CALCULATION OF CUMULATIVE DAMAGE OF TETRAPOD ARMOR LAYER

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ABSTRACT: In the performance-based design of a breakwater armor layer, it is often necessary to calculate its damage accumulated over the life cycle of the breakwater. Two methods for calculating the cumulative damage have been proposed; one by Melby and Kobayashi in 1998 and the other by Hanzawa et al. in 1996. In this paper, comparison is made between the two methods for a Tetrapod armor layer. For the damage progression of Tetrapod armor units, hydraulic experiments are made. In the case where a severe damage occurs at the beginning of the life cycle of the breakwater, the two methods do not show significant difference, but in general the latter predicts a larger cumulative damage than the former. For a Tetrapod armor layer, it is recommended to use the average of the two methods.

Keywords: Armor layer; Breakwaters; Cumulative damage; Tetrapod.

INTRODUCTION

The existing stability formulas for armor units calculate the degree of damage of the armor layer during the peak of a design storm. In the performance-based design of the breakwater armor layer, however, it is often necessary to calculate its damage accumulated over the life cycle of the breakwater. There are two methods appeared in literature to calculate the cumulative damage; Melby and Kobayashi (1998) and Hanzawa et al. (1996) (hereafter referred to as Method 1 and Method 2, respectively).

The Method 1 was proposed for stone armor layers for which the damage level $S$ was used, the physical description of which is the number of cubic stones with a side of $D_{s50}$, eroded within the width (along the breakwater alignment) of one nominal size $D_{n50}$. On the other hand, the Method 2 was proposed for Tetrapod armor layers for which the damage is expressed in terms of relative damage $N_0$, which is defined as the number of displaced Tetrapods within the width of one $D_s$. Since $S$ and $N_0$ have essentially the same physical description, the two methods could be used for both stone and concrete armor units.

Melby and Kobayashi (1998) conducted hydraulic experiments for damage progression of quarry stones. However, the experiments for concrete armor units are scarce. Suh and Chang (2003) compared the two methods with the experimental results of Melby and Kobayashi (1998) for stone armor units, but they compared only numerical results for Tetrapod armor units. In the present study, we conduct hydraulic experiments for damage progression of Tetrapod armor units. The two calculation methods are then compared for Tetrapod armor units.

CALCULATION METHODS

Method 1

Based on the stability formula of Van der Meer (1987), Melby and Kobayashi (1998) expressed the damage level as

$$ S = A_s N_w^b $$

(1)

where $A_s = a_s (H_s / (\Delta D_{s50}))^x$, $H_s$ = significant wave height, $\Delta = \rho_s / \rho - 1$ with $\rho_s$ = density of armor units and $\rho$ = fluid density, $N_w$ = number of attacking waves, and $a_s$ and $b$ = empirical coefficients. With the duration of wave attack given by $t = T_w N_w$, where $T_w$ = mean wave period, Eq. (1) can be written as

$$ S = A_s \left( \frac{T}{T_w} \right)^b $$

(2)

To calculate the cumulative damage level in real situations of $H_s$ and $T_w$ varying with time, Melby and Kobayashi (1998) proposed an empirical procedure, in
which the damage level at arbitrary time \( t_i \), \( S_i \), was expressed as

\[
S_i = S_{i-1} + A_k T_m^{-b} (t_i^b - t_{i-1}^b)
\]

(3)

where \( S_{i-1} \) = known damage level at \( t = t_{i-1} \), and \( T_m \) = mean wave period during the period from \( t = t_{i-1} \) to \( t = t_i \). Based on the empirical formulas of Suh and Kang (2012), the counterpart of Eq. (3) for Tetrapods can be written as

\[
N_n = N_{0,1} + A_k T_m^{-0.5} (t_i^{0.5} - t_{i-1}^{0.5})
\]

(4)

The coefficient \( A_k \) is estimated as

\[
A_k = \left( \frac{H_s}{\Delta D_n} \right)^{0.4 \tan \theta - 3.25} \frac{9.2}{\tan \theta - 3.25}
\]

if \( \xi_m < \xi_{mc} \)

(5)

\[
A_k = \left( \frac{H_s}{\Delta D_n} \right)^{0.4 \tan \theta - 3.25} \frac{9.2}{\tan \theta - 3.25}
\]

if \( \xi_m \geq \xi_{mc} \)

(6)

where \( \tan \theta \) = slope of the armor layer, \( \xi_m = \tan \theta / \sqrt{H_s/T_0} \) with \( T_0 = gT_m^3 / (2\pi) \), and

\[
\xi_{mc} = \left[ \frac{9.2N_0^{0.5} / N_0^{0.25} + 3.25}{5.0N_0^{0.5} / N_0^{0.25} + 0.85} \right]^{0.25}
\]

(7)

To calculate \( \xi_{mc} \) using Eq. (7), \( N_0 \) is necessary, which is a priori unknown. We calculated \( \xi_{mc} \) using \( N_{0,1} \) and \( N_{w,1} = t_{i-1} / T_m \) in Eq. (7) and calculated \( N_0 \) using Eqs. (4)-(6). This \( N_0 \) and \( N_{w,1} = t_i / T_m \) were then used in Eq. (7) to calculate \( \xi_{mc} \), and \( N_0 \) was re-calculated using Eqs. (4)-(6). Damage to the Tetrapod armor layer is assumed to occur under rough sea conditions of the significant wave height greater than a critical value, \( H_s \), which may be defined as the wave height corresponding to zero damage in a stability formula.

