pada Pemangsa Menggunakan Maple

a. Efek Allee Kuat

> ***#Patch 1***

> `restart : with(linalg) : with(DEtools) :`

> $k_1 := 5 : \theta_1 := 0.1 : \varepsilon_1 := 0.25 : \mu_1 := 0.2 : \eta_{11} := 2.95 : \eta_{12} := 2.85 : \eta_{13} := 2.5 :$

> $a1 := \left(\frac{x_1 \cdot (x_1 - \eta_{11})}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right)\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} :$

> $a2 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1 :$

> $b1 := \left(\frac{x_1 \cdot (x_1 - \eta_{12})}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right)\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} :$

> $b2 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 * y_1$:

> $c1 := \left(\frac{x_1 \cdot (x_1 - \eta_{13})}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right) \right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1}$:

> $c2 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 * y_1$:

> $T := solve([a1, a2], [x_1, y_1])$:

> $T2 := T[2]$;

$$T2 := [x_1 = 4., y_1 = 1.024390244]$$

> $P := solve([b1, b2], [x_1, y_1])$:

> $P2 := P[2]$

$$P2 := [x_1 = 4., y_1 = 1.121951220]$$

> $H := solve([c1, c2], [x_1, y_1])$:

> $H2 := H[2]$;

$$H2 := [x_1 = 4., y_1 = 1.463414634]$$

> with(VectorCalculus) : with(LinearAlgebra) :

> Jac1 := Jacobian([a1, a2] , $[x_1, y_1]$) :

> t2 := subs(T2, evalm(Jac1)); eigenvals(t2);

$$t2 := \begin{bmatrix} -0.01875074365 & -0.2000000000 \\ 0.01024390245 & 0. \end{bmatrix}$$

$-0.00937537182500000 + 0.0442818573813588\, I, -0.00937537182500000 - 0.0442818573813588\, I$

> Jaco1 := Jacobian([b1, b2] , $[x_1, y_1]$) :

> p2 := subs(P2, evalm(Jaco1)); eigenvals(p2);

$$p2 := \begin{bmatrix} -0.03911957180 & -0.2000000000 \\ 0.01121951220 & 0. \end{bmatrix}$$

$-0.0195597859000000 + 0.0431429857050502\, I, -0.0195597859000000 - 0.0431429857050502\, I$

> Jacoo1 := Jacobian([c1, c2] , $[x_1, y_1]$) :

> h2 := subs(H2, evalm(Jacoo1)); eigenvals(h2);

$$h2 := \begin{bmatrix} -0.1104104700 & -0.2000000000 \\ 0.01463414635 & 0. \end{bmatrix}$$

$-0.0661956263217512, -0.0442148436782488$

> **#Patch 2**

> $k_2 := 7 : \theta_2 := 0.18 : \varepsilon_2 := 0.24 : \mu_2 := 0.2 : \eta_{21} := 2.95 : r := 0.5 : \eta_{22} := 2.85 : \eta_{23} := 2.5$:

> $d1 := \left(\frac{x_2 \cdot r \cdot (x_2 - \eta_{21})}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2}\right)\right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2}$:

> $d2 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2$:

> $e1 := \left(\frac{x_2 \cdot r \cdot (x_2 - \eta_{22})}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2}\right)\right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2}$:

> $e2 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2$:

> $f1 := \left(\frac{x_2 \cdot r \cdot (x_2 - \eta_{23})}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2}\right)\right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2}$:

> $f2 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2$:

> $K := solve([d1, d2], [x_2, y_2])$:

> $K2 := K[2];$

$$K2 := [x_2 = 5., y_2 = 1.413403199]$$

> $R := solve([e1, e2], [x_2, y_2])$:

> $R2 := R[2]$

$$R2 := [x_2 = 5., y_2 = 1.482349697]$$

> $S := solve([f1, f2], [x_2, y_2])$:

> $S2 := S[2];$

$$S2 := [x_2 = 5., y_2 = 1.723662438]$$

> $Jac2 := Jacobian([d1,\ d2]\ , [x_2, y_2])$:

> $k2 := subs(K2,\ evalm(Jac2)); eigenvals(k2);$

$$k2 := \begin{bmatrix} -0.01090543697 & -0.2000000000 \\ 0.00942268797 & 0. \end{bmatrix}$$

$-0.00545271848500000 + 0.0430674523871954\,I,\ -0.00545271848500000 - 0.0430674523871954\,I$

> $Jaco2 := Jacobian([e1,\ e2]\ , [x_2, y_2])$:

> $r2 := subs(R2,\ evalm(Jaco2)); eigenvals(r2);$

$$r2 := \begin{bmatrix} -0.01816389709 & -0.2000000000 \\ 0.00988233129 & 0. \end{bmatrix}$$

$-0.00908194854500000 + 0.0435199318545649\,I,\ -0.00908194854500000 - 0.0435199318545649\,I$

> $Jacoo2 := Jacobian([f1,\ f2]\ , [x_2, y_2])$:

> $s2 := subs(S2,\ evalm(Jacoo2)); eigenvals(s2);$

$$s2 := \begin{bmatrix} -0.04356850751 & -0.2000000000 \\ 0.01149108293 & 0. \end{bmatrix}$$

