IRREGULAR WAVE BOTTOM BOUNDARY LAYER OVER ROUGH BED

T. Rachman ¹, Suntoyo ², Juswan ³ and Wahyuddin ⁴

ABSTRACT: An understanding of nature of the wave boundary layer above the seabed is of fundamental importance to coastal engineers and workers in the field of sediment transport. Waves in natural coastal environments are essentially irregular and the properties in the bottom boundary layer are different from those under purely sinusoidal waves. Therefore, it is appropriate to investigate boundary layer behavior over rough bed under irregular waves to achieve the most representative estimation of bottom shear stress in coastal sediment process. The bottom boundary layer in water wave propagation is important factor which influences the near shore morphodynamics and ecosystems in which bottom shear stress is responsible for sediment transport. In the present paper, the behaviors of the rough turbulent boundary layer for irregular wave has been investigated by the BSL $k-\omega$ turbulence model and validated by the available experimental data. The turbulence numerical models could predict well the mean velocity, turbulent intensity and time variation of bottom shear stress under irregular waves. This paper may be helpful in defining a suitable model for a specific practical application in coastal environments.

Keywords: Irregular wave, rough turbulent boundary layer, and BSL $k-\omega$ turbulence model

INTRODUCTION

An understanding of nature of the wave boundary layer above the seabed is of fundamental importance to coastal engineers and workers in the field of sediment transport. At the seabed, there is a thin flow region–bottom boundary layer- that is dominated by friction arising from bottom roughness (Holmedal et al. 2000). Due to the roughness of the sea bottom, the bottom boundary layer flow is mostly rough turbulent. The flow is governed by: the near-bed wave velocity, the corresponding orbital displacement, the near-bed current velocity, the bottom roughness, the water depth and the relative angle between the waves and the current (Holmedal and Myrhaug 2004). The accurate prediction of turbulence is important when considering the initiation of the sediment motions.

The turbulent bottom boundary layer induced by non-linear waves over a rough bed has been examined in many experimental and numerical studies (Fuhrman et al., 2013; Thompson et al., 2012; Tanaka et al., 2011; Suntoyo and Tanaka, 2009; Suntoyo et al., 2008). The wave boundary layer is normally a few centimeters thick. Although the thickness of the wave turbulent boundary layers is quite small compared with the water depth (Fig. 1), it still plays a very important role in determining the rate of sediment transport, the rate of wave energy dissipation, and the magnitude of bottom shear stress associated with large scale slowly varying currents. Therefore, a quantitative understanding of the mechanism of wave induced bottom boundary layers is of primary importance in predicting coastal or continental shelf processes (Ozdemir et al., 2010; Hsu and Ou, 1997). The boundary layer flow determines the bottom shear stresses, which are of fundamental interest for sediment transport and thereby the evolution of coastal morphology.

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Studies on turbulent boundary layers and bottom friction have been made by many researchers through laboratory experiments as well as numerical models. From the experimental studies it has given new contributions to understanding of turbulent behavior of oscillatory flow over both smooth and rough bed for irregular waves. The wave boundary layer and the bottom friction studies induced by wave motion for irregular waves is very rarely done, although there, but they are mostly limited to a smooth bed condition (e.g. Samad and Tanaka, 1999; Tanaka and Samad, 2006), which are very different from an actual situation on a sea bottom with roughness bed. Reviews on turbulence model of bottom boundary layers for irregular waves are given in Tanaka et al. (2002), Holmedal et al. (2003) and Sana et al. (2004). Tanaka et al. (2002) has studied one of turbulent boundary layer characteristics, namely the bottom shear stress, over rough bed conditions under irregular waves have been carried out through experimental and proposed a new the estimation method to determine the bottom shear stress, but once the results are not so good agreement with the experiment. Holmedal et al. (2003) investigated the boundary layer under irregular waves alone as well as under irregular waves plus current, using a dynamic turbulent boundary layer model and the turbulence closure provided by a high Reynold number \( k\epsilon \) model. Sana et al. (2004) has presented the wave boundary layer properties under irregular waves using the original version of \( k\epsilon \) model and two-versions of two-layer \( k\epsilon \) models. Recently, Thompson et al. (2012) also has conducted studies the use of high-frequency turbulence data to estimate of bottom shear stress and wave friction factor in a broad range of different water depths and irregular wave conditions. Estimates of wave friction factor are then compared with results from a number of widely-used predictive formulae. To overcome the significant data scatter a new simple expression for wave friction factor is presented that combines the wave Reynolds number, wave steepness and relative depth to provide improved prediction of wave friction factor for the present range of wave, bottom roughness and water depths.

