DAFTAR PUSTAKA

Alman, Kusuma J., Amiruddin. 2013. Penyelesaian Numerik Persamaan Adveksi-difusi 2-D untuk Transfer Polutan Dengan Menggunakan Metode Beda Hingga DuFort-Frankel. Fakultas MIPA Universitas Hasanuddin Makassar.

Anderson J.D. 2010. *Fundamentals of Aerodynamics Fifth Edition.* New York: McGraw-Hill Companies, Inc.

Biringen S., & Chow C. 2011. *An Introduction to Computational Fluid Mechanics by Example.*, Hoboken, New Jersey: John Wiley & Sons, Inc.

Gerald C. F., & Wheatley P.O. 1970. *Applied Numerical Analysis Fifth Edition*. San Luis Obispo: California Polytechnic University.

Gilberto E. & Urroz. 2004. *Convergence, Stability, and Consistency of Finite Difference Schemes in the Solution of Partial Differential Equation*. Diakses 23 Februari 2020. Tersedia oleh: http://www2.myoops.org/twocw/usu/Civil_and_Environmental_Engineering/Numerical_Methods_in_Civil_Engineering/usufiles/StabilityNumericalSchemes.pdf

Hoffmann K.A., & Chiang S.T. 1995. *Computational Fluid Duynamics For Engineers Volume I*. Kansas: Engineering Education System.

Hundsdorfer W., & Verwer J. 2003. *Numerical Solution of Time-Dependent Advection-Diffusion Reaction Equation*. New York: Springer-Verlag Berlin Heidenberg

Hutomo G.D., Kusuma J., Ribal A., Mahie A.G., Aris N. 2019. *Numerical Solution of 2-D advection-diffusion equation with variable coefficient using du-fort frankel method*. IOP Conference Series 1180(2019)012009 doi:10.1088/1742-6596/1180/1/012009.

Krisdiantoro. 2012. Model Sebaran Tumpahan Minyak di Perairan Indramayu Jawa Barat. Bogor. Institut Pertanian Bogor.

Morton K.W., & Mayers, D.F. 2005. *Numerical Solution of Partial Differential Equations Second Edition*. New York: Cambridge University Press.

or. 2007. Pencemaran Pesisir dan Laut. P.T. Pradya Paramita. Jakarta.

983. *Computational Techniques For Differential Equations.* New York: orth-Holland Publishing Company.

Sampera H., & Apriansyah. 2016. Aplikasi Metode Beda Hingga Crank-Nicolson Implisit Untuk Menentukan Kasus Adveksi-difusi 2D pada Sebaran Polutan di Suatu Perairan. Pontianak: Universitas Tanjungpura.

Strikwerda J.C. 2004. *Finite difference schemes and partial differential equations. Second edition.* Philadelphia, PA : Society for Industrial and Applied Mathematics (SIAM).

Zaureder E. 2006. *Partial Differential Equation of Applied Mathematics*. New York: John Wiley & Sons, Inc.