$-0.0217842537550000 + 0.0427043659868376\,I, -0.0217842537550000 - 0.0427043659868376\,I$

```
> #Gambar 4.2 (a) dan Gambar 4.2 (b)
> with(plots) :
> dx1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), a1) :
> dy1 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), a2) :
> du1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), b1) :
> dv1 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), b2) :
> dw1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), c1) :
> dz1 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), c2) :
>
> ivs1 := [x_1(0) = 3.9, y_1(0) = 0.7]
```

$$ivs1 := [x_1(0) = 3.9, y_1(0) = 0.7]$$

```
> A11 := DEplot([dx1, dy1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor = "red",
      arrows = medium, scene = [t, x_1(t)]) :

> A12 := DEplot([du1, dv1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor
      = "MidnightBlue", arrows = medium, scene = [t, x_1(t)]) :

> A13 := DEplot([dw1, dz1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor
      = "DarkGreen", arrows = medium, scene = [t, x_1(t)]) :

> A21 := DEplot([dx1, dy1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor = "red",
      arrows = medium, scene = [t, y_1(t)]) :

> A22 := DEplot([du1, dv1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor
      = "MidnightBlue", arrows = medium, scene = [t, y_1(t)]) :

> A23 := DEplot([dw1, dz1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor
      = "DarkGreen", arrows = medium, scene = [t, y_1(t)]) :

> display([A11, A12, A13], labels = [Waktu (t), Kepadatan Populasi x_1(t)], labeldirections
      = [horizontal, vertical]) :
  display([A21, A22, A23], labels = [Waktu (t), Kepadatan Populasi y_1(t)], labeldirections
      = [horizontal, vertical]) :

>
> #Gambar 4.2 (c) dan Gambar 4.2 (d)
> dx2 := diff(x_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), d1) :
> dy2 := diff(y_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), d2) :
```

> $du2 := diff(x_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), e1) :$

> $dv2 := diff(y_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), e2) :$

> $dw2 := diff(x_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), f1) :$

> $dz2 := diff(y_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), f2) :$

>

> $ivs2 := [x_2(0) = 4.9, y_2(0) = 1.3]$

$$ivs2 := [x_2(0) = 4.9, y_2(0) = 1.3]$$

> $B11 := DEplot([dx2, dy2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"red"}, arrows = medium, scene = [t, x_2(t)]) :$

> $B12 := DEplot([du2, dv2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"MidnightBlue"}, arrows = medium, scene = [t, x_2(t)]) :$

> $B13 := DEplot([dw2, dz2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"DarkGreen"}, arrows = medium, scene = [t, x_2(t)]) :$

> $B21 := DEplot([dx2, dy2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"red"}, arrows = medium, scene = [t, y_2(t)]) :$

> $B22 := DEplot([du2, dv2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"MidnightBlue"}, arrows = medium, scene = [t, y_2(t)]) :$

> $B23 := DEplot([dw2, dz2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"DarkGreen"}, arrows = medium, scene = [t, y_2(t)]) :$

> $display([B11, B12, B13], labels = [Waktu\ (t), Kepadatan\ Populasi\ x_2(t)], labeldirections = [horizontal, vertical]) :$
$display([B21, B22, B23], labels = [Waktu\ (t), Kepadatan\ Populasi\ y_2(t)], labeldirections = [horizontal, vertical]) :$

>

> #Nilai θ yang divariasikan

> ***#Patch 1***

> `restart : with(linalg) : with(DEtools) :`

> $k_1 := 5 : \theta_{11} := 0.1 : \varepsilon_1 := 0.25 : \mu_1 := 0.2 : \eta_1 := 2.95 : r := 1 : \theta_{12} := 0.5 : \theta_{13} := 1 :$

> $a1 := \left(\frac{x_1 \cdot r \cdot (x_1 - \eta_1)}{x_1 + \theta_{11}} \cdot \left(1 - \frac{x_1}{k_1}\right)\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} :$

> $a2 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1 :$

> $b1 := \left(\frac{x_1 \cdot r \cdot (x_1 - \eta_1)}{x_1 + \theta_{12}} \cdot \left(1 - \frac{x_1}{k_1}\right)\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} :$

> $b2 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 * y_1 :$

> $c1 := \left(\frac{x_1 \cdot r \cdot (x_1 - \eta_1)}{x_1 + \theta_{13}} \cdot \left(1 - \frac{x_1}{k_1}\right)\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1}$:

> $c2 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 * y_1$:

> $T := solve([a1, a2], [x_1, y_1])$:

> $T2 := T[2]$;

$$T2 := [x_1 = 4., y_1 = 1.024390244]$$

> $P := solve([b1, b2], [x_1, y_1])$:

> $P2 := P[2]$

$$P2 := [x_1 = 4., y_1 = 0.9333333333]$$

> $H := solve([c1, c2], [x_1, y_1])$:

> $H2 := H[3]$;

$$H2 := [x_1 = 4., y_1 = 0.8400000000]$$

> with(VectorCalculus) : with(LinearAlgebra) :

> $Jac1 := Jacobian([a1, a2], [x_1, y_1])$:

> t2 := subs(T2, evalm(Jac1)); eigenvals(t2);

$$t2 := \begin{bmatrix} -0.01875074365 & -0.2000000000 \\ 0.01024390245 & 0. \end{bmatrix}$$

$-0.00937537182500000 + 0.0442818573813588\,I, -0.00937537182500000 - 0.0442818573813588\,I$

> $Jaco1 := Jacobian([b1, b2], [x_1, y_1])$:

> p2 := subs(P2, evalm(Jaco1)); eigenvals(p2);

$$p2 := \begin{bmatrix} -0.01303703706 & -0.2000000000 \\ 0.00933333336 & 0. \end{bmatrix}$$

$-0.00651851853000000 + 0.0427103686260614\,I, -0.00651851853000000 - 0.0427103686260614\,I$

> $Jacoo1 := Jacobian([c1, c2], [x_1, y_1])$:

> h2 := subs(H2, evalm(Jacoo1)); eigenvals(h2);

$$h2 := \begin{bmatrix} -0.00800000000 & -0.2000000000 \\ 0.00840000000 & 0. \end{bmatrix}$$

$-0.00400000000000000 + 0.0407921561087423\,I, -0.00400000000000000 - 0.0407921561087423\,I$

> ***#Patch 2***

> $k_2 := 7 : \theta_{21} := 0.18 : \varepsilon_2 := 0.24 : \mu_2 := 0.2 : \eta_2 := 2.95 : r := 0.5 : \theta_{22} := 0.05 : \theta_{23} := 0.6$:

> $d1 := \left(\frac{x_2 \cdot r \cdot (x_2 - \eta_2)}{x_2 + \theta_{21}} \cdot \left(1 - \frac{x_2}{k_2}\right) \right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$

