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Lampiran 1. Titik Kesetimbangan Non-Endemik dan Endemik

```
> restart;
> with(DEtools) :
> with(linalg) :
> a1 := (β + ψ) : a2 := ρ·(L + theta·C) : a3 := η1 + β : a4 := β + η2 : a5 := u2 + η3 : a6 := η4 + β;
```

$$a6 := \eta 4 + \beta \tag{1}$$

```
> f1 := c·β − a1·M;
```

$$f1 := -M(\beta + \psi) + c\beta \tag{2}$$

```
> f2 := (1 − c)·β + ψ·M − β·S − a2·S − u1·S + η4·R;
```

$$f2 := (1 - c)\,\beta + \psi M - \beta S - \rho\,(C\,\theta + L)\,S - u1\,S + \eta 4\,R \tag{3}$$

```
> f3 := a2·S − a3·E;
```

$$f3 := -E\,(\eta 1 + \beta) + \rho\,(C\,\theta + L)\,S \tag{4}$$

```
> f4 := η1·E − a4·L;
```

$$f4 := \eta 1\,E - L\,(\beta + \eta 2) \tag{5}$$

```
> f5 := p·η2·L − β·C − a5·C;
```

$$f5 := p\,\eta 2\,L - C\,(u2 + \eta 3) - \beta\,C \tag{6}$$

```
> f6 := a5·C + (1 − p)·η2·L + u1·S − a6·R;
```

$$f6 := C\,(u2 + \eta 3) + (1 - p)\,\eta 2 L + u1\,S - (\eta 4 + \beta)\,R \tag{7}$$

```
> fixpoint := solve({f1, f2, f3, f4, f5, f6}, {M, S, E, L, C, R})
```

```
> fix1 := fixpoint[1];
```

$$fix1 := \left\{C = 0, E = 0, L = 0, M = \frac{c\beta}{\beta + \psi}, R = -\frac{(c\beta - \psi - \beta)\,u1}{\psi u1 + \psi\beta + \psi\eta 4 + u1\beta + \beta^2 + \beta\eta 4}, S = -\frac{c\beta^2 + c\beta\eta 4 - \psi\beta - \psi\eta 4 - \beta^2 - \beta\eta 4}{\psi u1 + \psi\beta + \psi\eta 4 + u1\beta + \beta^2 + \beta\eta 4}\right\} \tag{9}$$

```
> fix2 := fixpoint[2];
```

$$fix2 := \Big\{C = -\big(p\,\eta 2\,(c\beta^2\eta 4\rho\eta 1 - \beta\psi\eta 4\rho\eta 1 - \psi p\eta 2\eta 4\rho\eta 1\theta - \psi\eta 4\rho\eta 1 u2 - \psi\eta 4\rho\eta 1\eta 3 + u1\psi\beta\eta 1 u2 + u1\psi\beta\eta 1\eta 3 + c\beta^2 p\eta 2\rho\eta 1\theta + c\beta^2\rho\eta 1 u2 \tag{10}$$

$$+ c\beta^2\rho\eta 1\eta 3 - \beta\psi p\eta 2\rho\eta 1\theta - \beta\psi\rho\eta 1 u2 - \beta\psi\rho\eta 1\eta 3 - \beta p\eta 2\eta 4\rho\eta 1\theta - \beta\eta 4\rho\eta 1 u2 - \beta\eta 4\rho\eta 1\eta 3 + \psi\beta^3 u2 + \psi\beta^3\eta 1 + \psi\beta^3\eta 2 + \psi\beta^3\eta 3 + \beta^3\eta 4 u2$$

$$+ \beta^3\eta 4\eta 1 + \beta^3\eta 4\eta 2 + \beta^3\eta 4\eta 3 + \psi\beta^3\eta 4 + \beta^3\eta 1 u2 + \beta^3\eta 2 u2 + \beta^3\eta 1\eta 2 + \beta^3\eta 1\eta 3 + \beta^3\eta 2\eta 3 + u1\beta^4 + u1\beta^3\eta 2 + c\beta p\eta 2\eta 4\rho\eta 1\theta + c\beta\eta 4\rho\eta 1 u2$$