In this paper, one of the two-equation turbulence models, namely the base line (BSL) \( k\epsilon \) model proposed by Menter (1994) is applied to predict the turbulent properties for irregular waves over rough beds. This model is validated by comparison with the experimental data in Tanaka et al. (2002). The characteristics of the turbulent boundary layer under irregular waves (including mean velocity, turbulent intensity and bottom shear stress) are presented for experimental as well as turbulent numerical models, BSL \( k\epsilon \) model. Turbulent properties predictions for irregular waves from turbulent models are compared from experimental results.

**ROUGH TURBULENT FLOW EXPERIMENTS**

Turbulent flow experiments over rough bed for irregular waves were carried out in an oscillating tunnel by using air as the working fluid. The waves that are generated in the natural environment are irregular. The definition sketch for irregular wave is given by Holthuijsen as shown in Fig. 2 (Nielsen, 2009). They have different lengths and different height. Large waves usually come in groups, known to surfers as set. There are two possible of the wave heights, namely zero downcrossing height (measure from a trough to following crest), \( H_d \), and zero upcrossing height (measure from a crest to following trough), \( H_u \), wave periods, \( T_u \) and \( T_d \), and \( \eta(t) \) is the surface elevation time series. The averages are invariant with respect to the choice of upcrossing versus downcrossing: \( H_{av} = H_u \) and \( T_{av} = T_u \). The average zero crossing period for a record is often referred to as \( T_c = \frac{T_d}{T_u} \).

![Fig. 2 Definition sketch for irregular wave](image)

Experiment has been carried in (Tanaka et al., 2002) under irregular waves. The experimental conditions are given in Table 1, in which \( U_{1/3} \) is significant wave free-stream velocity, \( T_{1/3} \) is the significant wave period, \( a_{sw}/k_i \) is the roughness parameter, \( k_i \) is Nikuradse’s equivalent roughness defined as \( k_i = 30\zeta_i \) in which \( \zeta_i \) is the roughness height and \( S \) is the reciprocal of Strouhal number. The present experiment was carried out for the case with the Reynold number \( Re_{1/3} = 5.0 \times 10^3 \) to reach a fully turbulent regime.

<table>
<thead>
<tr>
<th>Exp.</th>
<th>( U_{1/3} ) (cm/s)</th>
<th>( T_{1/3} ) (s)</th>
<th>( Re_{1/3} )</th>
<th>( a_{sw}/k_i )</th>
<th>( S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>392.348</td>
<td>3.0</td>
<td>5.0 \times 10^3</td>
<td>69.38</td>
<td>18.73</td>
</tr>
</tbody>
</table>

The oscillatory motion was generated by a computer-controlled mechanism. The Bretschneider-Mistuyasu spectrum was used to generate an input signal in this experiment. The aluminum balls roughness having a diameter of 1.0cm, similar idea used by Justesen (1988),
Past over the bottom surface of the wind tunnel without spacing along the wind tunnel, as shown in Fig. 3. In the wind tunnel, Laser Doppler Velocimeter (LDV) was installed for the flow measurement. Velocities were measured at 20 point in the vertical direction at the center part of wind tunnel. Thus, the velocity distribution near a rough bed is logarithmic. It can be therefore assumed that log-law can be used to estimate bottom shear stress over rough bed.