> $d2 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 :$

> $e1 := \left(\frac{x_2 \cdot r \cdot (x_2 - \eta_2)}{x_2 + \theta_{22}} \cdot \left(1 - \frac{x_2}{k_2}\right) \right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$

> $e2 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 :$

> $f1 := \left(\frac{x_2 \cdot r \cdot (x_2 - \eta_2)}{x_2 + \theta_{23}} \cdot \left(1 - \frac{x_2}{k_2}\right) \right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$

> $f2 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 :$

> $K := solve([d1, d2], [x_2, y_2]) :$

> $K2 := K[2];$

$$K2 := [x_2 = 5., y_2 = 1.413403199]$$

> $R := solve([e1, e2], [x_2, y_2]) :$

> $R2 := R[2]$

$$R2 := [x_2 = 5., y_2 = 1.449787836]$$

> $S := solve([f1, f2], [x_2, y_2]) :$

> $S2 := S[2];$

$$S2 := [x_2 = 5., y_2 = 1.307397959]$$

> with(VectorCalculus) : with(LinearAlgebra) :

> $Jac2 := Jacobian([d1, d2], [x_2, y_2]) :$

> k2 := subs(K2, evalm(Jac2)); eigenvals(k2);

$$k2 := \begin{bmatrix} -0.01090543697 & -0.2000000000 \\ 0.00942268797 & 0. \end{bmatrix}$$

$-0.00545271848500000 + 0.0430674523871954\,I, -0.00545271848500000 - 0.0430674523871954\,I$

> $Jaco2 := Jacobian([e1, e2], [x_2, y_2]) :$

> r2 := subs(R2, evalm(Jaco2)); eigenvals(r2);

$$r2 := \begin{bmatrix} -0.01262714663 & -0.2000000000 \\ 0.00966525223 & 0. \end{bmatrix}$$

$-0.00631357331500000 + 0.0435107944997115\,I, -0.00631357331500000 - 0.0435107944997115\,I$

> $Jacoo2 := Jacobian([f1, f2], [x_2, y_2]) :$

> s2 := subs(S2, evalm(Jacoo2)); eigenvals(s2);

$$s2 := \begin{bmatrix} -0.00630162777 & -0.2000000000 \\ 0.00871598637 & 0. \end{bmatrix}$$

$-0.00315081388500000 + 0.0416325551205075\,\mathrm{I}, -0.00315081388500000 - 0.0416325551205075\,\mathrm{I}$

```
> #Gambar 4.3 (a) dan Gambar 4.3 (b)
> with(plots) :
> dx1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), a1) :
> dy1 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), a2) :
> du1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), b1) :
> dv1 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), b2) :
> dw1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), c1) :
> dz1 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), c2) :
>
> ivs1 := [x_1(0) = 3.9, y_1(0) = 0.7]
```

$$ivs1 := [x_1(0) = 3.9, y_1(0) = 0.7]$$

```
> A11 := DEplot([dx1, dy1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor = "red",
      arrows = medium, scene = [t, x_1(t)]) :

> A12 := DEplot([du1, dv1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor
      = "MidnightBlue", arrows = medium, scene = [t, x_1(t)]) :

> A13 := DEplot([dw1, dz1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor
      = "DarkGreen", arrows = medium, scene = [t, x_1(t)]) :

> A21 := DEplot([dx1, dy1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor = "red",
      arrows = medium, scene = [t, y_1(t)]) :

> A22 := DEplot([du1, dv1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor
      = "MidnightBlue", arrows = medium, scene = [t, y_1(t)]) :

> A23 := DEplot([dw1, dz1], [x_1(t), y_1(t)], t = 0 ..800, [ivs1], stepsize = 0.9, linecolor
      = "DarkGreen", arrows = medium, scene = [t, y_1(t)]) :

> display([A11, A12, A13], labels = [Waktu (t), Kepadatan Populasi x_1(t)], labeldirections
      = [horizontal, vertical]) :
  display([A21, A22, A23], labels = [Waktu (t), Kepadatan Populasi y_1(t)], labeldirections
      = [horizontal, vertical]) :

> #Gambar 4.3 (c) dan Gambar 4.3 (d)
> dx2 := diff(x_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), d1) :
> dy2 := diff(y_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), d2) :
> du2 := diff(x_2(t), t) = subs(x_2 = x_2(t), y_2 = y_2(t), e1) :
```

> $dv2 := diff(y_2(t), t) = subs(x_2 = x_2(t),\ y_2 = y_2(t), e2)$:

> $dw2 := diff(x_2(t), t) = subs(x_2 = x_2(t),\ y_2 = y_2(t), f1)$:

> $dz2 := diff(y_2(t), t) = subs(x_2 = x_2(t),\ y_2 = y_2(t), f2)$:

>

> $ivs2 := [x_2(0) = 4.9, y_2(0) = 1.3]$

$$ivs2 := [x_2(0) = 4.9, y_2(0) = 1.3]$$

> $B11 := DEplot([dx2, dy2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"red"}, arrows = medium,\ scene = [t, x_2(t)])$:

> $B12 := DEplot([du2, dv2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"MidnightBlue"}, arrows = medium,\ scene = [t, x_2(t)])$:

> $B13 := DEplot([dw2, dz2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"DarkGreen"}, arrows = medium,\ scene = [t, x_2(t)])$:

> $B21 := DEplot([dx2, dy2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"red"}, arrows = medium,\ scene = [t, y_2(t)])$:

> $B22 := DEplot([du2, dv2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"MidnightBlue"}, arrows = medium,\ scene = [t, y_2(t)])$:

> $B23 := DEplot([dw2, dz2], [x_2(t), y_2(t)], t = 0..800, [ivs2], stepsize = 0.9, linecolor = \text{"DarkGreen"}, arrows = medium,\ scene = [t, y_2(t)])$:

> $display([B11, B12, B13], labels = [Waktu\ (t),\ Kepadatan\ Populasi\ x_2(t)], labeldirections = [horizontal, vertical])$:
$display([B21, B22, B23], labels = [Waktu\ (t),\ Kepadatan\ Populasi\ y_2(t)], labeldirections = [horizontal, vertical])$:

b. Efek Allee Lemah

> **#Patch 1**

> `restart : with(linalg) : with(DEtools) :`

> $k_1 := 5 : \theta_1 := 0.5 : \varepsilon_1 := 0.25 : \mu_1 := 0.2 : \eta_1 := -0.3$:

> $a1 := \left(\frac{x_1 \cdot (x_1 - \eta_1)}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right)\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1}$:

> $a2 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1$:

> $T := solve([a1, a2], [x_1, y_1])$:

> $T2 := T[2];$

$$T2 := [x_1 = 4., y_1 = 3.822222222]$$

> `with(VectorCalculus)` : $with(LinearAlgebra)$:

```
> Jac1 := Jacobian([a1, a2] , [x1, y1]) :
> t2 := subs(T2, evalm(Jac1)); eigenvals(t2);
```

$$t2 := \begin{bmatrix} -0.6036543209 & -0.2000000000 \\ 0.0382222222 & 0. \end{bmatrix}$$

$$-0.590713280478827, -0.0129410404211726$$

> ***#Patch 2***

> $k_2 := 7 : \theta_2 := 0.18 : \varepsilon_2 := 0.26 : \mu_2 := 0.2 : \eta_2 := -0.1 : r := 0.3 :$

> $d1 := \left(\frac{x_2 \cdot r \cdot (x_2 - \eta_2)}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2} \right) \right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$

> $d2 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 :$

> $K := solve([d1, d2], [x_2, y_2]) :$

> $K2 := K[2];$

$$K2 := [x_2 = 3.333333333, y_2 = 2.559410861]$$

```
> Jac2 := Jacobian([d1, d2] , [x2, y2]) :
> k2 := subs(K2, evalm(Jac2)); eigenvals(k2);
```

$$k2 := \begin{bmatrix} -0.0180826917 & -0.2000000000 \\ 0.0354379965 & 0. \end{bmatrix}$$

$$-0.00904134585000000 + 0.0837009758916865\,I, -0.00904134585000000 - 0.0837009758916865\,I$$

> **#Gambar 4.4 (a) dan Gambar 4.4 (b)**

```
> with(plots) :
```

> $dx1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), a1) :$

> $dy1 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), a2) :$

>

> $A1 := [x_1(0) = 3.9, y_1(0) = 3];$

$$A1 := [x_1(0) = 3.9, y_1(0) = 3]$$

> $A2 := [x_1(0) = 3.9, y_1(0) = 2.5];$

$$A2 := [x_1(0) = 3.9, y_1(0) = 2.5]$$

> $A3 := [x_1(0) = 2.5, y_1(0) = 0.7];$

$$A3 := [x_1(0) = 2.5, y_1(0) = 0.7]$$

> $A11 := DEplot([dx1, dy1], [x_1(t), y_1(t)], t = 0..800, [A1], stepsize = 0.9, linecolor = \text{"DarkGreen"}, arrows = medium, scene = [t, x_1(t)]) :$

> $A12 := DEplot([dx1, dy1], [x_1(t), y_1(t)], t = 0..800, [A2], stepsize = 0.9, linecolor = \text{"MidnightBlue"}, arrows = medium, scene = [t, x_1(t)]) :$

```
> A13 := DEplot([dx1, dy1], [x1(t), y1(t)], t = 0 ..800, [A3], stepsize = 0.9, linecolor = "red",
    arrows = medium, scene = [t, x1(t)]) :

> A21 := DEplot([dx1, dy1], [x1(t), y1(t)], t = 0 ..800, [A1], stepsize = 0.9, linecolor
    = "DarkGreen", arrows = medium, scene = [t, y1(t)]) :

> A22 := DEplot([dx1, dy1], [x1(t), y1(t)], t = 0 ..800, [A2], stepsize = 0.9, linecolor
    = "MidnightBlue", arrows = medium, scene = [t, y1(t)]) :

> A23 := DEplot([dx1, dy1], [x1(t), y1(t)], t = 0 ..800, [A3], stepsize = 0.9, linecolor = "red",
    arrows = medium, scene = [t, y1(t)]) :

> display([A11, A12, A13], labels = [Waktu (t), Kepadatan Populasi x1(t)], labeldirections
    = [horizontal, vertical]) :
  display([A21, A22, A23], labels = [Waktu (t), Kepadatan Populasi y1(t)], labeldirections
    = [horizontal, vertical]) :

> #Gambar 4.4 (c) dan Gambar 4.4 (d)
> dx2 := diff(x2(t), t) = subs(x2 = x2(t), y2 = y2(t), d1) :
> dy2 := diff(y2(t), t) = subs(x2 = x2(t), y2 = y2(t), d2) :
> B1 := [x2(0) = 1.5, y2(0) = 0.5];
                          B1 := [x2(0) = 1.5, y2(0) = 0.5]
> B2 := [x2(0) = 3, y2(0) = 1];
                          B2 := [x2(0) = 3, y2(0) = 1]
> B3 := [x2(0) = 3, y2(0) = 2];
                          B3 := [x2(0) = 3, y2(0) = 2]
> B11 := DEplot([dx2, dy2], [x2(t), y2(t)], t = 0 ..800, [B1], stepsize = 0.9, linecolor = "red",
    arrows = medium, scene = [t, x2(t)]) :

> B12 := DEplot([dx2, dy2], [x2(t), y2(t)], t = 0 ..800, [B2], stepsize = 0.9, linecolor
    = "MidnightBlue", arrows = medium, scene = [t, x2(t)]) :

> B13 := DEplot([dx2, dy2], [x2(t), y2(t)], t = 0 ..800, [B3], stepsize = 0.9, linecolor
    = "DarkGreen", arrows = medium, scene = [t, x2(t)]) :

> B21 := DEplot([dx2, dy2], [x2(t), y2(t)], t = 0 ..800, [B1], stepsize = 0.9, linecolor = "red",
    arrows = medium, scene = [t, y2(t)]) :

> B22 := DEplot([dx2, dy2], [x2(t), y2(t)], t = 0 ..800, [B2], stepsize = 0.9, linecolor
    = "MidnightBlue", arrows = medium, scene = [t, y2(t)]) :
```

```
> B23 := DEplot([dx2, dy2], [x2(t), y2(t)], t = 0 ..800, [B3], stepsize = 0.9, linecolor
      = "DarkGreen", arrows = medium, scene = [t, y2(t)]) :
> display([B11, B12, B13], labels = [Waktu (t), Kepadatan Populasi x2(t)], labeldirections
      = [horizontal, vertical]) :
  display([B21, B22, B23], labels = [Waktu (t), Kepadatan Populasi y2(t)], labeldirections
      = [horizontal, vertical]) :
```