$$+ c\beta\eta 4\rho\eta 1\eta 3 + \psi\eta 2\eta 4\eta 1 u2 + \psi\eta 2\eta 4\eta 1\eta 3 + u1\psi\eta 2\eta 1 u2 + u1\psi\eta 2\eta 1\eta 3 + c\beta^3\rho\eta 1 - \beta^2\psi\rho\eta 1 - \beta^2\eta 4\rho\eta 1 + u1\psi\eta 2\beta\eta 3 - \beta^2 p\eta 2\rho\eta 1\theta - \beta^2\rho\eta 1 u2$$

$$- \beta^2\rho\eta 1\eta 3 + \beta\eta 2\eta 4\eta 1 u2 + \beta\eta 2\eta 4\eta 1\eta 3 + u1\beta^2\eta 1 u2 + u1\beta\eta 2\eta 1 u2 + u1\beta^2\eta 1\eta 3 + u1\beta\eta 2\eta 1\eta 3 + \beta\psi\eta 2\eta 4\eta 1 + u1\psi\beta^2 u2 + u1\psi\eta 2\beta u2 + u1\psi\beta^2\eta 1$$

$$+ u1\psi\beta^2\eta 3 + u1\psi\eta 2\beta\eta 1 + \psi\beta\eta 1\eta 2\eta 3 + \psi u2\eta 4\eta 1\beta + \psi u2\eta 4\beta\eta 2 + \psi\beta\eta 4\eta 1\eta 3 + \psi\beta\eta 4\eta 2\eta 3 + \psi\beta\eta 1\eta 2 u2 + \beta^5 + \beta^4 u2 + \beta^4\eta 1 + \beta^4\eta 2 + \beta^4\eta 3$$

$$+ \psi\beta^4 + \beta^4\eta 4 + \beta^2\eta 2\eta 4\eta 1 + u1\beta^3 u2 + u1\beta^2\eta 2 u2 + u1\beta^3\eta 1 + u1\beta^3\eta 3 + u1\beta^2\eta 2\eta 1 + u1\beta^2\eta 2\eta 3 + u1\psi\beta^3 + u1\psi\eta 2\beta^2 - \beta^3\rho\eta 1 + \beta^2\eta 1\eta 2 u2$$

$$+ \beta^2\eta 1\eta 2\eta 3 + \psi\beta^2\eta 1 u2 + \psi\beta^2\eta 2 u2 + \psi\beta^2\eta 1\eta 2 + \psi\beta^2\eta 1\eta 3 + \psi\beta^2\eta 2\eta 3 + \beta^2\eta 4\eta 1 u2 + \beta^2\eta 4\eta 2 u2 + \beta^2\eta 4\eta 1\eta 3 + \beta^2\eta 4\eta 2\eta 3 + \psi u2\eta 4\beta^2 + \psi\beta^2\eta 4\eta 1$$

$$+ \psi\beta^2\eta4\eta2 + \psi\beta^2\eta4\eta3)) \Big/ (\rho\,(\theta\beta^2 p\eta2\eta4\eta3 + \theta\beta p\eta2^2\eta4\eta3 + \theta\beta^2 p\eta2\eta1 u2 + \theta\beta p\eta2^2\eta1 u2 + \theta\beta^2 p\eta2\eta1\eta3 + \theta\beta p\eta2^2\eta1\eta3 + \theta\psi p\eta2^2\eta4 u2$$

$$+ \theta\psi p\eta2^2\eta4\eta3 + \theta\psi p\eta2^2\eta1 u2 + \theta\psi p\eta2^2\eta1\eta3 + \theta\psi p\eta2\eta4\beta^2 + \theta\psi p\eta2^2\eta4\beta + \theta\psi p\eta2\beta^2 u2 + \theta\psi p\eta2^2\beta u2 + \theta\psi p\eta2\beta^2\eta1 + \theta\psi p\eta2\beta^2\eta3$$

