Transmission management for congested power system: A review of concepts, technical challenges and development of a new methodology

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Abstract

Transmission networks have some constraints that should be addressed in order to ensure sufficient control to maintain the security level of a power system while maximising market efficiency. The most obvious drawback of transmission constraints is a congestion problem that becomes an obstacle to perfect competition among the market participants since it can influence spot market pricing. As the power flow violates transmission constraints, redispatching generating units is required and this will cause the price at every node to vary. This manuscript presents concepts, technical challenges and methodology for investigating an alternative solution to the redispatch mechanism and then formulates LMP scheme using an optimisation technique that may well control congestion as the main issue. The LMP scheme are varied and improved to take into account the energy price, congestion revenue, cost of losses, as well as the transmission usage tariff by utilising shift factor-based optimal power flow (SF-OPF), which is derived from the well-known DC optimal power flow (DC-OPF) model.

Keywords:
Transmission congestion and management
Network losses
Economic dispatch
Optimal power flow
Shift factor
Electricity market
Locational marginal price (LMP)
1. Introduction

In deregulated electricity markets, transmission networks hold an essential role in supporting interaction between producers and customers because it should provide an unbiased environment for all participants [1]. Furthermore, bottlenecks in transmission lines are an obstacle for perfect competition among market participants [2]. One obvious drawback of transmission constraint is congestion. The system is said to be congested when both parties agree to produce and consume a particular amount of electric power but fails to do this because of a transmission network exceeding its thermal limits. This eventually brings further impacts to exercise market power that can cause price volatility beyond the marginal costs [3]. Therefore, an efficient congestion management becomes one of the most important indicators of the design of a power market to display the network capability in a competitive environment.

The fundamental issue in a competitive market is the market clearing price (MCP) mechanism. As the power flow violates transmission constraints, redispach of generating output is required and this causes different prices at every node. This phenomenon is defined as locational marginal prices (LMPs), also known as load pocket [4]. Based on these facts, the relationship between MCP and transmission management has a strong relationship, which needs to be assessed in order to obtain an efficient and transparent price to satisfy all market participants.

The DC-OPF has been used widely to primarily manage congestion through computing LMPs, due to its speed and strength. However when the LMP scheme takes line losses into account the advantageous features of the DC-OPF based LMP mode are diminished.

Therefore, this paper examines an advanced solution for the redispach mechanism, which not only improves the computation of congestion and losses, but also formulates a new LMP scheme using an optimisation technique that considers congestion as the crucial problem. The LMP schemes are adapted and improved to take into account the cost of losses, congestion revenue, energy prices, as well as an embedded cost, called the transmission usage tariff (TUT). The objective is to support the development of a standard market design in managing transmission systems which promotes economic efficiency, lowers delivered energy costs, maintains power system reliability and mitigates exercising market power. Accordingly, three schemes of the LMP are introduced, namely LMP-lossless, LMP-loss and LMP–TUT. The LMP-lossless model has two components, namely, energy price and transmission congestion revenue. The second scheme in this LMP modeling, LMP-loss, includes three components: energy price, transmission congestion revenue and transmission losses cost. LMP–TUT is formulated based on the LMP-loss but takes into account a tariff for transmission usage as well. These schemes are developed to evaluate the performance of the proposed method of shift factor-based optimal power flow [10]. Transmission usage tariff is defined as embedded cost in [6], while [11] classified it as the use of transmission system charge. This is to convert standard operating and maintenance costs into a transmission charge cost, which refers to the previous capital cost acquired in the transmission infrastructure development and maintenance. The last aspect in transmission management is the cost of losses. Transmission losses are simply defined as the difference between the total power supply from generation and the total power accepted (demand) by customers in the system. Even though the impact of losses may be small compared to other potential sources of market inefficiency they must be considered as well [12]. To meet the required demand optimal power flow. Section 4 explores the congestion and supply/demand equilibrium and Section 5 presents a generic LMP scheme for congestion management. Section 6 proposes an improved method of OPF for LMP. The basic tasks of the improved method and its conceptual flowchart are presented in Section 7. Finally, the conclusion of this paper is presented in Section 8.
market operators have to make sure that there is a specific supply for network losses. In terms of cost minimisation this is a challenging problem to find an appropriate generator responsible for the supply of losses while still keeping the price to a minimum. Besides, in the planning of integration of renewable distributed generation (DG) into the grid, it is essential evaluating a proper assessment on optimal allocation of DGs penetration in order to achieve a high efficient electricity market through minimising total energy losses and maximising social welfare [13–15].