LAMPIRAN

Lampiran 1. Program MATLAB penyebaran polutan yang diinjeksikan sesaat pada danau dengan 2 *inlet* dan 2 *outlet*

```
clear all; clc;                 %Pseudocode MATLAB

%KONSTANTA DAN GRID
panjang=1;                      %Panjang Danau
lebar=panjang;                  %Lebar Danau
waktu=600;                      %Perhitungan Waktu Maksimal

Dx=0.0004;                      %Koefisien difusi arah x
Dy=Dx;                          %Koefisien difusi arah y

dx=0.05;                        %Lebar Grid dalam sumbu x
dy=dx;                          %Lebar Grid dalam sumbu y
dt=0.025;                       %Lebar Grid dalam ruang waktu t

x=0:dx:panjang;                 %Grid sumbu x
y=0:dy:lebar;                   %Grid sumbu y
t=0:dt:waktu;                   %Grid waktu t

L=(panjang/dx)+1;               %Jumlah Grid sumbu x
M=(lebar/dy)+1;                 %Jumlah Grid sumbu y
N=(waktu/dt)+1;                 %Jumlah Grid waktu t
%=================================================================
==============================
%                   TITIK KOORDINAT ARUS MASUK DAN KELUAR DANAU
PERSEGI
%=================================================================
==============================
x1=8;                           %9,1  (Bawah)
x2=11;                          %1,12 (Kiri)
x3=10;                          %11,41(Atas)
x4=9;                           %41,10(Kanan)

error=1e-20;                    %Error Konvergensi Metode Gauss_Seidel
T=2000;                         %Jumlah Maksimal Iterasi Gauss Seidel

k1=0.0025;                      %Konstanta Nilai Batas
k2=0;                           %Konstanta Nilai Batas
%=================================================================
==============================
%                                       NILAI BATAS ALIRAN
%=================================================================
==============================
%NILAI BATAS BAWAH
for j=1;
    for k=1:T;
        for i=1:x1-1;
            X(i,j,k)=k1;
        end
        for i=x1+1:L;
            X(i,j,k)=k2;
        end


BATAS KIRI
```

```
    for k=1:T;
        for j=1:x2-1;
            X(i,j,k)=k1;
        end
        for j=x2+1:M;
            X(i,j,k)=k2;
        end
    end
end
%NILAI BATAS ATAS
for j=M;
    for k=1:T;
        for i=1:x3-1;
            X(i,j,k)=k2;
        end
        for i=x3+1:L;
            X(i,j,k)=k1;
        end
    end
end
%NILAI BATAS KANAN
for i=L;
    for k=1:T;
        for j=1:x4-1;
            X(i,j,k)=k2;
        end
        for j=x4+1:L;
            X(i,j,k)=k1;
        end
    end
end
%=================================================================
==============================
%                                          METODE GAUSS SEIDEL
%=================================================================
==============================
%NILAI AWAL ITERASI GAUSS SEIDEL
for i=2:L-1
    for j=2:M-1
        for k=1
            X(i,j,k)=0.1;
        end
    end
end
X(x1,1,1)=0.1;
X(1,x2,1)=0.1;
X(x3,M,1)=0.1;
X(L,x4,1)=0.1;

%PERHITUNGAN ITERASI

for k=2:T
    X(x1,1,k)=(1/4)*(X(x1-1,1,k-1)+X(x1+1,1,k-1)+2*X(x1,2,k-1));
    x2,k)=(1/4)*(X(1,x2-1,k-1)+X(1,x2+1,k-1)+2*X(2,x2,k-1));
    3,M,k)=(1/4)*(X(x3+1,M,k-1)+X(x3-1,M,k-1)+2*X(x3,M-1,k-1));
    x4,k)=(1/4)*(X(L,x4+1,k-1)+X(L,x4-1,k-1)+2*X(L-1,x4,k-1));

    j=2:L-1
```

```
        for i=2:L-1

            X(i,j,k)=(1/4)*(X(i+1,j,k-1)+X(i-1,j,k)+X(i,j+1,k-
1)+X(i,j-1,k));

            a(i,j)=X(i,j,k)-X(i,j,k-1);
            b(i,j)=abs(a(i,j));
        end
    end
            c=max(max(b));
            if c < error;
                X(:,:,k);
                r=k;
                break
            else
                r=k;
                continue
            end
end

%TRANSFORMASI X KE NILAI PSI(Aliran)
for i=1:L
    for j=1:M
        Psi(i,j)=X(i,j,r);
    end
end
%=================================================================
==============================
%                              PENENTUAN NILAI KECEPATAN ARUS DI
TIAP TITIK
%=================================================================
==============================
for i=1:L
    for j=1;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
    end
end
for i=1
    for j=1:M;
        if j>1 && j<M
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
        L
        j=M;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
```

```
            vy(i,j)=0;
        end
    end
end
for i=L
    for j=1:M;
        if j>1 && j<M
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=2:L-1
    for j=2:M-1
        ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
    end
end
%=================================================================
==============================
%                                       METODE DuFort-Frankel
%=================================================================
==============================
%NILAI KONSTANTA SKEMA DuFort-Frankel
Bx=Dx*dt/dx^2;
By=Dy*dt/dy^2;

for i=2:1:L-1
    for j=2:1:M-1
        Ax(i,j) =ux(i,j)*(dt/dx);
        Ay(i,j) =vy(i,j)*(dt/dy);
    end
end

%TITIK KOORDINAT DAN NILAI KONSENTRASI AWAL
c11=8;                      %titik sumbu x
c22=2;                      %titik sumbu y

c1=10;                       %Nilai Konsentrasi awal di titik
spesifik (c11,c22)
c0=1;                        %Nilai Konsentrasi awal di titik lain

%NILAI KONSENTRASI PADA BATAS BATAS DANAU
for n=1:1:N;
    for j=1;
        for i=1:L;
            C(i,j,n)=c0;     %Bawah
        end
    end
end
:1:N;
j=M;
for i=1:1:L;
    C(i,j,n)=c0;      %Atas
end
```

```
end
for n=1:1:N;
    for j=2:1:M;
        for i=1;
            C(i,j,n)=c0;     %Kiri
        end
    end
end
for n=1:1:N;
    for j=1:1:M-1;
        for i=L;
            C(i,j,n)=c0;     %Kanan
        end
    end
end

%NILAI KONSENTRASI AWAL
for n=1:1:N;
    for j=2:1:M-1;
        for i=2:1:L-1;
            C(i,j,n)=c0;
        end
    end
end

C(c11,c22,1)=c1;             %Nilai Konsentrasi di Titik Spesifik
%=================================================================
==============================
%   Perhitungan skema FTCS untuk menentukan nilai konsentrasi pada
grid langkah waktu kedua
%=================================================================
==============================
for n=1;
    for j=2:1:M-1
        for i=2:1:L-1
            C(i,j,n+1)=((-
Ax(i,j)+2*Bx)/(2))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(2))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(2))*C(i,j+1,n)+((Ay(i,j)+2*By)/(2))*C(i,j-1,n)+(1-
2*Bx-2*By)*C(i,j,n);
        end
    end
end
%=================================================================
==============================
%                           Perhitungan dengan skema DuFort-Frankel
%=================================================================
==============================
for n=2:1:N
    for j=2:1:M-1
        for i=2:1:L-1
            C(i,j,n+1)=((1-2*Bx-2*By)/(1+2*Bx+2*By))*C(i,j,n-
1)+((-
Ax(i,j)+2*Bx)/(1+2*Bx+2*By))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(1+2*Bx+2*
i-1,j,n)+((-
2*By)/(1+2*Bx+2*By))*C(i,j+1,n)+((Ay(i,j)+2*By)/(1+2*Bx+2*
i,j-1,n);
        end
```

```
end
%=================================================================
=============================
%                                                PLOTING
%=================================================================
=============================
disp('Pilih Jenis Ploting')
disp('1. Ploting Waktu Sesaat ')
disp('2. Ploting Animasi Perubahan Terhadap Waktu ')
z1=input('Pilih= ');
disp('==============================================================
====');
if z1==1
%PLOTING
ts=input('          Input Time Step (ex:60)= ');
subplot(1,3,1);
surf(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');
cb = colorbar;

subplot(1,3,2);
for i=1:L
    for j=1:M
        Ux(i,j)=ux(i,j);
        Uy(i,j)=vy(i,j);
    end
end
quiver(x,y,transpose(Ux),transpose(Uy));
xlabel('Lebar Danau');
ylabel('Panjang Danau');

subplot(1,3,3);
[D,h]=contour(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
colormap jet

else
    if z1==2
%OUTPUT GRAFIS ANIMASI
disp('          Pilih Jenis Animasi')
disp('          1. Animasi Contour Konsentrasi ')
disp('          2. Animasi Hamburan Warna Konsentrasi Spesifik ')
z11=input('          Pilih= ');
disp('
=====================================================');
if z11==1
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
1:(1/dt):Tmax+1

x,y,transpose(C(:,:,am)));
[0 90]);
l('Lebar Danau');
```

```
% ylabel('Panjang Danau');
% zlabel('Consentration');

[D,h]=contour(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
else
    if z11==2
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

surf(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');

% [D,h]=contour(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
    else
        disp('Tidak Ada Tampilan!')
    end
end
    else
    disp('Tidak Ada Tampilan!');
    end
end
```