**Method 2**

Hanzawa et al. (1996) proposed a slightly different method. Supposing the wave height during the period from \( t = t_{i-1} \) to \( t = t_i \) is \( H_s \), and the cumulative damage level up to \( t = t_i \) is \( S_{i-1} \), the number of waves which attacked the breakwater up to \( t = t_{i-1} \), \( N_w' \), is determined using \( H_s \) and \( S_{i-1} \), respectively, in places of \( H_s \) and \( S \) in Eq. (1) as

\[
N_w' = \frac{S_{i-1}}{A_{k'}}^{1/b}
\]

(8)

The cumulative damage level up to \( t = t_i \), \( S_i \), is calculated by Eq. (1) with \( N_w = N_w' + N_w \) and \( H_s = H_s \), where \( N_w \) = number of waves between \( t = t_{i-1} \) and \( t = t_i \). With \( N_w = (t_i - t_{i-1}) / T_m \), \( S_i \) is given by

\[
S_i = A_k \left( \frac{S_{i-1}}{A_{k'}} \right)^{1/b} + \frac{t_i - t_{i-1}}{T_m}
\]

(9)

Using the similar procedure, the cumulative relative damage of Tetrapods is calculated by

\[
N_0 = A_k \left( \frac{N_{0,1}}{A_{k'}} \right)^{0.5} + \frac{t_i - t_{i-1}}{T_m}
\]

(10)

**HYDRAULIC EXPERIMENT**

Hydraulic experiments for damage progression of Tetrapods were conducted in the wave flume at the Hydraulic and Coastal Engineering Laboratory in Seoul National University that was 36-m long, 1.0-m wide, and 1.2-m deep. Fig. 1 shows the experimental setup. A horizontal bed with a 1/25 foreshore slope was installed at the elevation of 20 cm from the bottom of the flume. The breakwater model was placed at a distance of 25 m from the wave maker with the breakwater toe at a few centimeters from the beginning of the horizontal bed. The test section was divided into two channels by a vertical wall along the wave flume, each having a width of 0.6 m and 0.4 m, respectively. The breakwater was installed in the wider channel and the other channel was left empty. Irregular waves based on the modified Bretschneider-Mitsuyasu spectrum (Goda 2010) were generated with a piston-type wave maker. The water depth was 0.6 m at the wave paddle and 0.4 m at the toe of the structure. To measure the incident waves, three wave gauges were installed in the empty channel. The free surface displacements measured by these wave gauges were used to separate the incident and reflected waves using the method of Suh et al. (2001). Even though the channel is empty, wave reflection occurs from the sloping bed and the wave absorber located at the downstream end of the flume. The method of Suh et al. (2001) estimates the time series of surface elevation of the incident and reflected waves. The significant wave height \( H_s \) was calculated by the zero-crossing analysis of the time series of the incident wave profile.
Fig. 1 Sketch of wave flume and experimental setup.

Fig. 2 Cross-section of breakwater
Fig. 2 shows the cross-section of the breakwater. The main characteristics of the Tetrapods were: height $H_w = 6.2$ cm; nominal size $D_s = 4.03$ cm; $\rho_s = 2.3$ g/cm$^3$; weight $W = 150.5$ g; and layer thickness 8.0 cm. The underlayer consisted of stones of diameter 2.0-2.5 cm and thickness of 6.0 cm, while the core consisted of stones of nominal size $D_{50} = 1.3$ cm. The slope of the structure was $\cot \theta = 1.5$. A little wave overtopping occurred when the significant wave height was greater than 18 cm. Tetrapods were placed in two layers: randomly placed upper layer on the regularly placed lower layer. Since the Tetrapods in contact with the sidewalls of the flume have less degree of interlocking, they were fixed not to move and were not included in the calculation of damage.

Eight different irregular wave trains of 15 min duration with unique combinations of wave height and wave period were used to simulate storms of several hours’ duration. Table 1 summarizes the incident wave characteristics at the toe of the structure. The number of waves in the 15 min burst was approximately 450 to 710 depending on the wave period. The surf similarity parameter based on $T_m$ and $H_s$, 

$$\xi_m = \tan \theta g T_m^2 / (2 \pi H_s)^{1/2},$$

was in the range 3.04-4.40. On the other hand, the surf similarity parameter for collapsing waves, $\xi_{mc}$, calculated by Eq. (7) using various combinations of $N_g = 0.0$, 0.5, 1.0 and $N_g = 1500$, 3000, 4500 was in the range of 2.94-4.26. This proves that the waves listed in Table 1 produce intense collapsing breakers at the structure.