Lampiran 4. Simulasi Numerik Model dengan Migrasi pada Pemangsa Menggunakan Maple

a. Efek Allee Kuat

```
> restart : with(linalg) : with(DEtools) : with(plots) :
    with(VectorCalculus) : with(LinearAlgebra) :
```

> $k_1 := 5 : \eta_1 := 2.95 : \mu_1 := 0.2 : \varepsilon_1 := 0.25 : \theta_1 := 0.1 :$

> $k_2 := 7 : \eta_2 := 2.95 : \mu_2 := 0.2 : \varepsilon_2 := 0.24 : \theta_2 := 0.18 : r := 0.5 :$

> #$\boldsymbol{\rho_1 = 0.25}$ **dan** $\boldsymbol{\rho_2 = 0.6}$

> $\rho_{11} := 0.25 :$

> $\rho_{21} := 0.6 :$

> $dx11 := \dfrac{x_1 \cdot (x_1 - \eta_1)}{x_1 + \theta_1} \cdot \left(1 - \dfrac{x_1}{k_1}\right) - \dfrac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} :$

> $dy11 := \dfrac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1 + \rho_{11} \cdot \left(\dfrac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 - \dfrac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1\right) :$

> $dx21 := \dfrac{r \cdot x_2 \cdot (x_2 - \eta_2)}{x_2 + \theta_2} \cdot \left(1 - \dfrac{x_2}{k_2}\right) - \dfrac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$

> $dy21 := \dfrac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 + \rho_{21} \cdot \left(\dfrac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 - \dfrac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2\right) :$

> $S := solve([dx11, dy11, dx21, dy21], [x_1, y_1, x_2, y_2]) :$

> $S16 := S[16];$

$$S16 := [x_1 = 4., y_1 = 1.024390244, x_2 = 5., y_2 = 1.413403199]$$

> $Jac1 := Jacobian([dx11, dy11, dx21, dy21], [x_1, y_1, x_2, y_2]) :$

> $J1 := subs(S16, evalm(Jac1));$

$$J1 := \begin{bmatrix} -0.01875074365 & -0.2000000000 & 0 & 0 \\ 0.01386359357 & 0. & -0.00241312741 & 0. \\ 0 & 0 & -0.01090543697 & -0.2000000000 \\ -0.00868725870 & 1.\,10^{-10} & 0.01521419376 & 0. \end{bmatrix}$$

> $eigenvals(J1);$

$-0.00705918974859727 + 0.0612574796234946\,I,\ -0.00705918974859727 - 0.0612574796234946\,I,\ -0.00776890056140273 + 0.0440228027339183\,I,\ -0.00776890056140273 - 0.0440228027339183\,I$

> #$\rho_1 = 0.25$ **dan** $\rho_2 = 0.9$

> $\rho_{12} := 0.25 :$

> $\rho_{22} := 0.9 :$

> $dx12 := \frac{x_1 \cdot (x_1 - \eta_1)}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} :$

> $dy12 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1 + \rho_{12} \cdot \left(\frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 \right) :$

> $dx22 := \frac{r \cdot x_2 \cdot (x_2 - \eta_2)}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2}\right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$

> $dy22 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 + \rho_{22} \cdot \left(\frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 \right) :$

> $T := solve([dx12, dy12, dx22, dy22], [x_1, y_1, x_2, y_2]) :$

> $T16 := T[16];$

$$T16 := [x_1 = 4., y_1 = 1.024390244, x_2 = 5., y_2 = 1.413403199]$$

> $Jac2 := Jacobian([dx12, dy12, dx22, dy22], [x_1, y_1, x_2, y_2]) :$

> $J2 := subs(T16, evalm(Jac2));$

$$J2 := \begin{bmatrix} -0.01875074365 & -0.2000000000 & 0 & 0 \\ 0.01386359357 & 0. & -0.00241312741 & 0. \\ 0 & 0 & -0.01090543697 & -0.2000000000 \\ -0.01303088805 & 0. & 0.01810994665 & 0. \end{bmatrix}$$

> $eigenvals(J2);$

$-0.00666788557259425 + 0.0657865872629796\,I,\ -0.00666788557259425 - 0.0657865872629796\,I,\ -0.00816020473740576 + 0.0440751987878514\,I,\ -0.00816020473740576 - 0.0440751987878514\,I$

> #$\rho_1 = 0.025$ **dan** $\rho_2 = 0.9$

> $\rho_{13} := 0.025 :$

> $\rho_{23} := 0.9 :$

> $dx13 := \frac{x_1 \cdot (x_1 - \eta_1)}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} :$

> $dy13 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1 + \rho_{13} \cdot \left(\frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 \right) :$

> $dx23 := \frac{r \cdot x_2 \cdot (x_2 - \eta_2)}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2}\right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$

> $dy23 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 + \rho_{23} \cdot \left(\frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 \right)$:

> $V := solve([dx13, dy13, dx23, dy23], [x_1, y_1, x_2, y_2])$:

> $V16 := V[16];$

$$V16 := [x_1 = 4., y_1 = 1.024390244, x_2 = 5., y_2 = 1.413403199]$$

> $Jac3 := Jacobian([dx13, dy13, dx23, dy23], [x_1, y_1, x_2, y_2])$:

> $J3 := subs(T16, evalm(Jac3));$

$$J3 := \begin{bmatrix} -0.01875074365 & -0.2000000000 & 0 & 0 \\ 0.01060587156 & 0. & -0.000241312741 & 0. \\ 0 & 0 & -0.01090543697 & -0.2000000000 \\ -0.01303088805 & 0. & 0.01810994665 & 0. \end{bmatrix}$$

> $eigenvals(J3);$

$$-0.00920795775267717 + 0.0442707955033706\,I, -0.00920795775267717 - 0.0442707955033706\,I, -0.00562013255732280 + 0.0605340665408197\,I, -0.00562013255732280 - 0.0605340665408197\,I$$

> $ivs1 := [x_1(0) = 3.9, y_1(0) = 0.7, x_2(0) = 4.9, y_2(0) = 1.3];$

$$ivs1 := [x_1(0) = 3.9, y_1(0) = 0.7, x_2(0) = 4.9, y_2(0) = 1.3]$$

> **# Gambar 4.5**

> $f1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dx11)$:

> $f2 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dy11)$:

> $f3 := diff(x_2(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dx21)$:

> $f4 := diff(y_2(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dy21)$:

> $A1 := DEplot([f1, f2, f3, f4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0..800, [ivs1], stepsize = 0.01, linecolor = [orange], arrows = medium, scene = [t, x_1(t)])$:

> $A2 := DEplot([f1, f2, f3, f4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0..800, [ivs1], stepsize = 0.01,$ linecolor = "MidnightBlue", $arrows = medium, scene = [t, y_1(t)])$:

> $A3 := DEplot([f1, f2, f3, f4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0..800, [ivs1], stepsize = 0.01,$ linecolor = "DarkGreen", $arrows = medium, scene = [t, x_2(t)])$:

> $A4 := DEplot([f1, f2, f3, f4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0..800, [ivs1], stepsize = 0.01, linecolor = [red], arrows = medium, scene = [t, y_2(t)])$:

```
>
  display([A1], labels = [Waktu (t), Kepadatan Populasi x_1(t)], labeldirections = [horizontal,
      vertical]) :
  display([A3], labels = [Waktu (t), Kepadatan Populasi x_2(t)], labeldirections = [horizontal,
      vertical]) :
  display([A2], labels = [Waktu (t), Kepadatan Populasi y_1(t)], labeldirections = [horizontal,
      vertical]) :
  display([A4], labels = [Waktu (t), Kepadatan Populasi y_2(t)], labeldirections = [horizontal,
      vertical]) :
> # Gambar 4.6
> g1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dx12) :
> g2 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dy12) :
> g3 := diff(x_2(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dx22) :
> g4 := diff(y_2(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dy22) :
> B1 := DEplot([g1, g2, g3, g4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = [orange], arrows = medium, scene = [t, x_1(t)]) :
> B2 := DEplot([g1, g2, g3, g4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = "MidnightBlue", arrows = medium, scene = [t, y_1(t)]) :
> B3 := DEplot([g1, g2, g3, g4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = "DarkGreen", arrows = medium, scene = [t, x_2(t)]) :
> B4 := DEplot([g1, g2, g3, g4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = [red], arrows = medium, scene = [t, y_2(t)]) :
>
  display([B1], labels = [Waktu (t), Kepadatan Populasi x_1(t)], labeldirections = [horizontal,
      vertical]) :
  display([B3], labels = [Waktu (t), Kepadatan Populasi x_2(t)], labeldirections = [horizontal,
      vertical]) :
  display([B2], labels = [Waktu (t), Kepadatan Populasi y_1(t)], labeldirections = [horizontal,
      vertical]) :
  display([B4], labels = [Waktu (t), Kepadatan Populasi y_2(t)], labeldirections = [horizontal,
      vertical]) :
> # Gambar 4.7
> h1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dx13) :
> h2 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dy13) :
> h3 := diff(x_2(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dx23) :
> h4 := diff(y_2(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dy23) :
> C1 := DEplot([h1, h2, h3, h4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = [orange], arrows = medium, scene = [t, x_1(t)]) :
```

> $C2 := DEplot([h1, h2, h3, h4], [x_1(t), y_1(t), x_2(t), y_2(t)], \mathrm{t} = 0..800, [\mathrm{ivs1}], stepsize = 0.01, linecolor = \text{"MidnightBlue"}, arrows = medium, scene = [t, y_1(t)]) :$

> $C3 := DEplot([h1, h2, h3, h4], [x_1(t), y_1(t), x_2(t), y_2(t)], \mathrm{t} = 0..800, [\mathrm{ivs1}], stepsize = 0.01, linecolor = \text{"DarkGreen"}, arrows = medium, scene = [t, x_2(t)]) :$

> $C4 := DEplot([h1, h2, h3, h4], [x_1(t), y_1(t), x_2(t), y_2(t)], \mathrm{t} = 0..800, [\mathrm{ivs1}], stepsize = 0.01, linecolor = [red], arrows = medium, scene = [t, y_2(t)]) :$

>
display([C1], labels = [Waktu (t), Kepadatan Populasi $x_1(t)$], labeldirections = [horizontal, vertical]) :
display([C3], labels = [Waktu (t), Kepadatan Populasi $x_2(t)$], labeldirections = [horizontal, vertical]) :
display([C2], labels = [Waktu (t), Kepadatan Populasi $y_1(t)$], labeldirections = [horizontal, vertical]) :
display([C4], labels = [Waktu (t), Kepadatan Populasi $y_2(t)$], labeldirections = [horizontal, vertical]) :

b. Efek Allee Lemah

> `restart : with(linalg) : with(DEtools) : with(plots) : with(VectorCalculus)` : *with*(*LinearAlgebra*) :

> $k_1 := 5 : \eta_1 := -0.3 : \mu_1 := 0.2 : \varepsilon_1 := 0.25 : \theta_1 := 0.5 :$

> $k_2 := 7 : \eta_2 := -0.1 : \mu_2 := 0.2 : \varepsilon_2 := 0.26 : \theta_2 := 0.18 : r := 0.3 :$