Lampiran 2. Program MATLAB penyebaran polutan yang diinjeksikan sesaat pada danau dengan *inlet* di bagian bawah dan *outlet* di bagian atas domain

```
clear all; clc;                 %Pseudocode MATLAB

%KONSTANTA DAN GRID
panjang=1;                      %Panjang Danau
lebar=panjang;                  %Lebar Danau
waktu=600;                      %Perhitungan Waktu Maksimal

Dx=0.0004;                      %Koefisien difusi arah x
Dy=Dx;                          %Koefisien difusi arah y

dx=0.05;                        %Lebar Grid dalam sumbu x
dy=dx;                          %Lebar Grid dalam sumbu y
dt=0.025;                       %Lebar Grid dalam ruang waktu t

x=0:dx:panjang;                 %Grid sumbu x
y=0:dy:lebar;                   %Grid sumbu y
t=0:dt:waktu;                   %Grid waktu t

L=(panjang/dx)+1;               %Jumlah Grid sumbu x
M=(lebar/dy)+1;                 %Jumlah Grid sumbu y
N=(waktu/dt)+1;                 %Jumlah Grid waktu t
%=================================================================
==============================
%                   TITIK KOORDINAT ARUS MASUK DAN KELUAR DANAU
PERSEGI
%=================================================================
==============================
x1=8;                           %9,1  (Bawah)
x3=10;                          %11,41(Atas)

error=1e-20;                    %Error Toleransi Konvergensi Metode
Gauss_Seidel
T=2000;                         %Jumlah Maksimal Iterasi Gauss Seidel

k1=0.0025;                      %Konstanta Nilai Batas
k2=0;                           %Konstanta Nilai Batas

%=================================================================
==============================
%                                       NILAI BATAS ALIRAN
%=================================================================
==============================
%NILAI BATAS BAWAH
for j=1;
    for k=1:T;
        for i=1:x1-1;
            X(i,j,k)=k1;
        end
        for i=x1+1:L;
            X(i,j,k)=k2;
        end
```

```
%NILAI BATAS KIRI
for i=1;
    for k=1:T;
        for j=1:M;
            X(i,j,k)=k1;
        end
    end
end
%NILAI BATAS ATAS
for j=M;
    for k=1:T;
        for i=1:x3-1;
            X(i,j,k)=k1;
        end
        for i=x3+1:L;
            X(i,j,k)=k2;
        end
    end
end
%NILAI BATAS KANAN
for i=L;
    for k=1:T;
        for j=1:L;
            X(i,j,k)=k2;
        end
    end
end
%=================================================================
==============================
%                                        METODE GAUSS SEIDEL
%=================================================================
==============================
%NILAI AWAL GAUSS POINT ITERATION
for i=2:L-1
    for j=2:M-1
        for k=1
            X(i,j,k)=0.1;
        end
    end
end
X(x1,1,1)=0.1;
X(x3,M,1)=0.1;

%PERHITUNGAN ITERASI

for k=2:T
    X(x1,1,k)=(1/4)*(X(x1-1,1,k-1)+X(x1+1,1,k-1)+2*X(x1,2,k-1));
    X(x3,M,k)=(1/4)*(X(x3+1,M,k-1)+X(x3-1,M,k-1)+2*X(x3,M-1,k-1));
    for j=2:L-1
        for i=2:L-1

            X(i,j,k)=(1/4)*(X(i+1,j,k-1)+X(i-1,j,k)+X(i,j+1,k-
j-1,k));

            a(i,j)=X(i,j,k)-X(i,j,k-1);
            b(i,j)=abs(a(i,j));
        end
```

```
            c=max(max(b));
            if c < error;
                X(:,:,k);
                r=k;
                break
            else
                r=k;
                continue
            end
end

%TRANSFORMASI X KE NILAI PSI(Aliran)
for i=1:L
    for j=1:M
        Psi(i,j)=X(i,j,r);
    end
end
Psi1(:,:)=transpose(Psi);

%=================================================================
=============================
%                            PENENTUAN NILAI KECEPATAN ARUS DI
TIAP TITIK
%=================================================================
=============================
for i=1:L
    for j=1;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
    end
end
for i=1
    for j=1:M;
        if j>1 && j<M
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=1:L
    for j=M;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end


        j=1:M;
        if j>1 && j<M
```

```
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=2:L-1
    for j=2:M-1
        ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dx);
        vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
    end
end
%=================================================================
==============================
%                                    METODE DuFort-Frankel
%=================================================================
==============================
%NILAI KONSTANTA SKEMA DuFort-Frankel
Bx=Dx*dt/dx^2;
By=Dy*dt/dy^2;

for i=2:1:L-1
    for j=2:1:M-1
        Ax(i,j) =ux(i,j)*(dt/dx);
        Ay(i,j) =vy(i,j)*(dt/dy);
    end
end

%TITIK KOORDINAT DAN NILAI KONSENTRASI AWAL
c11=8;                      %titik sumbu x
c22=2;                      %titik sumbu y

c1=10;                       %Nilai Konsentrasi awal di titik
spesifik (c11,c22)
c0=1;                        %Nilai Konsentrasi awal di titik lain

%NILAI KONSENTRASI PADA BATAS BATAS DANAU
for n=1:1:N;
    for j=1;
        for i=1:L;
            C(i,j,n)=c0; %Bawah
        end
    end
end
for n=1:1:N;
    for j=M;
        for i=1:1:L;
            C(i,j,n)=1; %Atas
        end
    end
end
:1:N;
j=1:1:M;
for i=1;
    C(i,j,n)=c0; %Kiri
end
```

```
end
for n=1:1:N;
    for j=1:1:M;
        for i=L;
            C(i,j,n)=c0; %Kanan
        end
    end
end

%NILAI KONSENTRASI AWAL
for n=1:1:N;
    for j=2:1:M;
        for i=2:1:L;
            C(i,j,n)=c0;
        end
    end
end

C(c11,c22,1)=c1;            %Nilai Konsentrasi Awal di Titik
Spesifik
%=================================================================
=============================
%   Perhitungan skema FTCS untuk menentukan nilai konsentrasi pada
grid langkah waktu kedua
%=================================================================
=============================
for n=1;
    for j=2:1:M-1
        for i=2:1:L-1
            C(i,j,n+1)=((-
Ax(i,j)+2*Bx)/(2))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(2))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(2))*C(i,j+1,n)+((Ay(i,j)+2*By)/(2))*C(i,j-1,n)+(1-
2*Bx-2*By)*C(i,j,n);
        end
    end
end
%=================================================================
=============================
%                           Perhitungan dengan skema DuFort-Frankel
%=================================================================
=============================
for n=2:1:N
    for j=2:1:M-1
        for i=2:1:L-2
            C(i,j,n+1)=((1-2*Bx-2*By)/(1+2*Bx+2*By))*C(i,j,n-
1)+((-
Ax(i,j)+2*Bx)/(1+2*Bx+2*By))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(1+2*Bx+2*
By))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(1+2*Bx+2*By))*C(i,j+1,n)+((Ay(i,j)+2*By)/(1+2*Bx+2*
By))*C(i,j-1,n);
        end
    end
end
%=================================================================
=======================
                                            PLOTING
%=================================================================
=======================
```

```
disp('Pilih Jenis Ploting')
disp('1. Ploting Waktu Sesaat ')
disp('2. Ploting Animasi Perubahan Terhadap Waktu ')
z1=input('Pilih= ');
disp('==========================================================
====');
if z1==1
%PLOTING
ts=input('          Input Time Step (ex:60)= ');
subplot(1,3,1);
surf(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');
cb = colorbar;

subplot(1,3,2);
for i=1:L
    for j=1:M
        Ux(i,j)=ux(i,j);
        Uy(i,j)=vy(i,j);
    end
end
quiver(x,y,transpose(Ux),transpose(Uy));
xlabel('Lebar Danau');
ylabel('Panjang Danau');

subplot(1,3,3);
[D,h]=contour(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
colormap jet

else
    if z1==2
%OUTPUT GRAFIS ANIMASI
disp('          Pilih Jenis Animasi')
disp('          1. Animasi Contour Konsentrasi ')
disp('          2. Animasi Hamburan Warna Konsentrasi Spesifik ')
z11=input('          Pilih= ');
disp('
=====================================================');
if z11==1
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

% surf(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
l('Panjang Danau');
l('Consentration');

ontour(x,y,transpose(C(:,:,am)));
 90]);
'Lebar Danau');
```

```
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
else
    if z11==2
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

surf(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');

% [D,h]=contour(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
    else
        disp('Tidak Ada Tampilan!')
    end
end
    else
    disp('Tidak Ada Tampilan!');
    end
end
```