### Table 1 Test wave conditions.

<table>
<thead>
<tr>
<th>Wave</th>
<th>$H_s$ (cm)</th>
<th>$T_m$ (s)</th>
<th>$N_{mc}$</th>
<th>$\xi_{mc}$</th>
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<tr>
<td>1</td>
<td>12.7</td>
<td>1.82</td>
<td>495</td>
<td>4.26</td>
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<td>2</td>
<td>11.8</td>
<td>1.27</td>
<td>710</td>
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<td>3</td>
<td>12.7</td>
<td>1.51</td>
<td>596</td>
<td>3.53</td>
</tr>
<tr>
<td>4</td>
<td>12.7</td>
<td>1.43</td>
<td>630</td>
<td>3.34</td>
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<td>5</td>
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<tr>
<td>6</td>
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<td>1.46</td>
<td>617</td>
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<td>7</td>
<td>15.5</td>
<td>2.00</td>
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</tr>
<tr>
<td>8</td>
<td>15.6</td>
<td>1.61</td>
<td>560</td>
<td>3.39</td>
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</table>

Five test series were conducted as shown in Table 2. Each series consists of the eight wave cases listed in Table 1 in different sequences, lasting 2 h with approximately 4550 waves. The test series were intended to compare cumulative damage caused by different sequences of storms as listed in Table 2. In series A, the storm intensity increased monotonically from wave 1 to wave 8. In series B and C, strong storms occur in the middle of the sequence, while in series D and E, strong storms occur at the beginning and end of the sequence. For each test series listed in Table 2, tests were repeated nine times. The tests of the maximum and minimum final damage were discarded, and the remaining seven tests were used to calculate the mean and standard deviation of damage.

### Table 2 Wave sequences in test series.

<table>
<thead>
<tr>
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### COMPARISON WITH EXPERIMENTAL RESULTS

The calculation methods are applied to simulate the cumulative damage of the Tetrapod armor layer in the present experiment. Fig. 3 shows the relative damage calculated by Eqs. (4) and (10) along with the experimental data for series A to E. The experimental data are represented by error bars indicating ±1 standard deviation. The data of zero standard deviation indicate no damage progression from the previous damage. The time interval of calculation was 15 min, which is the same as the duration of each wave case in the experiment.

In general, the Method 2 predicts larger damages than the Method 1 except series E in which the first wave is the largest so that severe damage occurred at the beginning of the simulation. This trend is the same as that for stone armor layer (See Fig. 5 in Suh and Chang 2003). In series A where the storm intensity increases monotonically, both methods initially over-predict the cumulative damages, but the agreement between prediction and measurement becomes better in the last stage where strong storms occur. In series B and C where strong storms occur in the middle of the sequence, the measured damage locates somewhere between the two predictions. In series D and E where strong storms occur at the beginning and end of the sequence, both methods well predict the cumulative damage, especially in series E where the first wave is the largest so that severe damage occurred at the beginning of the simulation. As a whole, it is difficult to judge which method better predicts the measurement. Since the measured damage locates somewhere between the two predictions except the case of large initial damage in which the two methods yield similar predictions, it is recommended to use the average of the two methods.
Fig. 3 Comparison between calculated and measured relative damages of Tetrapod armors: (a) series A; (b) series B; (c) series C; (d) series D; (e) series E.
In the stability formulas for armor units, the damage level $S$ or the relative damage $N_0$ is proportional to $\sqrt{N_0}$ so that the coefficient $b$ must be 0.5. These equations were developed to calculate the damage of armor units during a single storm of relatively short duration. For rock armors, Van der Meer (1987) and Melby and Kobayashi (1998) showed that $b < 0.5$ for long-duration tests. Melby and Kobayashi (1998) obtained $b = 0.25$ by calibrating Eq. (3) with their long-duration experimental data. In the comparison shown in Fig. 3 where severe damages occurred three or four times in each series, $b = 0.5$ was used. This implies that the stability formula for Tetrapods developed for a single storm can be used for calculation of cumulative damage by several storms. If the methods are used for more number of storms, a calibration for the coefficient $b$ may be necessary. However, the probability that a rubble mound breakwater will be severely damaged more than several times during its lifetime must be very low.

**CONCLUSION**

Comparison was made between the two methods for calculating the cumulative damage of Tetrapod armor units; Method 1 (Melby and Kobayashi 1998) and Method 2 (Hanzawa et al. 1996). In the case where severe damage occurred at the beginning of the lifetime of the breakwater, the two methods did not show significant difference, but in general the Method 2 predicted larger cumulative damage than the Method 1.

The two methods were compared against the experimental data of the present study. Depending on the time of occurrence of large storms, each method over- or under-predicts the measurement so that it is difficult to judge which method makes a better prediction. Since the measured damage locates somewhere between the two predictions except the case of large initial damage in which the two methods yield similar predictions, it is recommended to use the average of the two methods for a Tetrapod armor layer.

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**REFERENCES**