Transmission management is faced dealing with many problems, however the transmission congestion issue remains the central problem in the new electricity structure [16,17]. The objective of managing congestion is the cost of re-dispatch of generation as constraints are violated. It also guarantees that there is a sufficient revenue to cover the cost of transmission system operators. The revenue can be used in various ways to address congestion, such as to promote efficient transmission construction planning which is not readily done, like installing new generation, expansion of networks or employing FACTS devices [18]. A study in [19] shows that by using FACTS devices the transmission congestion is mitigated without disturbing economic matters.

Independent system operators (ISOs) usually observe the transactions and control the state of the system, taking part in handling the network congestion management [20,21]. ISOs are being challenged to develop a set of regulations to control the security level of power systems and ensure that they are at acceptable level while keeping the efficiency of the power market high [22]. This implies that market operators should alleviate network congestion; maintain the security and efficiency of power system operation [23], in order to ensure that all market participants have the same rights to access a transmission system without any discrimination [24]. Congestion levels typically determine the security of a power system which would have further consequences on market transaction and energy prices.

3. Economic dispatch and optimal power flow

3.1. Economic dispatch

The definition of economic dispatch as cited in [25] is “the operation of generation facilities to produce energy at the lowest cost to reliably serve customers, recognising any operational limits of generation and transmission facilities”. While reference [26] defines economic dispatch as “the process of allocating generation levels to the generating unit in the mix, so that the system load may be supplied entirely and most economically”. The production cost of generation is analysed during the dispatch, subject to data which is concerning fuel cost and electrical power output. A quadratic equation is used to approximate the cost function along with several cost coefficients. The key objective of the economic dispatch problem is to find a set of active power delivered by the committed generators to satisfy at any time the required demand subject to the unit technical limits and at the lowest production cost. For this reason it is of great importance to solve this problem as fast and precisely as possible.

3.2. Optimal power flow

The development of electricity markets all over the world has brought new challenges to the scientific community, to the players and to the regulatory authorities. One of these challenges is the uncertainty which became a structural element in new environment [27]. All market participants have to be able to deal with it to guarantee the appropriate power system planning and operation as well as its own economical liquidity [28]. A well accepted tool used for controlling and assessing the appropriate and secure operation of a power system is optimal power flow (OPF). This economic dispatch technique is already applied in many energy management systems (EMS) but has some security constraints. The concept of OPF was defined by Carpentier in 1962 [29] as an extension of traditional economic dispatch of power to resolve the optimal settings for control variables while considering various constraints [30].

The OPF comprises mainly of an economic dispatch mechanism, a power flow program and a redispatch generation algorithm as violation of security constraints occurs. Such security constraints are primarily due to the thermal limits of transmission lines and capacity limits of generation [31]. The economic dispatch yields a single incremental cost known as lambda when the security constraints are not violated. Once the violation occurs, redispatch is needed and economic dispatch will produce a set of different lambda at different nodes. Lambda generated at this stage could be considered as a marginal cost for an appropriate generating bus to reschedule power supply such as to meet all required demand of the bus at minimum cost. Marginal cost is referred to as the incremental cost subject to the cost of producing additional output or one more unit of output, 1 MW h.