![Fig. 3 Definition sketch for roughness](image)

### TURBULENCE MODEL DESCRIPTION

For the 1-D incompressible unsteady flow within the boundary layer can be expressed as

\[ \frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]  
(1)

where \( u \) is the instantaneous horizontal velocity, \( t \) is time, \( \rho \) is water density, \( p \) is pressure. At the axis of symmetry or outside boundary layer \( u = U \) and \( \tau = 0 \), therefore, \( \partial U/\partial t = -1/\rho \partial p/\partial x \) and Eq. (1) became,

\[ \frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{1}{\rho} \frac{\partial \tau}{\partial z} \]  
(2)

By introducing the eddy viscosity model, the total shear stress for turbulence flow can be expressed as

\[ \tau = (v + \nu_t) \frac{\partial u}{\partial z} \]  
(3)

where \( \nu_t \) is the eddy viscosity describing the Reynolds stress and \( v \) is the kinematics viscosity. Substitution of Eq. (3) into Eq. (2) gives the simplified equation for the turbulent flow motion in the bottom boundary layer,

\[ \frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{1}{\rho} \frac{\partial}{\partial z} \left( (v + \nu_t) \frac{\partial u}{\partial z} \right) \]  
(4)

### Baseline (BSL) \( k-\omega \) Model

Turbulence models can be used to predict the turbulent properties under any waves motion. The baseline (BSL) \( k-\omega \) model is one of the two-equation turbulence models proposed by Menter (1994) to determine some unknown quantities in Eq. (4). The BSL \( k-\omega \) model is a two-equation model that gives results similar to the \( k-\omega \) model of Wilcox (1988) in the inner of boundary layer but changes gradually to the \( k-\epsilon \) model of Jones-Lauder (1972) towards the outer boundary layer and the free stream velocity. In order to be able to perform the computations within one set of equations, the Jones-Lauder model was first transformed into the \( k-\omega \) formulation. The blending between the two regions is done by a blending function \( F_1 \) changing gradually from one to zero in the desired region. The BSL \( k-\omega \) model has the different formulations with the original \( k-\omega \) model i.e. an additional cross-diffusion term appears in the \( \omega \)-equation and the modeling constants are different.

The functions \( F_1 \) and \( (1 - F_1) \) are multiplied by the original \( k-\omega \) model and the transformed \( k-\epsilon \) model, respectively and both are added together. In the near the wall the function \( F_1 \) is designed to be one for activating the original \( k-\omega \) model, while in the outer region of boundary layer is to be zero for activating the \( k-\epsilon \) model. Originall \( k-\omega \) model:

\[ \frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left( \frac{\partial k}{\partial z} \right) + \frac{\nu_t}{k} (\frac{\partial u}{\partial z})^2 - \beta^* k \]  
(5)

\[ \frac{\partial \omega}{\partial t} = \frac{\partial}{\partial z} \left( \frac{\partial \omega}{\partial z} \right) + \gamma \left( \frac{\partial u}{\partial z} \right)^2 - \beta^* \omega \]  
(6)

Transformed \( k-\epsilon \) model:

\[ \frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left( \frac{\partial \omega}{\partial z} \right) + \frac{1}{\omega} \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial z} \right)^2 - \beta^* k \]  
(7)

\[ \frac{\partial \omega}{\partial t} = \frac{\partial}{\partial z} \left( \frac{\partial \omega}{\partial z} \right) + \gamma \left( \frac{\partial u}{\partial z} \right)^2 - \beta^* \omega \]  
(8)