Lampiran 3. Program MATLAB penyebaran polutan yang diinjeksikan sesaat pada danau dengan *inlet* di bagian bawah dan *outlet* di bagian kanan domain

```
clear all; clc;                 %Pseudocode MATLAB

%KONSTANTA DAN GRID
panjang=1;                      %Panjang Danau
lebar=panjang;                  %Lebar Danau
waktu=600;                      %Perhitungan Waktu Maksimal

Dx=0.0004;                      %Koefisien difusi arah x
Dy=Dx;                          %Koefisien difusi arah y

dx=0.05;                        %Lebar Grid dalam sumbu x
dy=dx;                          %Lebar Grid dalam sumbu y
dt=0.025;                       %Lebar Grid dalam ruang waktu t

x=0:dx:panjang;                 %Grid sumbu x
y=0:dy:lebar;                   %Grid sumbu y
t=0:dt:waktu;                   %Grid waktu t

L=(panjang/dx)+1;               %Jumlah Grid sumbu x
M=(lebar/dy)+1;                 %Jumlah Grid sumbu y
N=(waktu/dt)+1;                 %Jumlah Grid waktu t
%=================================================================
==============================
%                   TITIK KOORDINAT ARUS MASUK DAN KELUAR DANAU
PERSEGI
%=================================================================
==============================
x1=8;                           %9,1  (Bawah)
x3=10;                          %11,41(Atas)

error=1e-20;                    %Error Toleransi Konvergensi Metode
Gauss_Seidel
T=2000;                         %Jumlah Maksimal Iterasi Gauss Seidel

k1=0.0025;                      %Konstanta Nilai Batas
k2=0;                           %Konstanta Nilai Batas

%=================================================================
==============================
%                                       NILAI BATAS ALIRAN
%=================================================================
==============================
%NILAI BATAS BAWAH
for j=1;
    for k=1:T;
        for i=1:x1-1;
            X(i,j,k)=k1;
        end
        for i=x1+1:L;
            X(i,j,k)=k2;
        end
```

```
%NILAI BATAS KIRI
for i=1;
    for k=1:T;
        for j=1:M;
            X(i,j,k)=k1;
        end
    end
end
%NILAI BATAS ATAS
for j=M;
    for k=1:T;
        for i=1:L;
            X(i,j,k)=k1;
        end
    end
end
%NILAI BATAS KANAN
for i=L;
    for k=1:T;
        for j=1:x3-1;
            X(i,j,k)=k2;
        end
        for j=x3+1:L;
            X(i,j,k)=k1;
        end
    end
end
%=================================================================
=============================
%                                        METODE GAUSS SEIDEL
%=================================================================
=============================
%NILAI AWAL GAUSS POINT ITERATION
for i=2:L-1
    for j=2:M-1
        for k=1
            X(i,j,k)=0.1;
        end
    end
end
X(x1,1,1)=0.1;
X(x3,M,1)=0.1;

%PERHITUNGAN ITERASI

for k=2:T
    X(x1,1,k)=(1/4)*(X(x1-1,1,k-1)+X(x1+1,1,k-1)+2*X(x1,2,k-1));
    X(x3,M,k)=(1/4)*(X(x3+1,M,k-1)+X(x3-1,M,k-1)+2*X(x3,M-1,k-1));
    for j=2:L-1
        for i=2:L-1

            X(i,j,k)=(1/4)*(X(i+1,j,k-1)+X(i-1,j,k)+X(i,j+1,k-
j-1,k));

            a(i,j)=X(i,j,k)-X(i,j,k-1);
            b(i,j)=abs(a(i,j));
        end
```

```
            c=max(max(b));
            if c < error;
                X(:,:,k);
                r=k;
                break
            else
                r=k;
                continue
            end
end

%TRANSFORMASI X KE NILAI PSI(Aliran)
for i=1:L
    for j=1:M
        Psi(i,j)=X(i,j,r);
    end
end
Psi1(:,:)=transpose(Psi);

%=================================================================
=============================
%                            PENENTUAN NILAI KECEPATAN ARUS DI
TIAP TITIK
%=================================================================
=============================
for i=1:L
    for j=1;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
    end
end
for i=1
    for j=1:M;
        if j>1 && j<M
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=1:L
    for j=M;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
```



```
    j=1:M;
    if j>1 && j<M
```

```
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=2:L-1
    for j=2:M-1
        ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dx);
        vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
    end
end
%=================================================================
=============================
%                                    METODE DuFort-Frankel
%=================================================================
=============================
%NILAI KONSTANTA SKEMA DuFort-Frankel
Bx=Dx*dt/dx^2;
By=Dy*dt/dy^2;

for i=2:1:L-1
    for j=2:1:M-1
        Ax(i,j) =ux(i,j)*(dt/dx);
        Ay(i,j) =vy(i,j)*(dt/dy);
    end
end

%TITIK KOORDINAT DAN NILAI KONSENTRASI AWAL
c11=8;                      %titik sumbu x
c22=2;                      %titik sumbu y

c1=10;                       %Nilai Konsentrasi awal di titik
spesifik (c11,c22)
c0=1;                        %Nilai Konsentrasi awal di titik lain

%NILAI KONSENTRASI PADA BATAS BATAS DANAU
for n=1:1:N;
    for j=1;
        for i=1:L;
            C(i,j,n)=c0; %Bawah
        end
    end
end
for n=1:1:N;
    for j=M;
        for i=1:1:L;
            C(i,j,n)=1; %Atas
        end
    end
end
:1:N;
j=1:1:M;
for i=1;
    C(i,j,n)=c0; %Kiri
end
```

```
end
for n=1:1:N;
    for j=1:1:M;
        for i=L;
            C(i,j,n)=c0; %Kanan
        end
    end
end

%NILAI KONSENTRASI AWAL
for n=1:1:N;
    for j=2:1:M;
        for i=2:1:L;
            C(i,j,n)=c0;
        end
    end
end

C(c11,c22,1)=c1;            %Nilai Konsentrasi Awal di Titik
Spesifik
%=================================================================
=============================
%   Perhitungan skema FTCS untuk menentukan nilai konsentrasi pada
grid langkah waktu kedua
%=================================================================
=============================
for n=1;
    for j=2:1:M-1
        for i=2:1:L-1
            C(i,j,n+1)=((-
Ax(i,j)+2*Bx)/(2))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(2))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(2))*C(i,j+1,n)+((Ay(i,j)+2*By)/(2))*C(i,j-1,n)+(1-
2*Bx-2*By)*C(i,j,n);
        end
    end
end
%=================================================================
=============================
%                           Perhitungan dengan skema DuFort-Frankel
%=================================================================
=============================
for n=2:1:N
    for j=2:1:M-1
        for i=2:1:L-2
            C(i,j,n+1)=((1-2*Bx-2*By)/(1+2*Bx+2*By))*C(i,j,n-
1)+((-
Ax(i,j)+2*Bx)/(1+2*Bx+2*By))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(1+2*Bx+2*
By))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(1+2*Bx+2*By))*C(i,j+1,n)+((Ay(i,j)+2*By)/(1+2*Bx+2*
By))*C(i,j-1,n);
        end
    end
end
=================================================================
=======================
                                                PLOTING
=================================================================
=======================
```

```
disp('Pilih Jenis Ploting')
disp('1. Ploting Waktu Sesaat ')
disp('2. Ploting Animasi Perubahan Terhadap Waktu ')
z1=input('Pilih= ');
disp('==========================================================
====');
if z1==1
%PLOTING
ts=input('          Input Time Step (ex:60)= ');
subplot(1,3,1);
surf(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');
cb = colorbar;

subplot(1,3,2);
for i=1:L
    for j=1:M
        Ux(i,j)=ux(i,j);
        Uy(i,j)=vy(i,j);
    end
end
quiver(x,y,transpose(Ux),transpose(Uy));
xlabel('Lebar Danau');
ylabel('Panjang Danau');

subplot(1,3,3);
[D,h]=contour(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
colormap jet

else
    if z1==2
%OUTPUT GRAFIS ANIMASI
disp('          Pilih Jenis Animasi')
disp('          1. Animasi Contour Konsentrasi ')
disp('          2. Animasi Hamburan Warna Konsentrasi Spesifik ')
z11=input('          Pilih= ');
disp('
=====================================================');
if z11==1
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

% surf(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
l('Panjang Danau');
l('Consentration');

ontour(x,y,transpose(C(:,:,am)));
 90]);
'Lebar Danau');
```

```
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
else
    if z11==2
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

surf(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');

% [D,h]=contour(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
    else
        disp('Tidak Ada Tampilan!')
    end
end
    else
    disp('Tidak Ada Tampilan!');
    end
end
```