$$+ \theta\psi p\eta2^2\beta\eta1 + \theta\psi p\eta2^2\beta\eta3 + \theta\beta^2 p\eta2\eta4 u2 + \theta\beta p\eta2^2\eta4 u2 + \theta\beta^2 p\eta2\eta4\eta1 + 2\psi\beta^3 u2 + \psi\beta^3\eta1 + \psi\beta^3\eta2 + 2\psi\beta^3\eta3 + u2^2\beta^2\eta1 + u2^2\beta^2\eta2$$

$$+ 2u2\beta^3\eta3 + 2\beta^3\eta4 u2 + \beta^3\eta4\eta1 + \beta^3\eta4\eta2 + 2\beta^3\eta4\eta3 + \beta^2\eta3^2\eta1 + \beta^2\eta3^2\eta2 + \psi u2^2\beta^2 + \psi\beta^3\eta4 + \psi\eta3^2\beta^2 + u2^2\beta^2\eta4 + \beta^2\eta3^2\eta4 + 2\beta^3\eta1 u2$$

$$+ 2\beta^3\eta2 u2 + \beta^3\eta1\eta2 + 2\beta^3\eta1\eta3 + 2\beta^3\eta2\eta3 + \theta\psi p\eta2\eta4\eta1\eta3 + \theta\psi p\eta2\eta4\beta u2 + \theta\psi p\eta2\eta4\eta1\beta + \theta\psi p\eta2\eta4\beta\eta3 + \theta\psi p\eta2\beta\eta1 u2 + \theta\psi p\eta2\beta\eta1\eta3$$

$$+ \theta\beta p\eta2\eta4\eta1 u2 + \theta\beta p\eta2\eta4\eta1\eta3 + \theta\psi p\eta2\eta4\eta1 u2 + 2\psi\beta\eta1\eta2\eta3 + 2u2\beta\eta1\eta2\eta3 + \theta\beta^3 p\eta2 u2 + \theta\beta^2 p\eta2^2 u2 + \theta\beta^3 p\eta2\eta1 + \theta\beta^3 p\eta2\eta3$$

$$+ \theta\beta^2 p\eta2^2\eta1 + \theta\beta^2 p\eta2^2\eta3 + 2\psi u2\eta4\eta1\beta + 2\psi u2\eta4\beta\eta2 + 2\psi u2\eta4\beta\eta3 + 2\psi u2\beta\eta1\eta3 + 2\psi u2\eta4\eta1\eta3 + 2\psi u2\eta4\eta2\eta3 + 2\psi u2\eta1\eta2\eta3$$

$$+ 2\psi u2\beta\eta2\eta3 + 2\psi\beta\eta4\eta1\eta3 + 2\psi\beta\eta4\eta2\eta3 + 2u2\beta\eta4\eta1\eta3 + 2u2\beta\eta4\eta2\eta3 + \theta\psi p\eta2\beta^3 + \theta\psi p\eta2^2\beta^2 + \theta\beta^3 p\eta2\eta4 + \theta\beta^2 p\eta2^2\eta4 + 2\psi\beta\eta1\eta2 u2$$

$$+ \beta^5 + 2\beta^4 u2 + \beta^4\eta1 + \beta^4\eta2 + u2^2\beta^3 + 2\beta^4\eta3 + \psi\beta^4 + \beta^4\eta4 + \beta^3\eta3^2 + \psi\eta3^2\eta4\eta2 + \psi\eta3^2\eta1\eta2 + \psi u2^2\eta4\eta1 + \psi u2^2\eta4\eta2 + \psi u2^2\eta1\eta2 + 2\beta^2\eta1\eta2 u2$$

$$+ 2\beta^2\eta1\eta2\eta3 + 2\psi\beta^2\eta1 u2 + 2\psi\beta^2\eta2 u2 + \psi\beta^2\eta1\eta2 + 2\psi\beta^2\eta1\eta3 + 2\psi\beta^2\eta2\eta3 + u2^2\beta\eta1\eta2 + 2u2\beta^2\eta1\eta3 + 2u2\beta^2\eta2\eta3 + 2\beta^2\eta4\eta1 u2 + 2\beta^2\eta4\eta2 u2$$