3.3. DC optimal power flow

The optimal power flow is a very large and complex mathematical program. An OPF can be described as the minimisation of real power generation cost in an interconnected power system while real and reactive power, transformer taps and phase-shift angles are controllable and various inequality constraints are required [32]. Its procedure consists of methods of employing power flow techniques for the economic dispatch while definite controllable variables are adjusted to minimise the objective functions such as the cost of active power generation or the power losses while satisfying physical and operating limits on various controls, dependent variables and function of variables.

While some authors [33–39] have used the AC power flow model, others [39–46] have used the DC optimal power flow model. The AC optimal load flow problem such as the OPF based on Gradient and Newton’s methods consists of finding the active and reactive power outputs and the voltage magnitudes of any generator unit in order to minimise the operating cost while meeting various security constraints. Whereas the DC OPF approach is applied to compute the load flow in the model characterised by ignoring losses and the focus is only on real power. The DC Optimal Power Flow (DC-OPF) is a Quadratic Separable Program with the aim of resolving the interests of both, suppliers (who are wishing to sell energy as expensive as possible, and buyers (who are wishing to buy energy as cheap as possible) [47]. This solution is widely used as this is the conversion of the ac approximation to a more simple linear circuit analysis problem. Even though using a full AC power flow is the most precise computation [48], however owing to its complexity and non-linearity features it may obscure parameters correlation. On the other hand the DC OPF methods and software are simple, its models can be optimised efficiently as well as minimal network data required is fairly easy to acquire [42]. Thus the DC power flow approximation is preferred and has been used extensively in many literatures when calculating the nodal prices and analysing bottlenecks over transmission lines [40,49].

In [29] the DC power flow model is developed with some considerations such as: voltage magnitudes are assumed to be constant at every bus, the different phase angles of bus voltages between two buses are small and transmission resistance is very small compared to its admittance, therefore there are no losses in this model. These considerations construct a model that is a reasonable first approximation for a real power system which is
only slightly nonlinear in normal steady state operation. The model has the benefit of speed of computation and other useful properties such as [50]: linearity feature, this means the flows that are directly attributable to any transaction are doubled if the MW in that transaction from one zone to another is also doubled and the superposition feature, means the flows on the networks can be divided into a sum of components each directly attributable to a transaction on the system.

4. Congestion and supply-demand equilibrium

Social welfare holds a significant role when assessing the condition of market economics in a power system, especially at the supply-demand equilibrium under congestion circumstances [50,51]. Social welfare is a function of the combination of the cost of energy production and the cost of customer benefit based on their ability to pay for it. In other words it is generally a function of all utilities in society [52]. Both supplier g and customer d will have supplier surplus SS(g) and customer surplus CS(d) respectively [53].

Nevertheless, these well-organised systems most likely become imperfect when congestion occurs over a certain transmission line. It may lead to market inefficiency due to the thermal limit of the network. An example of this phenomenon is shown in Fig. 2 for a two zone system connected by a tie line and ignoring the line loss. Let both zone 1 and zone 2 have a constant load of 125 MW where their generating capacities are alike and as much as 300 MW. It is assumed that all generators use their marginal cost for their bidding.

As shown in Fig. 2(a), in order to maximise their own profits each supplier involves bidding its marginal cost when a generator is a price taker. Whilst there is no congestion all 250 MW of demand will be directly bought from zone 2 since its incremental cost is cheaper than zone 1. The total purchase cost at $ 10/MW h will be $2500/h. Yet if power flow over the tie line is limited to 75 MW as in Fig. 2 (b) only 200 MW will then be bought from generator 2 at $ 10/MW h. Now the total purchase cost will be $3000/h as the remaining 50 MW will be ordered from generator 1 at $ 20/MW h. It is obvious that congestion has created an inefficiency in the power market by $500/h or equal to 20% of the optimum costs.