Both Eq. (5) and Eq. (6) are multiplied by \( F_1 \) whereas both Eq. (7) and Eq. (8) are multiplied by \( (1 - F_1) \) and then the corresponding equations of each set are added together to give the new model known as the BSL \( k-\omega \) model. The new governing equations of the transport
equation for turbulent kinetic energy \( k \) and the dissipation of the turbulent kinetic energy \( \omega \) from the BSL \( k-\omega \) model as mentioned before are,

\[
\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left[ \left( v + v_I \sigma_k \frac{\partial k}{\partial z} \right) \frac{\partial k}{\partial z} \right] + v_t \left( \frac{\partial u}{\partial z} \right)^2 - \beta^* \omega k \tag{9}
\]

\[
\frac{\partial \omega}{\partial t} = \frac{\partial}{\partial z} \left[ \left( v + v_I \sigma_\omega \frac{\partial \omega}{\partial z} \right) \frac{\partial \omega}{\partial z} \right] + \gamma \left( \frac{\partial u}{\partial z} \right)^2 - \beta \omega^2 \frac{\partial \omega}{\partial z} + 2(1 - F_j) \sigma_w \frac{1}{\omega} \frac{\partial k}{\partial z} \frac{\partial \omega}{\partial z} \tag{10}
\]

\[
v_t = \frac{k}{\omega} \tag{11}
\]

where, \( \sigma_k, \beta^*, \sigma_\omega, \gamma \) and \( \beta \) are model constants, \( F_j \) is a blending function.

A Crank-Nicolson type implicit finite-difference scheme was used to solve numerically the non-linear governing equations of the boundary layer for turbulence models. In order to achieve better accuracy near the wall, the grid spacing was allowed to increase exponentially. In space 100 and in time 7200 steps per wave cycle were used. The convergence was achieved through two stages; the first stage of convergence was based on the dimensionless values of \( u, k, \) and \( \omega \) at every time instant during a wave cycle. Second stage of convergence was based on the maximum wall shear stress in a wave cycle. The convergence limit was set to \( 1 \times 10^{-6} \) for both the stages. Complete description of the turbulence model, governing equations, numerical technique, boundary conditions and model parameters are provided in Suntoyo (2006).

RESULTS AND DISCUSSIONS

The characteristics of turbulent oscillatory boundary layers over a rough bed under irregular waves is investigated that includes the mean velocity distribution, turbulence intensity and bottom shear stress using laboratory experiments as well as turbulence numerical models. Moreover, a quantitative comparison among turbulence models and experimental data was made.

Mean Velocity Profiles

Mean velocity profiles in the rough turbulent boundary layer for irregular waves at selected phases were compared with the BSL numerical model as shown in Fig. 4. The solid line showed the BSL \( k-\omega \) model while marks (\( \circ, \Delta, \Theta, \Theta, ^*, v, + \) and \( \Theta \)) showed the experimental results of mean velocity profile distribution.

As seen that both for the BSL \( k-\omega \) model and experimental results, the velocity overshoot is much influenced by the effect of acceleration and the velocity magnitude. The velocity overshoot at phases of A, D, E, F and I are higher than that of at phases of B, C, G and H. The higher velocity overshoot occurs due to the higher acceleration and the velocity overshoot is much smaller due to the smaller acceleration. The BSL \( k-\omega \) model give an excellent agreement with the experimental data across the depth at phases of B, C, G and H.

Turbulent Intensity Profiles

The fluctuating velocity in x direction from numerical modeling or the turbulent intensity, \( u' \) can be calculated using Eq. (12) that is derived from experimental data for steady flow by Nezu (1977), where \( k \) is the turbulent kinetic energy provided in the BSL \( k-\omega \) model.

\[
u' = 1.052 \sqrt{k} \tag{12}
\]

Comparison of the turbulent intensity from BSL \( k-\omega \) model prediction and experimental data at selected phases are given in Fig. 5. The BSL \( k-\omega \) model can predict very well the turbulent intensity across the depth at almost all phases. The turbulence numerical model has given slightly the underestimates value with the experimental data especially near the wall of the turbulent intensity at phases B and D. The BSL \( k-\omega \) model also shows a good prediction for the region far from bed, while for the region under free stream velocity the prediction is not so much in a good agreement especially at phases A and F. However, the prediction...
models qualitatively produce very good indication of the pattern of turbulence generation and it mixing.