> #$\rho_1 = 1$ **dan** $\boldsymbol{\rho_2 = 0.6}$

> $\rho_{11} := 1 :$

> $\rho_{21} := 0.6 :$

> $$dx11 := \frac{x_1 \cdot (x_1 - \eta_1)}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} :$$

> $$dy11 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1 + \rho_{11} \cdot \left(\frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1\right) :$$

> $$dx21 := \frac{r \cdot x_2 \cdot (x_2 - \eta_2)}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2}\right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$$

> $$dy21 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 + \rho_{21} \cdot \left(\frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2\right) :$$

> $S := solve([dx11, dy11, dx21, dy21], [x_1, y_1, x_2, y_2]) :$

> $S16 := S[16];$

$$S16 := [x_1 = 4., y_1 = 3.822222222, x_2 = 3.333333333, y_2 = 2.559410861]$$

> $Jac1 := Jacobian([dx11, dy11, dx21, dy21], [x_1, y_1, x_2, y_2]) :$

> $J1 := subs(S16, evalm(Jac1));$

$$J1 := \begin{bmatrix} -0.6036543209 & -0.2000000000 & 0 & 0 \\ 0.1360485929 & 1.\,10^{-10} & -0.1354518977 & 1.\,10^{-10} \\ 0 & 0 & -0.0180826917 & -0.2000000000 \\ -0.0586958225 & -1.\,10^{-10} & 0.1167091350 & -1.\,10^{-10} \end{bmatrix}$$

> $eigenvals(J1);$

$-0.556531842980759, -0.0208288804337902 + 0.154154833706846\,I,$
$-0.0208288804337902 - 0.154154833706846\,I, -0.0235474087516608$

> **#$\rho_1 = 0.6$ dan $\rho_2 = 0.6$**

> $\rho_{12} := 0.6:$

> $\rho_{22} := 0.6:$

> $dx12 := \frac{x_1 \cdot (x_1 - \eta_1)}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1}:$

> $dy12 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1 + \rho_{12} \cdot \left(\frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 \right):$

> $dx22 := \frac{r \cdot x_2 \cdot (x_2 - \eta_2)}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2}\right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2}:$

> $dy22 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 + \rho_{22} \cdot \left(\frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 \right):$

> $T := solve([dx12, dy12, dx22, dy22], [x_1, y_1, x_2, y_2]):$

> $T16 := T[16];$

$$T16 := [x_1 = 4., y_1 = 3.822222222, x_2 = 3.333333333, y_2 = 2.559410861]$$

> $Jac2 := Jacobian([dx12, dy12, dx22, dy22], [x_1, y_1, x_2, y_2]):$

> $J2 := subs(T16, evalm(Jac2));$

$$J2 := \begin{bmatrix} -0.6036543209 & -0.2000000000 & 0 & 0 \\ 0.0969180447 & 1.\,10^{-10} & -0.0812711385 & 1.\,10^{-10} \\ 0 & 0 & -0.0180826917 & -0.2000000000 \\ -0.0586958225 & -1.\,10^{-10} & 0.1167091350 & -1.\,10^{-10} \end{bmatrix}$$

> $eigenvals(J2);$

$-0.570675445915976, -0.0191909544683318, -0.0159353061078461$
$+ 0.153739998481117\,I, -0.0159353061078461 - 0.153739998481117\,I$

> **#$\rho_1 = 0.25$ dan $\rho_2 = 0.6$**

> $\rho_{13} := 0.25:$

> $\rho_{23} := 0.6:$

> $dx13 := \frac{x_1 \cdot (x_1 - \eta_1)}{x_1 + \theta_1} \cdot \left(1 - \frac{x_1}{k_1}\right) - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1}:$

$$> dy13 := \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} - \mu_1 \cdot y_1 + \rho_{13} \cdot \left(\frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 \right) :$$

$$> dx23 := \frac{r \cdot x_2 \cdot (x_2 - \eta_2)}{x_2 + \theta_2} \cdot \left(1 - \frac{x_2}{k_2}\right) - \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} :$$

$$> dy23 := \frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} - \mu_2 \cdot y_2 + \rho_{23} \cdot \left(\frac{\varepsilon_2 \cdot x_2 \cdot y_2}{1 + x_2} \cdot y_1 - \frac{\varepsilon_1 \cdot x_1 \cdot y_1}{1 + x_1} \cdot y_2 \right) :$$

$> V := solve([dx13, dy13, dx23, dy23], [x_1, y_1, x_2, y_2]) :$

$> V16 := V[16];$

$$V16 := [x_1 = 4., y_1 = 3.822222222, x_2 = 3.333333333, y_2 = 2.559410861]$$

$> Jac3 := Jacobian([dx13, dy13, dx23, dy23], [x_1, y_1, x_2, y_2]) :$

$> J3 := subs(T16, evalm(Jac3));$

$$J3 := \begin{bmatrix} -0.6036543209 & -0.2000000000 & 0 & 0 \\ 0.06267881491 & 1.\ 10^{-10} & -0.0338629744 & 0. \\ 0 & 0 & -0.0180826917 & -0.2000000000 \\ -0.0586958225 & -1.\ 10^{-10} & 0.1167091350 & -1.\ 10^{-10} \end{bmatrix}$$

$> eigenvals(J3);$

$$-0.582522096639785, -0.0155129987935773, -0.0118509585833183 + 0.153106488640279\,I, -0.0118509585833183 - 0.153106488640279\,I$$

$> ivs1 := [x_1(0) = 3.9, y_1(0) = 3, x_2(0) = 3, y_2(0) = 2];$

$$ivs1 := [x_1(0) = 3.9, y_1(0) = 3, x_2(0) = 3, y_2(0) = 2]$$

> **# Gambar 4.8**

$> f1 := diff(x_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dx11) :$

$> f2 := diff(y_1(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dy11) :$

$> f3 := diff(x_2(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dx21) :$

$> f4 := diff(y_2(t), t) = subs(x_1 = x_1(t), y_1 = y_1(t), x_2 = x_2(t), y_2 = y_2(t), dy21) :$