In the above case no strategic bidding from the generators is employed. Indeed market inefficiency due to the congestion problem would suffer further from applying strategic behaviour of generators [54]. This strategy is defined as an effort to maximise profits due to imperfections in the market, especially when congestion exists. Strategic bidding is generally characterised by generators’ actions, which bid energy price not at their actual incremental cost. Once a generator can advantageously raise its profits by strategic bidding as a logical consequence, market power will be created [55].

5. Generic LMP scheme for congestion management

5.1. Definition of locational marginal prices (LMP)

LMP is the key factor to identify the spot price and to manage transmission congestion [56]. The LMP is utilised to calculate power dispatch schedules by maximising the social welfare function [57]. LMP methodology has been implemented at some independent system operators, for example: California ISO, PJM, New York ISO, ISO-New England, Midwest ISO, ERCOT, etc [58–60]. LMP is defined in [53] as: “the marginal cost of supplying the next increment of electric energy at a specific bus considering the generation marginal cost and the physical aspects of the transmission system”. In other words, the LMP is the cost to serve one additional MW of load at a specific location using the lowest production cost of all available generators while observing all transmission constraints. From the viewpoint of generation and transmission planning it is fundamental to always calculate and forecast the value of LMP which may be obtained using the traditional production cost optimisation model [38,61]. However, the implementation of LMP must be able to manage various impracticalities such as determining prices when systems run out of controls and transmission constraints are violated or there is an excess of generation, i.e. total generation is greater than total load [59].

5.2. Approaches of DC optimal power flow model

The DC-OPF model is a common optimisation based technique that has been proposed by many market operators and is being widely used in various ways both for dispatching power and clearing energy markets to decide the LMP. This methodology has become the leading approach in electrical power markets. The DC optimal power flow model has been utilised in the electricity industry to compute LMPs due to its high speed of convergence, simplicity and robustness [41]. It is particularly used in market simulation and planning [62,63].

Generally the DC-OPF is used for security constrained economic dispatch and redispatch when controlling transmission congestion while maximising the economic power transfer capability of the transmission system without violating its constraints [64,65]. These auction models perform system security by means of simple power flow constraints and hence can be stated as security constraint OPF problems.

Ref. [66] gives a tutorial review of the use of optimal power flow approximation to calculate LMPs and congestion costs. The standard DC-OPF scheme for LMP calculation can be modeled as the minimisation of the total production cost subject to energy balance and transmission constraints. This may be written as follows:

5.2.1. Objective function

Let the total production cost and the total customer benefit are given by:

\[ C_i(P_{ci}) = a_iP_{ci}^2 + b_iP_{ci} + c_i, \]  \hspace{1cm} (1)

\[ B_i(P_{di}) = d_iP_{di}^2 + e_iP_{di} + f_i, \]  \hspace{1cm} (2)

where \( C_i \) is production cost at bus i ($/MW h), \( B_i \) is customer benefit at bus i ($/MW h), \( P_{ci} \) is active power output of ith generator (MW), \( P_{di} \) is active power demand of ith load (MW), \( a_i, b_i, c_i \) are coefficients for power production cost function and \( d_i, e_i, f_i \) are coefficients for customer benefit cost function.

![Fig. 2. A simple transaction between two zone systems (a) without congestion (b) congestion with thermal limit.](image-url)
The objective function is then to maximise the total social welfare (TSW) which also equals to minimise the total social cost:

$$\max \sum_{i=1}^{n_l} B_i (P_i) - \sum_{i=1}^{n_g} C_i (P_i)$$

(3)

where \(n_l\) is number of load and \(n_g\) is number of generator.

TSW is the difference between total customer benefit \(B_i (P_i)\) and total production cost \(C_i (P_i)\), which is going to be maximised in the pool.