Lampiran 4. Program MATLAB penyebaran polutan yang diinjeksikan terus menerus pada danau dengan 2 *inlet* dan 2 *outlet*

```
clear all; clc;                 %Pseudocode MATLAB

%KONSTANTA DAN GRID
panjang=1;                      %Panjang Danau
lebar=panjang;                  %Lebar Danau
waktu=600;                      %Perhitungan Waktu Maksimal

Dx=0.0004;                      %Koefisien difusi arah x
Dy=Dx;                          %Koefisien difusi arah y

dx=0.05;                        %Lebar Grid dalam sumbu x
dy=dx;                          %Lebar Grid dalam sumbu y
dt=0.025;                       %Lebar Grid dalam ruang waktu t

x=0:dx:panjang;                 %Grid sumbu x
y=0:dy:lebar;                   %Grid sumbu y
t=0:dt:waktu;                   %Grid waktu t

L=(panjang/dx)+1;               %Jumlah Grid sumbu x
M=(lebar/dy)+1;                 %Jumlah Grid sumbu y
N=(waktu/dt)+1;                 %Jumlah Grid waktu t
%=================================================================
==============================
%                   TITIK KOORDINAT ARUS MASUK DAN KELUAR DANAU
PERSEGI
%=================================================================
==============================
x1=8;                           %9,1  (Bawah)
x2=11;                          %1,12 (Kiri)
x3=10;                          %11,41(Atas)
x4=9;                           %41,10(Kanan)

error=1e-20;                    %Error Konvergensi Metode Gauss_Seidel
T=2000;                         %Jumlah Maksimal Iterasi Gauss Seidel

k1=0.0025;                      %Konstanta Nilai Batas
k2=0;                           %Konstanta Nilai Batas
%=================================================================
==============================
%                                       NILAI BATAS ALIRAN
%=================================================================
==============================
%NILAI BATAS BAWAH
for j=1;
    for k=1:T;
        for i=1:x1-1;
            X(i,j,k)=k1;
        end
        for i=x1+1:L;
            X(i,j,k)=k2;
        end


BATAS KIRI
```

```
    for k=1:T;
        for j=1:x2-1;
            X(i,j,k)=k1;
        end
        for j=x2+1:M;
            X(i,j,k)=k2;
        end
    end
end
%NILAI BATAS ATAS
for j=M;
    for k=1:T;
        for i=1:x3-1;
            X(i,j,k)=k2;
        end
        for i=x3+1:L;
            X(i,j,k)=k1;
        end
    end
end
%NILAI BATAS KANAN
for i=L;
    for k=1:T;
        for j=1:x4-1;
            X(i,j,k)=k2;
        end
        for j=x4+1:L;
            X(i,j,k)=k1;
        end
    end
end
%=================================================================
==============================
%                                          METODE GAUSS SEIDEL
%=================================================================
==============================
%NILAI AWAL ITERASI GAUSS SEIDEL
for i=2:L-1
    for j=2:M-1
        for k=1
            X(i,j,k)=0.1;
        end
    end
end
X(x1,1,1)=0.1;
X(1,x2,1)=0.1;
X(x3,M,1)=0.1;
X(L,x4,1)=0.1;

%PERHITUNGAN ITERASI

for k=2:T
    X(x1,1,k)=(1/4)*(X(x1-1,1,k-1)+X(x1+1,1,k-1)+2*X(x1,2,k-1));
    x2,k)=(1/4)*(X(1,x2-1,k-1)+X(1,x2+1,k-1)+2*X(2,x2,k-1));
    3,M,k)=(1/4)*(X(x3+1,M,k-1)+X(x3-1,M,k-1)+2*X(x3,M-1,k-1));
    x4,k)=(1/4)*(X(L,x4+1,k-1)+X(L,x4-1,k-1)+2*X(L-1,x4,k-1));

    j=2:L-1
```

```
        for i=2:L-1

            X(i,j,k)=(1/4)*(X(i+1,j,k-1)+X(i-1,j,k)+X(i,j+1,k-
1)+X(i,j-1,k));

            a(i,j)=X(i,j,k)-X(i,j,k-1);
            b(i,j)=abs(a(i,j));
        end
    end
            c=max(max(b));
            if c < error;
                X(:,:,k);
                r=k;
                break
            else
                r=k;
                continue
            end
end

%TRANSFORMASI X KE NILAI PSI(Aliran)
for i=1:L
    for j=1:M
        Psi(i,j)=X(i,j,r);
    end
end
%=================================================================
==============================
%                              PENENTUAN NILAI KECEPATAN ARUS DI
TIAP TITIK
%=================================================================
==============================
for i=1:L
    for j=1;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
    end
end
for i=1
    for j=1:M;
        if j>1 && j<M
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
        L
        j=M;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
```

```
            vy(i,j)=0;
        end
    end
end
for i=L
    for j=1:M;
        if j>1 && j<M
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=2:L-1
    for j=2:M-1
        ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
    end
end
%=================================================================
==============================
%                                       METODE DuFort-Frankel
%=================================================================
==============================
%NILAI KONSTANTA SKEMA DuFort-Frankel
Bx=Dx*dt/dx^2;
By=Dy*dt/dy^2;

for i=2:1:L-1
    for j=2:1:M-1
        Ax(i,j) =ux(i,j)*(dt/dx);
        Ay(i,j) =vy(i,j)*(dt/dy);
    end
end

%TITIK KOORDINAT DAN NILAI KONSENTRASI AWAL
c11=8;                      %titik sumbu x
c22=2;                      %titik sumbu y

c1=10;                       %Nilai Konsentrasi awal di titik
spesifik (c11,c22)
c0=1;                        %Nilai Konsentrasi awal di titik lain

%NILAI KONSENTRASI PADA BATAS BATAS DANAU
for n=1:1:N;
    for j=1;
        for i=1:L;
            C(i,j,n)=c0;     %Bawah
        end
    end
end
:1:N;
j=M;
for i=1:1:L;
    C(i,j,n)=c0;      %Atas
end
```

```
end
for n=1:1:N;
    for j=2:1:M;
        for i=1;
            C(i,j,n)=c0;     %Kiri
        end
    end
end
for n=1:1:N;
    for j=1:1:M-1;
        for i=L;
            C(i,j,n)=c0;     %Kanan
        end
    end
end

%NILAI KONSENTRASI AWAL
for n=1:1:N;
    for j=2:1:M-1;
        for i=2:1:L-1;
            C(i,j,n)=c0;
        end
    end
end

C(c11,c22,1)=c1;             %Nilai Konsentrasi di Titik Spesifik
%=================================================================
==============================
%   Perhitungan skema FTCS untuk menentukan nilai konsentrasi pada
grid langkah waktu kedua
%=================================================================
==============================
for n=1;
    for j=2:1:M-1
        for i=2:1:L-1
            if i==c11 && j==c22
                C(c11,c22,n)=c1;
            else
            C(i,j,n+1)=((-
Ax(i,j)+2*Bx)/(2))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(2))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(2))*C(i,j+1,n)+((Ay(i,j)+2*By)/(2))*C(i,j-1,n)+(1-
2*Bx-2*By)*C(i,j,n);
            end
        end
    end
end
%=================================================================
==============================
%                           Perhitungan dengan skema DuFort-Frankel
%=================================================================
==============================
for n=2:1:N
    for j=2:1:M-1
        for i=2:1:L-2
            if i==c11 && j==c22
                C(c11,c22,n)=c1;
            else
```

```
            C(i,j,n+1)=((1-2*Bx-2*By)/(1+2*Bx+2*By))*C(i,j,n-
1)+((-
Ax(i,j)+2*Bx)/(1+2*Bx+2*By))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(1+2*Bx+2*
By))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(1+2*Bx+2*By))*C(i,j+1,n)+((Ay(i,j)+2*By)/(1+2*Bx+2*
By))*C(i,j-1,n);
            end
        end
    end
end
%=================================================================
==============================
%                                                PLOTING
%=================================================================
==============================
disp('Pilih Jenis Ploting')
disp('1. Ploting Waktu Sesaat ')
disp('2. Ploting Animasi Perubahan Terhadap Waktu ')
z1=input('Pilih= ');
disp('==============================================================
====');
if z1==1
%PLOTING
ts=input('          Input Time Step (ex:60)= ');
subplot(1,3,1);
surf(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');
cb = colorbar;

subplot(1,3,2);
for i=1:L
    for j=1:M
        Ux(i,j)=ux(i,j);
        Uy(i,j)=vy(i,j);
    end
end
quiver(x,y,transpose(Ux),transpose(Uy));
xlabel('Lebar Danau');
ylabel('Panjang Danau');

subplot(1,3,3);
[D,h]=contour(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
colormap jet

else
    if z1==2
%OUTPUT GRAFIS ANIMASI
                Pilih Jenis Animasi')
                1. Animasi Contour Konsentrasi ')
                2. Animasi Hamburan Warna Konsentrasi Spesifik ')
        ut('          Pilih= ');

        ==================================================');
```

```
if z11==1
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

% surf(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% zlabel('Consentration');

[D,h]=contour(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
else
    if z11==2
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

surf(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');

% [D,h]=contour(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
 colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
    else
        disp('Tidak Ada Tampilan!')

e
p('Tidak Ada Tampilan!');
```