$$+ u1\,\beta^3\,u2 + u1\,\beta^2\,\eta2\,u2 + u1\,\beta^3\,\eta1 + u1\,\beta^3\,\eta3 + u1\,\beta^2\,\eta2\,\eta1 + u1\,\beta^2\,\eta2\,\eta3 + u1\,\psi\,\beta^3 + u1\,\psi\,\eta2\,\beta^2 - \beta^3\,\rho\,\eta1 + \beta^2\,\eta1\,\eta2\,u2 + \beta^2\,\eta1\,\eta2\,\eta3 + \psi\,\beta^2\,\eta1\,u2 + \psi\,\beta^2\,\eta2\,u2$$

$$+ \psi\,\beta^2\,\eta1\,\eta2 + \psi\,\beta^2\,\eta1\,\eta3 + \psi\,\beta^2\,\eta2\,\eta3 + \beta^2\,\eta4\,\eta1\,u2 + \beta^2\,\eta4\,\eta2\,u2 + \beta^2\,\eta4\,\eta1\,\eta3 + \beta^2\,\eta4\,\eta2\,\eta3 + \psi\,u2\,\eta4\,\beta^2 + \psi\,\beta^2\,\eta4\,\eta1 + \psi\,\beta^2\,\eta4\,\eta2 + \psi\,\beta^2\,\eta4\,\eta3)\,(u2 + \beta$$

$$+ \eta3)\,(\beta + \eta2))\Big/\big(\rho\,(\theta\,\beta^2\,p\,\eta2\,\eta4\,\eta3 + \theta\,\beta\,p\,\eta2^2\,\eta4\,\eta3 + \theta\,\beta^2\,p\,\eta2\,\eta1\,u2 + \theta\,\beta\,p\,\eta2^2\,\eta1\,u2 + \theta\,\beta^2\,p\,\eta2\,\eta1\,\eta3 + \theta\,\beta\,p\,\eta2^2\,\eta1\,\eta3 + \theta\,\psi\,p\,\eta2^2\,\eta4\,u2 + \theta\,\psi\,p\,\eta2^2\,\eta4\,\eta3$$

$$+ \theta\,\psi\,p\,\eta2^2\,\eta1\,u2 + \theta\,\psi\,p\,\eta2^2\,\eta1\,\eta3 + \theta\,\psi\,p\,\eta2\,\eta4\,\beta^2 + \theta\,\psi\,p\,\eta2^2\,\eta4\,\beta + \theta\,\psi\,p\,\eta2\,\beta^2\,u2 + \theta\,\psi\,p\,\eta2^2\,\beta\,u2 + \theta\,\psi\,p\,\eta2\,\beta^2\,\eta1 + \theta\,\psi\,p\,\eta2\,\beta^2\,\eta3 + \theta\,\psi\,p\,\eta2^2\,\beta\,\eta1$$

$$+ \theta\,\psi\,p\,\eta2^2\,\beta\,\eta3 + \theta\,\beta^2\,p\,\eta2\,\eta4\,u2 + \theta\,\beta\,p\,\eta2^2\,\eta4\,u2 + \theta\,\beta^2\,p\,\eta2\,\eta4\,\eta1 + 2\,\psi\,\beta^3\,u2 + \psi\,\beta^3\,\eta1 + \psi\,\beta^3\,\eta2 + 2\,\psi\,\beta^3\,\eta3 + u2^2\,\beta^2\,\eta1 + u2^2\,\beta^2\,\eta2 + 2\,u2\,\beta^3\,\eta3 + 2\,\beta^3\,\eta4\,u2$$

$$+ \beta^3\,\eta4\,\eta1 + \beta^3\,\eta4\,\eta2 + 2\,\beta^3\,\eta4\,\eta3 + \beta^2\,\eta3^2\,\eta1 + \beta^2\,\eta3^2\,\eta2 + \psi\,u2^2\,\beta^2 + \psi\,\beta^3\,\eta4 + \psi\,\eta3^2\,\beta^2 + u2^2\,\beta^2\,\eta4 + \beta^2\,\eta3^2\,\eta4 + 2\,\beta^3\,\eta1\,u2 + 2\,\beta^3\,\eta2\,u2 + \beta^3\,\eta1\,\eta2$$