5.2.2. Constraints

In a pool based dispatch mechanism an optimal solution is required subject to a set of practical constraints as follows:

$$P_i = \sum_{i=1}^{Nbus} [b_i][\delta_i - \delta_j]$$

$$p_{Gi} - p_{Di} \leq p_{Gi} + p_{Di} \leq p_{Gi} + p_{Di}^{max} \quad (i = 1, \ldots, Nbus)$$

$$\delta_i - \delta_j \leq \delta_i - \delta_j \leq (1, Nbus) \quad (i = 1, 2, \ldots, Nbus)$$

(4)

where \(P_i\) is net injection at bus \(i\), \(P_{Gi}^{max}\) is upper limit of power flow between buses \(i-j\), \(b_i\) is susceptance of the branch connecting buses \(i-j\) (\(B_{ij} = -(1/x_{ij})\)), \(\delta_i, \delta_j\) are voltage angle of bus \(i\) and \(j\) and \(N_{bus}\) is number of bus.

5.2.3. Optimality condition

From the objective function and some constraints Lagrangian function is then performed after selecting a reference node. An optimality condition will result in a set of decision variables and multipliers, \(\lambda\), which may be written as follows:

$$z = [p_1, p_2, \ldots, p_n, \delta_1, \delta_2, \ldots, \delta_{n-1}, \lambda_1, \lambda_2, \ldots, \lambda_{n-1}, \mu_1, \mu_2, \ldots, \mu_m]$$

(5)

where \(\lambda_i, \mu_m\) are Lagrange multipliers.

If there is no transmission congestion it means there is no violation on transmission constraints and no redispatch is needed. In this case the economic dispatch program creates a single incremental cost (lambda or Lagrange multipliers), modelling the behavior of a basic economic dispatch algorithm. Once transmission congestion occurs security limits are violated and redispatch is required to meet the security constraints; the result is a set of different Lagrange multipliers at different buses in the power flow model. The optimal price is represented through the Lagrange multipliers \(\lambda\). The Lagrange multipliers may be considered to be the marginal cost or incremental cost at each generating bus to redispatch the generation in a manner to produce energy to serve load in the modeled area at minimum cost. Congestion cost in these models are formulated as \(\lambda_i = \lambda - \lambda_i\).

From the LMP scheme above, it can be seen that typically DC-OPF is developed for the production cost model given the information of generation, transmission and load data. The natural fit into the linear programming model for simplicity, robustness and efficiency is the main factor why a DC-OPF approach is the most popular one to be used in performing LMP formulation. The value of LMPs, energy price and congestion cost will follow a perfect linear model with zero loss prices, in particular if the line loss is ignored in the DC-OPF model.

Nevertheless challenges arise if loss must be considered. This is due to the non-linear or quadratic relationship between line loss and line current or line flow. These weaknesses are quite similar even when an ac optimal power flow approach is employed. Moreover, in [53] it is recommended to include the cost of marginal losses when performing the LMP model as it is known that resistance in transmission lines is inevitable.

This implies the LMP model should encompass generation marginal cost, congestion cost, as well as cost of marginal losses [67]. The LMP model with such a scheme has already been practiced in many ISOs such as PJM, CAISO, NYISO, etc. In [68] the authors give a solution of how to include line loss when using the DC-OPF model for economic dispatch mechanism. This solution applied to [29]’s work would result in a more comprehensive outcome.

5.3. Incorporating line loss in to the model

The line loss is important since it can cause major impact on optimal economic dispatch depending on the network [69]. The losses based LMP scheme can be modeled as the minimisation of the total production cost subject to energy balance and transmission constraints [70]. Thus LMP incorporating losses comprises of three components: marginal energy price, marginal congestion price and marginal loss price [62,71,72]. Hence the first constraint in Eq. (4), bus power balance, with line loss taken into account becomes:

$$P_i = \sum_{i=1}^{Nbus} [b_i][\delta_i - \delta_j] - \sum_{i=1}^{Nbus} \sum_{j=1}^{Nbus} \frac{\mu}{2} (\delta_i - \delta_j)^2$$

(6)

Since the line loss in this formulation is a function of voltage angle differences the total power loss does not depend on the selection of a reference node. One particular voltage angle may be chosen to become zero to represent a reference node yet the overall angle differences will always remain constant. The supply for power loss is distributed among all generating units. While distributing the power loss the angle difference over congested lines must be kept same to satisfy the binding constraint. The disadvantage of this technique is that such a procedure performs a uniform amount of total losses and ends up in a high cost. Hence an alternative solution is needed to perform an optimal LMP model which discovers a lower power loss so that network losses cost may also be reduced.

