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Analytical Study of the Existence of a Hopf Bifurcation in the Tumor Cell Growth Model with Time Delay

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Abstract

In this paper we study a mathematical model of an immune response system consisting of a number of immune cells that work together to protect the human body from invading tumor cells. The delay differential equation is used to model the immune system caused by a natural delay in the activation process of immune cells. Analytical studies are focused on finding conditions in which the system undergoes changes in stability near a tumor-free steady-state solution. We found that the existence of a tumor-free steady-state solution was warranted when the number of activated effector cells was sufficiently high. By considering the lag of stimulation of helper cell production as the bifurcation parameter, a critical lag is obtained that determines the threshold of the stability change of the tumor-free steady state. It is also leading the system undergoes a Hopf bifurcation to periodic solutions at the tumor-free steady-state solution.

Keywords: tumor–immune system; delay differential equation; transcendental function; Hopf bifurcation.

Abstrak

Dalam makalah ini, dikaji model matematika dari sistem respon imun yang terdiri dari sejumlah sel imun yang bekerja sama untuk melindungi tubuh manusia dari invasi sel tumor. Persamaan diferensial tunda digunakan untuk memodelkan sistem kekebalan yang disebabkan oleh keterlambatan alami dalam proses aktivasi sel-sel imun. Studi analitik difokuskan untuk menemukan kondisi di mana sistem mengalami perubahan stabilitas di sekitar solusi kesetimbangan bebas tumor. Diperoleh bahwa solusi kesetimbangan bebas tumor dijamin ada ketika jumlah sel efektor yang diaktifkan cukup tinggi. Dengan mempertimbangkan tundaan stimulasi produksi sel helper sebagai parameter bifurkasi, didapatkan lag kritis yang menentukan ambang batas perubahan stabilitas dari solusi kesetimbangan bebas tumor. Parameter tersebut juga mengakibatkan sistem mengalami percabangan Hopf untuk solusi periodik pada solusi kesetimbangan bebas tumor.

Kata kunci: *sistem tumor–imun; persamaan differensial tundaan; fungsi transedental; bifurkasi Hopf.*

1. INTRODUCTION

Tumor or cancer is a kind of neoplasm that is formed due to an abnormal growth or excessive growth of tissue caused by the rapid division of body cells that have undergone some form of mutation [1] [2]. Tumors are divided into two major groups, namely benign and malignant or cancerous tumors [1]. Benign tumors do not invade surrounding systems and do not spread to other parts of the body but can grow locally to become large. Meanwhile, malignant or cancerous tumors damage other normal cell systems and spread to other organs through connective system, blood, nerves, and system supporting organs [1][2]. Naturally in the human body, tumor cells can be controlled by the immune system as antigens because of the expression of viral proteins that these cells have not previously

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