$$+ 2\,\beta^3\,\eta1\,\eta3 + 2\,\beta^3\,\eta2\,\eta3 + \theta\,\psi\,p\,\eta2\,\eta4\,\eta1\,\eta3 + \theta\,\psi\,p\,\eta2\,\eta4\,\beta\,u2 + \theta\,\psi\,p\,\eta2\,\eta4\,\eta1\,\beta + \theta\,\psi\,p\,\eta2\,\eta4\,\beta\,\eta3 + \theta\,\psi\,p\,\eta2\,\beta\,\eta1\,u2 + \theta\,\psi\,p\,\eta2\,\beta\,\eta1\,\eta3 + \theta\,\beta\,p\,\eta2\,\eta4\,\eta1\,u2$$

$$+ \theta\,\beta\,p\,\eta2\,\eta4\,\eta1\,\eta3 + \theta\,\psi\,p\,\eta2\,\eta4\,\eta1\,u2 + 2\,\psi\,\beta\,\eta1\,\eta2\,\eta3 + 2\,u2\,\beta\,\eta1\,\eta2\,\eta3 + \theta\,\beta^3\,p\,\eta2\,u2 + \theta\,\beta^2\,p\,\eta2^2\,u2 + \theta\,\beta^3\,p\,\eta2\,\eta1 + \theta\,\beta^3\,p\,\eta2\,\eta3 + \theta\,\beta^2\,p\,\eta2^2\,\eta1 + \theta\,\beta^2\,p\,\eta2^2\,\eta3$$

$$+ 2\,\psi\,u2\,\eta4\,\eta1\,\beta + 2\,\psi\,u2\,\eta4\,\beta\,\eta2 + 2\,\psi\,u2\,\eta4\,\beta\,\eta3 + 2\,\psi\,u2\,\beta\,\eta1\,\eta3 + 2\,\psi\,u2\,\eta4\,\eta1\,\eta3 + 2\,\psi\,u2\,\eta4\,\eta2\,\eta3 + 2\,\psi\,u2\,\eta1\,\eta2\,\eta3 + 2\,\psi\,u2\,\beta\,\eta2\,\eta3 + 2\,\psi\,\beta\,\eta4\,\eta1\,\eta3$$

$$+ 2\,\psi\,\beta\,\eta4\,\eta2\,\eta3 + 2\,u2\,\beta\,\eta4\,\eta1\,\eta3 + 2\,u2\,\beta\,\eta4\,\eta2\,\eta3 + \theta\,\psi\,p\,\eta2^2\,\beta^2 + \theta\,\psi\,p\,\eta2^2\,\beta^2 + \theta\,\beta^3\,p\,\eta2\,\eta4 + \theta\,\beta^2\,p\,\eta2^2\,\eta4 + 2\,\psi\,\beta\,\eta1\,\eta2\,u2 + \beta^5 + 2\,\beta^4\,u2 + \beta^4\,\eta1 + \beta^4\,\eta2$$

$$+ 2\,\beta^2\,\eta4\,\eta1\,\eta3 + 2\,\beta^2\,\eta4\,\eta2\,\eta3 + \beta\,\eta3^2\,\eta1\,\eta2 + \theta\,\beta^4\,p\,\eta2 + \theta\,\beta^3\,p\,\eta2^2 + \psi\,u2^2\,\eta4\,\beta + 2\,\psi\,u2\,\eta4\,\beta^2 + \psi\,u2^2\,\beta\,\eta1 + \psi\,u2^2\,\beta\,\eta2 + 2\,\psi\,u2\,\beta^2\,\eta3 + \psi\,\beta^2\,\eta4\,\eta1 + \psi\,\beta^2\,\eta4\,\eta2$$

$$+ 2\,\psi\,\beta^2\,\eta4\,\eta3 + \psi\,\eta3^2\,\eta4\,\beta + \psi\,\eta3^2\,\beta\,\eta1 + \psi\,\eta3^2\,\beta\,\eta2 + u2^2\,\beta\,\eta4\,\eta1 + u2^2\,\beta\,\eta4\,\eta2 + 2\,u2\,\beta^2\,\eta4\,\eta3 + \beta\,\eta3^2\,\eta4\,\eta1 + \beta\,\eta3^2\,\eta4\,\eta2 + \psi\,\eta3^2\,\eta4\,\eta1 + \beta\,\psi\,p\,\eta2\,\eta4\,\eta1$$