Therefore in the following section a different approach will be introduced by using the shift factor method with the intention of maintaining the linearity and superposition features of the LMP model while this will be able to optimise for both congestion cost and losses cost.

6. An improved method of OPF for locational marginal prices

6.1. Basic concept of shift factor

From a power flow perspective a transaction can be seen as an amount of power that is injected by a generator into the system at one zone and received by a load at another zone. A DC power flow owns a linearity characteristic that can be occupied to calculate the transaction amount that would provide rise to a specific power flow, such as an interface limit. The coefficient of the linear correlation between a transaction quantity and the flow on a line is called the shift factor (SF). The shift factor is the actual value of the shift factor. Shift factor is also known as power transfer distribution factor (PTDF). The value of the shift factor of particular node, a particular line with reference direction and the value of the shift factor method with the intention of maintaining the linearity and superposition features of the LMP model while this will be able to optimise for both congestion cost and losses cost.

$$SF = \frac{\Delta \text{flow in line } i}{\Delta \text{injection at node } i}$$

(7)
6.2. Formulation of shift factor

Basically, the shift factor can result from three stages based on DC load flow network model, as follows [73]:

- Stage 1: Making sensitivity equation of phase angles as the function of node injections.
- Stage 2: Making sensitivity equation of branch flows as a function of phase angles.
- Stage 3: Substituting the result of stage 1 into stage 2.

7. Basic tasks of the improved method and conceptual flowchart of the schemes

The methodology of this research comprises of several basic objectives and is divided into a number of steps as follows.

7.1. Identification of primary data

The identification of primary data is divided into three data collection tasks. The first task is generation profile data including production cost parameters to perform a generator’s marginal bids, location of generators and a generator’s output limits. This is continued in the second task, namely identification of power system network profile data such as: number of branches, resistance and reactance values of transmission lines, thermal limits or maximum capacity of each branch and network charge rate of the branch. Identification of demand profile data is the third task. It contains number of loads connected to a certain bus and maximum demand of every load.

7.2. Assessment of branch power flow and branch loss

Assessments of branch power flow and branch loss are conducted in this step. This should be done in order to know whether or not power flow on the transmission line violates the maximum capacity of the branch. If the power flow is under the range of maximum capacity then the system will carry out uniform pricing stipulation. Once it exceeds the thermal limits a re-dispatch process is required to ensure that power system remains secured for transactions while keeping a balanced supply and demand function.

7.3. Modeling of LMP schemes

When redispatch is necessary, different model of LMP are formulated. Three schemes of the LMP are introduced: LMP-lossless, LMP-loss and LMP–TUT.

7.3.1. Scheme I: Named as LMP-lossless model, has two components which are energy price (EP) and transmission congestion revenue (CR), while transmission losses are ignored

\[
\text{LMP}_{\text{lossless}} = \text{EP} + \text{CR} 
\]

Objective function and its constraints are combined to formulate a Lagrange function using Lagrangian multipliers. These multipliers are referred to as dual variables or shadow prices:

\[
\begin{align*}
L(P_{Dj}, P_{Gi}, \lambda, \mu) &= \sum_{i=1}^{n} C_i(P_{Gi}) - \sum_{j=1}^{m} B_j(P_{Dj}) + \lambda \left( \sum_{j=1}^{m} P_{Dj} - \sum_{i=1}^{n} P_{Gi} \right) \\
&\quad - \sum_{i} \mu_{\min, Gi}(P_{Gi} - P_{\min, Gi}) + \sum_{i} \mu_{\max, Gi}(P_{Gi} - P_{\max, Gi}) \\
&\quad - \sum_{m} \mu_{\min, flow,m}(P_{flow,m} - P_{\min}) \\
&\quad + \sum_{m} \mu_{\max, flow,m}(P_{flow,m} - P_{\max}) 
\end{align*}
\]