Lampiran 5. Program MATLAB penyebaran polutan yang diinjeksikan terus menerus pada danau dengan *inlet* di bagian bawah dan *outlet* di bagian atas domain

```
clear all; clc;                 %Pseudocode MATLAB

%KONSTANTA DAN GRID
panjang=1;                      %Panjang Danau
lebar=panjang;                  %Lebar Danau
waktu=600;                      %Perhitungan Waktu Maksimal

Dx=0.0004;                      %Koefisien difusi arah x
Dy=Dx;                          %Koefisien difusi arah y

dx=0.05;                        %Lebar Grid dalam sumbu x
dy=dx;                          %Lebar Grid dalam sumbu y
dt=0.025;                       %Lebar Grid dalam ruang waktu t

x=0:dx:panjang;                 %Grid sumbu x
y=0:dy:lebar;                   %Grid sumbu y
t=0:dt:waktu;                   %Grid waktu t

L=(panjang/dx)+1;               %Jumlah Grid sumbu x
M=(lebar/dy)+1;                 %Jumlah Grid sumbu y
N=(waktu/dt)+1;                 %Jumlah Grid waktu t
%=================================================================
==============================
%                   TITIK KOORDINAT ARUS MASUK DAN KELUAR DANAU
PERSEGI
%=================================================================
==============================
x1=8;                           %9,1  (Bawah)
x3=10;                          %11,41(Atas)

error=1e-20;                    %Error Toleransi Konvergensi Metode
Gauss_Seidel
T=2000;                         %Jumlah Maksimal Iterasi Gauss Seidel

k1=0.0025;                      %Konstanta Nilai Batas
k2=0;                           %Konstanta Nilai Batas

%=================================================================
==============================
%                                       NILAI BATAS ALIRAN
%=================================================================
==============================
%NILAI BATAS BAWAH
for j=1;
    for k=1:T;
        for i=1:x1-1;
            X(i,j,k)=k1;
        end
        for i=x1+1:L;
            X(i,j,k)=k2;
        end
```

```
%NILAI BATAS KIRI
for i=1;
    for k=1:T;
        for j=1:M;
            X(i,j,k)=k1;
        end
    end
end
%NILAI BATAS ATAS
for j=M;
    for k=1:T;
        for i=1:x3-1;
            X(i,j,k)=k1;
        end
        for i=x3+1:L;
            X(i,j,k)=k2;
        end
    end
end
%NILAI BATAS KANAN
for i=L;
    for k=1:T;
        for j=1:L;
            X(i,j,k)=k2;
        end
    end
end
%=================================================================
==============================
%                                        METODE GAUSS SEIDEL
%=================================================================
==============================
%NILAI AWAL GAUSS POINT ITERATION
for i=2:L-1
    for j=2:M-1
        for k=1
            X(i,j,k)=0.1;
        end
    end
end
X(x1,1,1)=0.1;
X(x3,M,1)=0.1;

%PERHITUNGAN ITERASI

for k=2:T
    X(x1,1,k)=(1/4)*(X(x1-1,1,k-1)+X(x1+1,1,k-1)+2*X(x1,2,k-1));
    X(x3,M,k)=(1/4)*(X(x3+1,M,k-1)+X(x3-1,M,k-1)+2*X(x3,M-1,k-1));
    for j=2:L-1
        for i=2:L-1

            X(i,j,k)=(1/4)*(X(i+1,j,k-1)+X(i-1,j,k)+X(i,j+1,k-
j-1,k));

            a(i,j)=X(i,j,k)-X(i,j,k-1);
            b(i,j)=abs(a(i,j));
        end
```

```
            c=max(max(b));
            if c < error;
                X(:,:,k);
                r=k;
                break
            else
                r=k;
                continue
            end
end

%TRANSFORMASI X KE NILAI PSI(Aliran)
for i=1:L
    for j=1:M
        Psi(i,j)=X(i,j,r);
    end
end
Psi1(:,:)=transpose(Psi);

%=================================================================
=============================
%                            PENENTUAN NILAI KECEPATAN ARUS DI
TIAP TITIK
%=================================================================
=============================
for i=1:L
    for j=1;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
    end
end
for i=1
    for j=1:M;
        if j>1 && j<M
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=1:L
    for j=M;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
```



```
    j=1:M;
    if j>1 && j<M
```

```
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=2:L-1
    for j=2:M-1
        ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dx);
        vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
    end
end
%=================================================================
==============================
%                                    METODE DuFort-Frankel
%=================================================================
==============================
%NILAI KONSTANTA SKEMA DuFort-Frankel
Bx=Dx*dt/dx^2;
By=Dy*dt/dy^2;

for i=2:1:L-1
    for j=2:1:M-1
        Ax(i,j) =ux(i,j)*(dt/dx);
        Ay(i,j) =vy(i,j)*(dt/dy);
    end
end

%TITIK KOORDINAT DAN NILAI KONSENTRASI AWAL
c11=8;                      %titik sumbu x
c22=2;                      %titik sumbu y

c1=10;                       %Nilai Konsentrasi awal di titik
spesifik (c11,c22)
c0=1;                        %Nilai Konsentrasi awal di titik lain

%NILAI KONSENTRASI PADA BATAS BATAS DANAU
for n=1:1:N;
    for j=1;
        for i=1:L;
            C(i,j,n)=c0; %Bawah
        end
    end
end
for n=1:1:N;
    for j=M;
        for i=1:1:L;
            C(i,j,n)=1; %Atas
        end
    end
end
:1:N;
j=1:1:M;
for i=1;
    C(i,j,n)=c0; %Kiri
end
```

```
end
for n=1:1:N;
    for j=1:1:M;
        for i=L;
            C(i,j,n)=c0; %Kanan
        end
    end
end

%NILAI KONSENTRASI AWAL
for n=1:1:N;
    for j=2:1:M;
        for i=2:1:L;
            C(i,j,n)=c0;
        end
    end
end

C(c11,c22,1)=c1;            %Nilai Konsentrasi Awal di Titik
Spesifik
%=================================================================
=============================
%   Perhitungan skema FTCS untuk menentukan nilai konsentrasi pada
grid langkah waktu kedua
%=================================================================
=============================
for n=1;
    for j=2:1:M-1
        for i=2:1:L-1
            if i==c11 && j==c22
                C(c11,c22,n)=c1;
            else
            C(i,j,n+1)=((-
Ax(i,j)+2*Bx)/(2))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(2))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(2))*C(i,j+1,n)+((Ay(i,j)+2*By)/(2))*C(i,j-1,n)+(1-
2*Bx-2*By)*C(i,j,n);
            end
        end
    end
end
%=================================================================
=============================
%                           Perhitungan dengan skema DuFort-Frankel
%=================================================================
=============================
for n=2:1:N
    for j=2:1:M-1
        for i=2:1:L-2
            if i==c11 && j==c22
                C(c11,c22,n)=c1;
            else
            C(i,j,n+1)=((1-2*Bx-2*By)/(1+2*Bx+2*By))*C(i,j,n-
1)+((
+2*Bx)/(1+2*Bx+2*By))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(1+2*Bx+2*
i-1,j,n)+((-
+2*By)/(1+2*Bx+2*By))*C(i,j+1,n)+((Ay(i,j)+2*By)/(1+2*Bx+2*
i,j-1,n);
            end
```

```
        end
    end
end
%=================================================================
==============================
%                                                PLOTING
%=================================================================
==============================
disp('Pilih Jenis Ploting')
disp('1. Ploting Waktu Sesaat ')
disp('2. Ploting Animasi Perubahan Terhadap Waktu ')
z1=input('Pilih= ');
disp('==============================================================
====');
if z1==1
%PLOTING
ts=input('          Input Time Step (ex:60)= ');
subplot(1,3,1);
surf(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');
cb = colorbar;

subplot(1,3,2);
for i=1:L
    for j=1:M
        Ux(i,j)=ux(i,j);
        Uy(i,j)=vy(i,j);
    end
end
quiver(x,y,transpose(Ux),transpose(Uy));
xlabel('Lebar Danau');
ylabel('Panjang Danau');

subplot(1,3,3);
[D,h]=contour(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
colormap jet

else
    if z1==2
%OUTPUT GRAFIS ANIMASI
disp('          Pilih Jenis Animasi')
disp('          1. Animasi Contour Konsentrasi ')
disp('          2. Animasi Hamburan Warna Konsentrasi Spesifik ')
z11=input('          Pilih= ');
disp('
=====================================================');
if z11==1
    put('                    Input grid waktu maksimal(ex:100)=

    l:(1/dt):Tmax+1


    x,y,transpose(C(:,:,am)));
```

```
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% zlabel('Consentration');

[D,h]=contour(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
else
    if z11==2
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

surf(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');

% [D,h]=contour(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
    else
        disp('Tidak Ada Tampilan!')
    end
end
    else
    disp('Tidak Ada Tampilan!');
    end
end
```