$$+ \psi\,p^2\,\eta2^2\,\eta4\,\eta1\,\theta + \psi\,p\,\eta2\,\eta4\,\eta1\,u2 + \psi\,p\,\eta2\,\eta4\,\eta1\,\eta3 + \beta\,p^2\,\eta2^2\,\eta4\,\eta1\,\theta + \beta\,p\,\eta2\,\eta4\,\eta1\,u2 + \beta\,p\,\eta2\,\eta4\,\eta1\,\eta3 + \beta^2\,p\,\eta2\,\eta4\,\eta1)\big),\, E = -\big(\big(c\,\beta^2\,\eta4\,\rho\,\eta1$$

$$- \beta\,\psi\,\eta4\,\rho\,\eta1 - \psi\,p\,\eta2\,\eta4\,\rho\,\eta1\,\theta - \psi\,\eta4\,\rho\,\eta1\,u2 - \psi\,\eta4\,\rho\,\eta1\,\eta3 + u1\,\psi\,\beta\,\eta1\,u2 + u1\,\psi\,\beta\,\eta1\,\eta3 + c\,\beta^2\,p\,\eta2\,\rho\,\eta1\,\theta + c\,\beta^2\,\rho\,\eta1\,u2 + c\,\beta^2\,\rho\,\eta1\,\eta3 - \beta\,\psi\,p\,\eta2\,\rho\,\eta1\,\theta$$

$$- \beta\,\psi\,\rho\,\eta1\,u2 - \beta\,\psi\,\rho\,\eta1\,\eta3 - \beta\,p\,\eta2\,\eta4\,\rho\,\eta1\,\theta - \beta\,\eta4\,\rho\,\eta1\,u2 - \beta\,\eta4\,\rho\,\eta1\,\eta3 + \psi\,\beta^3\,u2 + \psi\,\beta^3\,\eta1 + \psi\,\beta^3\,\eta2 + \psi\,\beta^3\,\eta3 + \beta^3\,\eta4\,u2 + \beta^3\,\eta4\,\eta1 + \beta^3\,\eta4\,\eta2 + \beta^3\,\eta4\,\eta3$$

$$+ \psi\,\beta^3\,\eta4 + \beta^3\,\eta1\,u2 + \beta^3\,\eta2\,u2 + \beta^3\,\eta1\,\eta2 + \beta^3\,\eta1\,\eta3 + \beta^3\,\eta2\,\eta3 + u1\,\beta^4 + u1\,\beta^3\,\eta2 + c\,\beta\,p\,\eta2\,\eta4\,\rho\,\eta1\,\theta + c\,\beta\,\eta4\,\rho\,\eta1\,u2 + c\,\beta\,\eta4\,\rho\,\eta1\,\eta3 + \psi\,\eta2\,\eta4\,\eta1\,u2$$

$$+ \psi\,\eta2\,\eta4\,\eta1\,\eta3 + u1\,\psi\,\eta2\,\eta1\,u2 + u1\,\psi\,\eta2\,\eta1\,\eta3 + c\,\beta^3\,\rho\,\eta1 - \beta^2\,\psi\,\rho\,\eta1 - \beta^2\,\eta4\,\rho\,\eta1 + u1\,\psi\,\eta2\,\beta\,\eta3 - \beta^2\,p\,\eta2\,\rho\,\eta1\,\theta - \beta^2\,\rho\,\eta1\,u2 - \beta^2\,\rho\,\eta1\,\eta3 + \beta\,\eta2\,\eta4\,\eta1\,u2$$