By plotting $u$ against $\ln(z/z_0)$, a straight line is drawn from log-fitting to measured velocity profile through the experimental data, the value of friction velocity, $U'$ can be obtained from the slope of this line and bottom shear stress, $\tau_0$ can then be obtained from (14). The obtained values of $\Delta z$ and $z_0$ as the above mentioned have a sufficient accuracy for application of logarithmic law in a wide range of velocity profile near bottom region.

Bottom shear stress from experimental data can be estimated by the logarithmic relation between the friction velocity and the variation of velocity with height using Eq. (13), where, $u$ is the flow velocity in the boundary layer, $\kappa$ is the von Karman's constant ($\kappa=0.4$), $z$ is the cross-stream distance from theoretical bed level ($z=y+\Delta z$), and $z_0$ is the characteristic roughness length denoting the value of $z$ at which the logarithmic velocity profile predicts a velocity of zero. $z_0$ can be obtained by applying the Nikuradse's equivalent roughness in which $z_0 = k_s/z_0$ and $k_s$ is the bottom roughness.

$$u = \frac{U'}{\kappa} \ln \left( \frac{z}{z_0} \right)$$  \hspace{1cm} (13)$$

$$U' = \sqrt{\frac{\tau_0}{\rho}}$$  \hspace{1cm} (14)$$

Bottom Shear Stress

In this study, roughness elements that have the aluminum balls shape were used to definite the bed roughness. These roughness elements that cause a wake behind each roughness element, and the shear stress is transmitted to the bottom by the pressure drag on the roughness elements. Viscosity becomes irrelevant for determining either the velocity distribution or the overall drag on the surface. Thus, the velocity distribution near a rough bed for steady flow is logarithmic. It may be therefore assumed that log-law can be used to estimate the time variation of bottom shear stress $\tau_b(t)$ over rough bed as shown by previous studies i.e. Jonsson and Carlsen (1976). Hereafter, the bottom shear stress for experimental results can be evaluated with that of turbulence models.

Bottom shear stress from experimental data can be estimated by the logarithmic relation between the friction velocity and the variation of velocity with height using Eq. (13), where, $u$ is the flow velocity in the boundary layer, $\kappa$ is the von Karman's constant ($\kappa=0.4$), $z$ is the cross-stream distance from theoretical bed level ($z=y+\Delta z$), and $z_0$ is the characteristic roughness length denoting the value of $z$ at which the logarithmic velocity profile predicts a velocity of zero. $z_0$ can be obtained by applying the Nikuradse's equivalent roughness in which $z_0 = k_s/z_0$ and $k_s$ is the bottom roughness.

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$$U' = \sqrt{\frac{\tau_0}{\rho}}$$  \hspace{1cm} (14)$$

Fig. 6 shows a bottom shear stress comparison among experimental results and turbulence models prediction. The BSL $k-\omega$ model could predict well the bottom shear stress showing a good agreement with the experimental data along a wave cycle under irregular wave. The BSL $k-\omega$ model has given the underestimate and overestimate values of bottom shear stress with the experimental data especially at the trough part and the crest part. It’s caused by incorporating the acceleration term was not done in numerical method. It is confirmed that acceleration has significant role in the calculation of bottom shear stress under irregular waves. However, it can be concluded that BSL $k-\omega$ model can be used to predict well the bottom shear stress under irregular waves over rough beds.
CONCLUSIONS
The behaviors of the rough turbulent boundary layer for irregular wave has been investigated by the BSL $k-\omega$ turbulence model and validated by the available experimental data in Tanaka et al. (2002). The BSL $k-\omega$ model could predict well the mean velocity, turbulent intensity and time variation of bottom shear stress under irregular waves.

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