Lampiran 6. Program MATLAB penyebaran polutan yang diinjeksikan terus menerus pada danau dengan *inlet* di bagian bawah dan *outlet* di bagian kanan domain

```
clear all; clc;                 %Pseudocode MATLAB

%KONSTANTA DAN GRID
panjang=1;                      %Panjang Danau
lebar=panjang;                  %Lebar Danau
waktu=600;                      %Perhitungan Waktu Maksimal

Dx=0.0004;                      %Koefisien difusi arah x
Dy=Dx;                          %Koefisien difusi arah y

dx=0.05;                        %Lebar Grid dalam sumbu x
dy=dx;                          %Lebar Grid dalam sumbu y
dt=0.025;                       %Lebar Grid dalam ruang waktu t

x=0:dx:panjang;                 %Grid sumbu x
y=0:dy:lebar;                   %Grid sumbu y
t=0:dt:waktu;                   %Grid waktu t

L=(panjang/dx)+1;               %Jumlah Grid sumbu x
M=(lebar/dy)+1;                 %Jumlah Grid sumbu y
N=(waktu/dt)+1;                 %Jumlah Grid waktu t
%=================================================================
==============================
%                   TITIK KOORDINAT ARUS MASUK DAN KELUAR DANAU
PERSEGI
%=================================================================
==============================
x1=8;                           %9,1  (Bawah)
x3=10;                          %11,41(Atas)

error=1e-20;                    %Error Toleransi Konvergensi Metode
Gauss_Seidel
T=2000;                         %Jumlah Maksimal Iterasi Gauss Seidel

k1=0.0025;                      %Konstanta Nilai Batas
k2=0;                           %Konstanta Nilai Batas

%=================================================================
==============================
%                                       NILAI BATAS ALIRAN
%=================================================================
==============================
%NILAI BATAS BAWAH
for j=1;
    for k=1:T;
        for i=1:x1-1;
            X(i,j,k)=k1;
        end
        for i=x1+1:L;
            X(i,j,k)=k2;
        end
```

```
%NILAI BATAS KIRI
for i=1;
    for k=1:T;
        for j=1:M;
            X(i,j,k)=k1;
        end
    end
end
%NILAI BATAS ATAS
for j=M;
    for k=1:T;
        for i=1:L;
            X(i,j,k)=k1;
        end
    end
end
%NILAI BATAS KANAN
for i=L;
    for k=1:T;
        for j=1:x3-1;
            X(i,j,k)=k2;
        end
        for j=x3+1:L;
            X(i,j,k)=k1;
        end
    end
end
%=================================================================
=============================
%                                        METODE GAUSS SEIDEL
%=================================================================
=============================
%NILAI AWAL GAUSS POINT ITERATION
for i=2:L-1
    for j=2:M-1
        for k=1
            X(i,j,k)=0.1;
        end
    end
end
X(x1,1,1)=0.1;
X(x3,M,1)=0.1;

%PERHITUNGAN ITERASI

for k=2:T
    X(x1,1,k)=(1/4)*(X(x1-1,1,k-1)+X(x1+1,1,k-1)+2*X(x1,2,k-1));
    X(x3,M,k)=(1/4)*(X(x3+1,M,k-1)+X(x3-1,M,k-1)+2*X(x3,M-1,k-1));
    for j=2:L-1
        for i=2:L-1

            X(i,j,k)=(1/4)*(X(i+1,j,k-1)+X(i-1,j,k)+X(i,j+1,k-
j-1,k));

            a(i,j)=X(i,j,k)-X(i,j,k-1);
            b(i,j)=abs(a(i,j));
        end
```

```
            c=max(max(b));
            if c < error;
                X(:,:,k);
                r=k;
                break
            else
                r=k;
                continue
            end
end

%TRANSFORMASI X KE NILAI PSI(Aliran)
for i=1:L
    for j=1:M
        Psi(i,j)=X(i,j,r);
    end
end
Psi1(:,:)=transpose(Psi);

%=================================================================
=============================
%                            PENENTUAN NILAI KECEPATAN ARUS DI
TIAP TITIK
%=================================================================
=============================
for i=1:L
    for j=1;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
    end
end
for i=1
    for j=1:M;
        if j>1 && j<M
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=1:L
    for j=M;
        ux(i,j)=0;
        if i>1 && i<L
            vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
        else
            vy(i,j)=0;
        end
```



```
    j=1:M;
    if j>1 && j<M
```

```
            ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dy);
        else
            ux(i,j)=0;
        end
        vy(i,j)=0;
    end
end
for i=2:L-1
    for j=2:M-1
        ux(i,j)=(Psi(i,j+1)-Psi(i,j-1))/(2*dx);
        vy(i,j)=(Psi(i-1,j)-Psi(i+1,j))/(2*dx);
    end
end
%=================================================================
==============================
%                                    METODE DuFort-Frankel