7.3.2. Scheme II: Named as LMP-loss model, includes three components: energy price, transmission congestion revenue and transmission losses cost (LC)

\[
\text{LMP}_{\text{loss}} = \text{EP} + \text{CR} + \text{LC}
\]

Formulation of network losses:

\[
\begin{align*}
P_i(P_{Gi}) &= [P_{flow,m}]^T[R][P_{flow,m}] \\
P_{flow,m} &= \sum_{i} S_{mi} P_{Gi} - \sum_{j} S_{mj} P_{Dj} 
\end{align*}
\]

The branch flow may be stated in a general form:

\[
f_{flow} = S.P_i
\]

Hence

\[
P_i(P_{Gi}) = [P_{Gi}]^T[S]^T[R][S][P_{Gi}] = \frac{1}{2}[P_{Gi}]^T[G_i][P_{Gi}]
\]

\[
\text{For} \quad [G_i] = 2[S]^T[R][S]
\]

Both \(S_{mi(j)}\) and \(S_{mj(i)}\) are elements of the shift factor related to the network connection node between the generator or supplier \(i\) and load or customer \(j\). The Lagrange function is then formulated as:

\[
L(P_{Dj}, P_{Gi}, \lambda, \mu) = \sum_{i=1}^{n} C_i(P_{Gi}) - \sum_{j=1}^{m} B_j(P_{Dj}) \\
+ \lambda \left( \sum_{j=1}^{m} P_{Dj} - \sum_{i=1}^{n} P_{Gi} \right) - \sum_{i} \mu_{\min, Gi}(P_{Gi} - P_{\min, Gi}) \\
+ \sum_{i} \mu_{\max, Gi}(P_{Gi} - P_{\max, Gi}) - \sum_{m} \mu_{\min, flow,m}(P_{flow,m} - P_{\min}) \\
- \sum_{m} \mu_{\max, flow,m}(P_{flow,m} - P_{\max}) + \sum_{m} \mu_{\min, flow,m}(P_{flow,m} - P_{\min}) + \sum_{m} \mu_{\max, flow,m}(P_{flow,m} - P_{\max})
\]

The LMPs may be derived as the first order KKT optimality condition as follows:

\[
\rho_i = \lambda_i \left( 1 - \sum_{i=1}^{n} G_{i,n-r} \left( \sum_{j=1}^{m} P_{Gi} - \sum_{j=1}^{m} P_{Dj} \right) \right) + \sum_{m} \mu_{\min, flow,m - S_m} \\
- \sum_{m} \mu_{\max, flow,m - S_m}
\]

7.3.3. Scheme III: Named as LMP–TUT is formulated based on the LMP-loss but takes into account a tariff for transmission usage (TUT) as well

\[
\text{LMP}_{\text{TUT}} = \text{EP} + \text{CR} + \text{LC} + \text{TUT}
\]

Total transmission usage tariff is written as:

\[
T_m(P_{Gi}) = \alpha_m P_{flow,m}
\]