$$+ \beta\,\eta2\,\eta4\,\eta1\,\eta3 + u1\,\beta^2\,\eta1\,u2 + u1\,\beta\,\eta2\,\eta1\,u2 + u1\,\beta^2\,\eta1\,\eta3 + u1\,\beta\,\eta2\,\eta1\,\eta3 + \beta\,\psi\,\eta2\,\eta4\,\eta1 + u1\,\psi\,\beta^2\,u2 + u1\,\psi\,\eta2\,\beta\,u2 + u1\,\psi\,\beta^2\,\eta1 + u1\,\psi\,\beta^2\,\eta3 + u1\,\psi\,\eta2\,\beta\,\eta1$$

$$+ \psi\,\beta\,\eta1\,\eta2\,\eta3 + \psi\,u2\,\eta4\,\eta1\,\beta + \psi\,u2\,\eta4\,\beta\,\eta2 + \psi\,\beta\,\eta4\,\eta1\,\eta3 + \psi\,\beta\,\eta4\,\eta2\,\eta3 + \psi\,\beta\,\eta1\,\eta2\,u2 + \beta^5 + \beta^4\,u2 + \beta^4\,\eta1 + \beta^4\,\eta2 + \beta^4\,\eta3 + \psi\,\beta^4 + \beta^4\,\eta4 + \beta^2\,\eta2\,\eta4\,\eta1$$

$$+ u1\,\beta^3\,u2 + u1\,\beta^2\,\eta2\,u2 + u1\,\beta^3\,\eta1 + u1\,\beta^3\,\eta3 + u1\,\beta^2\,\eta2\,\eta1 + u1\,\beta^2\,\eta2\,\eta3 + u1\,\psi\,\beta^3 + u1\,\psi\,\eta2\,\beta^2 - \beta^3\,\rho\,\eta1 + \beta^2\,\eta1\,\eta2\,u2 + \beta^2\,\eta1\,\eta2\,\eta3 + \psi\,\beta^2\,\eta1\,u2 + \psi\,\beta^2\,\eta2\,u2$$

$$+ u2^2\beta^3 + 2\beta^4\eta3 + \psi\beta^4 + \beta^4\eta4 + \beta^3\eta3^2 + \psi\eta3^2\eta4\eta2 + \psi\eta3^2\eta1\eta2 + \psi u2^2\eta4\eta1 + \psi u2^2\eta4\eta2 + \psi u2^2\eta1\eta2 + 2\beta^2\eta1\eta2u2 + 2\beta^2\eta1\eta2\eta3 + 2\psi\beta^2\eta1u2$$

$$+ 2\psi\beta^2\eta2u2 + \psi\beta^2\eta1\eta2 + 2\psi\beta^2\eta1\eta3 + 2\psi\beta^2\eta2\eta3 + u2^2\beta\eta1\eta2 + 2u2\beta^2\eta1\eta3 + 2u2\beta^2\eta2\eta3 + 2\beta^2\eta4\eta1u2 + 2\beta^2\eta4\eta2u2 + 2\beta^2\eta4\eta1\eta3$$

$$+ 2\beta^2\eta4\eta2\eta3 + \beta\eta3^2\eta1\eta2 + \theta\beta^4p\eta2 + \theta\beta^3p\eta2^2 + \psi u2^2\eta4\beta + 2\psi u2\eta4\beta^2 + \psi u2^2\beta\eta1 + \psi u2^2\beta\eta2 + 2\psi u2\beta^2\eta3 + \psi\beta^2\eta4\eta1 + \psi\beta^2\eta4\eta2 + 2\psi\beta^2\eta4\eta3$$

$$+ \psi\eta3^2\eta4\beta + \psi\eta3^2\beta\eta1 + \psi\eta3^2\beta\eta2 + u2^2\beta\eta4\eta1 + u2^2\beta\eta4\eta2 + 2u2\beta^2\eta4\eta3 + \beta\eta3^2\eta4\eta1 + \beta\eta3^2\eta4\eta2 + \psi\eta3^2\eta4\eta1 + \beta\psi p\eta2\eta4\eta1 + \psi p^2\eta2^2\eta4\eta1\theta$$

$$+ \psi p\eta2\eta4\eta1u2 + \psi p\eta2\eta4\eta1\eta3 + \beta p^2\eta2^2\eta4\eta1\theta + \beta p\eta2\eta4\eta1u2 + \beta p\eta2\eta4\eta1\eta3 + \beta^2p\eta2\eta4\eta1\big)\eta1\big), L = -\Big(\big(c\beta^2\eta4\rho\eta1 - \beta\psi\eta4\rho\eta1$$