%=================================================================
==============================
%NILAI KONSTANTA SKEMA DuFort-Frankel
Bx=Dx*dt/dx^2;
By=Dy*dt/dy^2;

for i=2:1:L-1
    for j=2:1:M-1
        Ax(i,j) =ux(i,j)*(dt/dx);
        Ay(i,j) =vy(i,j)*(dt/dy);
    end
end

%TITIK KOORDINAT DAN NILAI KONSENTRASI AWAL
c11=8;                      %titik sumbu x
c22=2;                      %titik sumbu y

c1=10;                       %Nilai Konsentrasi awal di titik
spesifik (c11,c22)
c0=1;                        %Nilai Konsentrasi awal di titik lain

%NILAI KONSENTRASI PADA BATAS BATAS DANAU
for n=1:1:N;
    for j=1;
        for i=1:L;
            C(i,j,n)=c0; %Bawah
        end
    end
end
for n=1:1:N;
    for j=M;
        for i=1:1:L;
            C(i,j,n)=1; %Atas
        end
    end
end
:1:N;
j=1:1:M;
for i=1;
    C(i,j,n)=c0; %Kiri
end
```

```
end
for n=1:1:N;
    for j=1:1:M;
        for i=L;
            C(i,j,n)=c0; %Kanan
        end
    end
end

%NILAI KONSENTRASI AWAL
for n=1:1:N;
    for j=2:1:M;
        for i=2:1:L;
            C(i,j,n)=c0;
        end
    end
end

C(c11,c22,1)=c1;            %Nilai Konsentrasi Awal di Titik
Spesifik
%=================================================================
==============================
%   Perhitungan skema FTCS untuk menentukan nilai konsentrasi pada
grid langkah waktu kedua
%=================================================================
==============================
for n=1;
    for j=2:1:M-1
        for i=2:1:L-1
            if i==c11 && j==c22
                C(c11,c22,n)=c1;
            else
            C(i,j,n+1)=((-
Ax(i,j)+2*Bx)/(2))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(2))*C(i-1,j,n)+((-
Ay(i,j)+2*By)/(2))*C(i,j+1,n)+((Ay(i,j)+2*By)/(2))*C(i,j-1,n)+(1-
2*Bx-2*By)*C(i,j,n);
            end
        end
    end
end
%=================================================================
==============================
%                           Perhitungan dengan skema DuFort-Frankel
%=================================================================
==============================
for n=2:1:N
    for j=2:1:M-1
        for i=2:1:L-2
            if i==c11 && j==c22
                C(c11,c22,n)=c1;
            else
            C(i,j,n+1)=((1-2*Bx-2*By)/(1+2*Bx+2*By))*C(i,j,n-
1)+((
            +2*Bx)/(1+2*Bx+2*By))*C(i+1,j,n)+((Ax(i,j)+2*Bx)/(1+2*Bx+2*
            i-1,j,n)+((-
            +2*By)/(1+2*Bx+2*By))*C(i,j+1,n)+((Ay(i,j)+2*By)/(1+2*Bx+2*
            i,j-1,n);
            end
```

```
        end
    end
end
%=================================================================
=============================
%                                                PLOTING
%=================================================================
=============================
disp('Pilih Jenis Ploting')
disp('1. Ploting Waktu Sesaat ')
disp('2. Ploting Animasi Perubahan Terhadap Waktu ')
z1=input('Pilih= ');
disp('==============================================================
====');
if z1==1
%PLOTING
ts=input('          Input Time Step (ex:60)= ');
subplot(1,3,1);
surf(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');
cb = colorbar;

subplot(1,3,2);
for i=1:L
    for j=1:M
        Ux(i,j)=ux(i,j);
        Uy(i,j)=vy(i,j);
    end
end
quiver(x,y,transpose(Ux),transpose(Uy));
xlabel('Lebar Danau');
ylabel('Panjang Danau');

subplot(1,3,3);
[D,h]=contour(x,y,transpose(C(:,:,ts)));
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));
colormap jet

else
    if z1==2
%OUTPUT GRAFIS ANIMASI
disp('          Pilih Jenis Animasi')
disp('          1. Animasi Contour Konsentrasi ')
disp('          2. Animasi Hamburan Warna Konsentrasi Spesifik ')
z11=input('          Pilih= ');
disp('
=====================================================');
if z11==1
    put('                    Input grid waktu maksimal(ex:100)=

    l:(1/dt):Tmax+1


    x,y,transpose(C(:,:,am)));
```

```
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% zlabel('Consentration');

[D,h]=contour(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
else
    if z11==2
Tmax=input('                    Input grid waktu maksimal(ex:100)=
');
for am=1:(1/dt):Tmax+1
clf

surf(x,y,transpose(C(:,:,am)));
view([0 90]);
xlabel('Lebar Danau');
ylabel('Panjang Danau');
zlabel('Consentration');

% [D,h]=contour(x,y,transpose(C(:,:,am)));
% view([0 90]);
% xlabel('Lebar Danau');
% ylabel('Panjang Danau');
% set(h,'ShowText','on','TextStep',get(h,'LevelStep'));

colormap jet
cb = colorbar;
time=sprintf('Waktu : %0.1f || Langkah Waktu : %g',(am-1)*dt,(am-
1));
title(time)
drawnow
end
    else
        disp('Tidak Ada Tampilan!')
    end
end
    else
    disp('Tidak Ada Tampilan!');
    end
end
```