$> A1 := DEplot([f1, f2, f3, f4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0..800, [ivs1], stepsize = 0.01,$
$linecolor = [orange], arrows = medium, scene = [t, x_1(t)]) :$

$> A2 := DEplot([f1, f2, f3, f4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0..800, [ivs1], stepsize = 0.01,$
$linecolor = \text{"MidnightBlue"}, arrows = medium, scene = [t, y_1(t)]) :$

$> A3 := DEplot([f1, f2, f3, f4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0..800, [ivs1], stepsize = 0.01,$
$linecolor = \text{"DarkGreen"}, arrows = medium, scene = [t, x_2(t)]) :$

$> A4 := DEplot([f1, f2, f3, f4], [x_1(t), y_1(t), x_2(t), y_2(t)], t = 0..800, [ivs1], stepsize = 0.01,$
$linecolor = [red], arrows = medium, scene = [t, y_2(t)]) :$

```
> display([A1], labels = [Waktu(t), Kepadatan Populasi x1(t)], labeldirections = [horizontal, vertical]) :
  display([A3], labels = [Waktu(t), Kepadatan Populasi x2(t)], labeldirections = [horizontal, vertical]) :
  display([A2], labels = [Waktu(t), Kepadatan Populasi y1(t)], labeldirections = [horizontal, vertical]) :
  display([A4], labels = [Waktu(t), Kepadatan Populasi y2(t)], labeldirections = [horizontal, vertical]) :
> # Gambar 4.9
> g1 := diff(x1(t), t) = subs(x1 = x1(t), y1 = y1(t), x2 = x2(t), y2 = y2(t), dx12) :
> g2 := diff(y1(t), t) = subs(x1 = x1(t), y1 = y1(t), x2 = x2(t), y2 = y2(t), dy12) :
> g3 := diff(x2(t), t) = subs(x1 = x1(t), y1 = y1(t), x2 = x2(t), y2 = y2(t), dx22) :
> g4 := diff(y2(t), t) = subs(x1 = x1(t), y1 = y1(t), x2 = x2(t), y2 = y2(t), dy22) :
> B1 := DEplot([g1, g2, g3, g4], [x1(t), y1(t), x2(t), y2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = [orange], arrows = medium, scene = [t, x1(t)]) :
> B2 := DEplot([g1, g2, g3, g4], [x1(t), y1(t), x2(t), y2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = "MidnightBlue", arrows = medium, scene = [t, y1(t)]) :
> B3 := DEplot([g1, g2, g3, g4], [x1(t), y1(t), x2(t), y2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = "DarkGreen", arrows = medium, scene = [t, x2(t)]) :
> B4 := DEplot([g1, g2, g3, g4], [x1(t), y1(t), x2(t), y2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = [red], arrows = medium, scene = [t, y2(t)]) :
> display([B1], labels = [Waktu(t), Kepadatan Populasi x1(t)], labeldirections = [horizontal, vertical]) :
  display([B3], labels = [Waktu(t), Kepadatan Populasi x2(t)], labeldirections = [horizontal, vertical]) :
  display([B2], labels = [Waktu(t), Kepadatan Populasi y1(t)], labeldirections = [horizontal, vertical]) :
  display([B4], labels = [Waktu(t), Kepadatan Populasi y2(t)], labeldirections = [horizontal, vertical]) :
> # Gambar 4.10
> h1 := diff(x1(t), t) = subs(x1 = x1(t), y1 = y1(t), x2 = x2(t), y2 = y2(t), dx13) :
> h2 := diff(y1(t), t) = subs(x1 = x1(t), y1 = y1(t), x2 = x2(t), y2 = y2(t), dy13) :
> h3 := diff(x2(t), t) = subs(x1 = x1(t), y1 = y1(t), x2 = x2(t), y2 = y2(t), dx23) :
> h4 := diff(y2(t), t) = subs(x1 = x1(t), y1 = y1(t), x2 = x2(t), y2 = y2(t), dy23) :
> C1 := DEplot([h1, h2, h3, h4], [x1(t), y1(t), x2(t), y2(t)], t = 0 ..800, [ivs1], stepsize = 0.01,
      linecolor = [orange], arrows = medium, scene = [t, x1(t)]) :
```

> $C2 := DEplot\left([h1, h2, h3, h4], \left[x_1(t), y_1(t), x_2(t), y_2(t) \right], \mathrm{t} = 0 ..800, [\mathrm{ivs1}], stepsize = 0.01, linecolor = \text{"MidnightBlue"}, arrows = medium,\ scene = \left[t, y_1(t) \right] \right) :$

> $C3 := DEplot\left([h1, h2, h3, h4], \left[x_1(t), y_1(t), x_2(t), y_2(t) \right], \mathrm{t} = 0 ..800, [\mathrm{ivs1}], stepsize = 0.01, linecolor = \text{"DarkGreen"}, arrows = medium,\ scene = \left[t, x_2(t) \right] \right) :$

> $C4 := DEplot\left([h1, h2, h3, h4], \left[x_1(t), y_1(t), x_2(t), y_2(t) \right], \mathrm{t} = 0 ..800, [\mathrm{ivs1}], stepsize = 0.01, linecolor = [red], arrows = medium,\ scene = \left[t, y_2(t) \right] \right) :$

>

display([C1], labels = [Waktu (t), Kepadatan Populasi $\mathrm{x}_1(\mathrm{t})$], labeldirections = [horizontal, vertical]) :

display([C3], labels = [Waktu (t), Kepadatan Populasi $\mathrm{x}_2(\mathrm{t})$], labeldirections = [horizontal, vertical]) :

display([C2], labels = [Waktu (t), Kepadatan Populasi $\mathrm{y}_1(\mathrm{t})$], labeldirections = [horizontal, vertical]) :

display([C4], labels = [Waktu (t), Kepadatan Populasi $\mathrm{y}_2(\mathrm{t})$], labeldirections = [horizontal, vertical]) :