Formulation of network losses
The model for network losses refers to Eq. (12). The Lagrange function becomes:

\[
L(P_{Dj}, P_{Gi}, \lambda_i, \mu_i) = \sum_{m=1}^{M} T_m(P_{Gi}) + \sum_{i=1}^{nG} C_i(P_{Gi}) - \sum_{j=1}^{nD} B_j(P_{Dj}) + \lambda_i \left( \sum_j P_{Dj} - \sum_i P_{Gi} - P_{L}(P_{Gi}) \right) - \sum_j \mu_{\min,Dj}(P_{Dj} - P_{Dj}^{\min}) + \sum_j \mu_{\max,Dj}(P_{Dj} - P_{Dj}^{\max}) - \sum_i \mu_{\min,Gi}(P_{Gi} - P_{Gi}^{\min})
\]
\[
+ \sum_{m} \mu_{\text{max}} \cdot (P_{m} - P_{m}^{\text{max}}) - \sum_{m} \mu_{\text{min}} \cdot \text{flow}_{m} - \text{flow}_{m}^{\text{min}} \\
+ \sum_{m} \mu_{\text{max}} \cdot \text{flow}_{m} \cdot (\text{flow}_{m} - \text{flow}_{m}^{\text{max}})
\]

Thus, the suppliers’ and customers’ LMPs are stated as:

\[
\rho_{i} = \lambda_{i} \left\{ 1 - \sum_{f=1}^{I} G_{n-f} \cdot \left( i \cdot \sum_{j=1}^{d} P_{d} - j \cdot \sum_{f} P_{f} \right) \right\} + \sum_{m} \mu_{\text{min}} \cdot \text{flow}_{m} \cdot S_{m} \\
- \sum_{m} \mu_{\text{max}} \cdot \text{flow}_{m} \cdot S_{m} - \sum_{m} \alpha_{m} \cdot \text{flow}_{m} \cdot (P_{m}) \cdot S_{m} 
\]

where \( \rho_{i} \) is locational marginal price at bus \( i \), \( \lambda_{i}, \mu \) are Lagrange multipliers, \( i \) is supplier index \( (1, 2, 3, ..., I) \), \( j \) is consumer index \( (1, 2, 3, ..., J) \), \( S \) is shift factor of the transmission network, \( m \) and \( n \) are line number where the supplier \( i \) or customer \( j \) is connected, \( g_{i} \) is output of generator \( i \), \( d_{j} \) is demand of load \( j \), \( \text{flow}_{m} \) is power flow on line \( m \), \( n_{G}, n_{O} \) are number of generators and load, \( C_{i} \) is transmission network loss representation matrix, \( \alpha_{m} \) is transmission charge rate for line \( m \) ($/MW h$), \( C_{i} \) is production cost at bus \( i \), \( B_{i} \) is consumer benefit at bus \( i \), \( P_{0j} \) is active power output of \( j \)th generator and \( P_{0j} \) is active power demand of consumer \( j \).

### 7.4. Developing optimisation tools

After creating an LMP model for three schemes optimisation tools are developed. With reference to their specific models tools are made in such a way that they are able to examine and evaluate the optimal condition of every scheme through an iterative process.

### 7.5. Examining the most efficient reference node

As shift factor is a reference node based optimal power flow methodology, used for a particular node that can obtain the lowest objective value. Once an optimal condition is reached, the objective value of the schemes, generator’s output and total branch losses are calculated. Since market settlements are cleared at their competitive value of the schemes, generator’s output and total branch losses cost and transmission usage tariff. The transmission congestion issue is the central problem since it may lead to further impacts such as market inefficiency owing to maximum capacity of the network and the possibility of exercising market power by strategic behaviour of the producers which may create high prices in a congested area.

DC-OPF has been widely used for dispatching power and clearing energy to determine the LMP due to its speed and robustness, particularly in market simulation and planning. However, DC-OPF does not consider the network losses. As the DC-OPF is modified by incorporating the line losses, such advantageous features do not exist anymore due to the complexity. Therefore, a new approach of optimal power flow to maintain the linearity and superposition features of the LMPs model is proposed while it is still able to account for both congestion cost and losses cost. The proposed method uses shift factor based optimal power flow. Optimisation scenarios developed in this research is based on the perfect competition where each market participant is performed by their marginal cost. The shift factor is the actual component that determines the power flow over a given transmission line from the source node (generation) to the sink node (load) which can be obtained through three stages based on a DC load flow network model.

### References