$$- \psi p\eta2\eta4\rho\eta1\theta - \psi\eta4\rho\eta1u2 - \psi\eta4\rho\eta1\eta3 + u1\psi\beta\eta1u2 + u1\psi\beta\eta1\eta3 + c\beta^2p\eta2\rho\eta1\theta + c\beta^2\rho\eta1u2 + c\beta^2\rho\eta1\eta3 - \beta\psi p\eta2\rho\eta1\theta - \beta\psi\rho\eta1u2$$

$$- \beta\psi\rho\eta1\eta3 - \beta p\eta2\eta4\rho\eta1\theta - \beta\eta4\rho\eta1u2 - \beta\eta4\rho\eta1\eta3 + \psi\beta^3u2 + \psi\beta^3\eta1 + \psi\beta^3\eta2 + \psi\beta^3\eta3 + \beta^3\eta4u2 + \beta^3\eta4\eta1 + \beta^3\eta4\eta2 + \beta^3\eta4\eta3 + \psi\beta^3\eta4$$

$$+ \beta^3\eta1u2 + \beta^3\eta2u2 + \beta^3\eta1\eta2 + \beta^3\eta1\eta3 + \beta^3\eta2\eta3 + u1\beta^4 + u1\beta^3\eta2 + c\beta p\eta2\eta4\rho\eta1\theta + c\beta\eta4\rho\eta1u2 + c\beta\eta4\rho\eta1\eta3 + \psi\eta2\eta4\eta1u2 + \psi\eta2\eta4\eta1\eta3$$

$$+ u1\psi\eta2\eta1u2 + u1\psi\eta2\eta1\eta3 + c\beta^3\rho\eta1 - \beta^2\psi\rho\eta1 - \beta^2\eta4\rho\eta1 + u1\psi\eta2\beta\eta3 - \beta^2p\eta2\rho\eta1\theta - \beta^2\rho\eta1u2 - \beta^2\rho\eta1\eta3 + \beta\eta2\eta4\eta1u2 + \beta\eta2\eta4\eta1\eta3$$

$$+ u1\beta^2\eta1u2 + u1\beta\eta2\eta1u2 + u1\beta^2\eta1\eta3 + u1\beta\eta2\eta1\eta3 + \beta\psi\eta2\eta4\eta1 + u1\psi\beta^2u2 + u1\psi\eta2\beta u2 + u1\psi\beta^2\eta1 + u1\psi\beta^2\eta3 + u1\psi\eta2\beta\eta1 + \psi\beta\eta1\eta2\eta3$$

$$+ \psi u2\eta4\eta1\beta + \psi u2\eta4\beta\eta2 + \psi\beta\eta4\eta1\eta3 + \psi\beta\eta4\eta2\eta3 + \psi\beta\eta1\eta2u2 + \beta^5 + \beta^4u2 + \beta^4\eta1 + \beta^4\eta2 + \beta^4\eta3 + \psi\beta^4 + \beta^4\eta4 + \beta^2\eta2\eta4\eta1 + u1\beta^3u2$$

$$+ u1\beta^2\eta2u2 + u1\beta^3\eta1 + u1\beta^3\eta3 + u1\beta^2\eta2\eta1 + u1\beta^2\eta2\eta3 + u1\psi\beta^3 + u1\psi\eta2\beta^2 - \beta^3\rho\eta1 + \beta^2\eta1\eta2u2 + \beta^2\eta1\eta2\eta3 + \psi\beta^2\eta1u2 + \psi\beta^2\eta2u2$$

$$+ \psi\beta^2\eta1\eta2 + \psi\beta^2\eta1\eta3 + \psi\beta^2\eta2\eta3 + \beta^2\eta4\eta1u2 + \beta^2\eta4\eta2u2 + \beta^2\eta4\eta1\eta3 + \beta^2\eta4\eta2\eta3 + \psi u2\eta4\beta^2 + \psi\beta^2\eta4\eta1 + \psi\beta^2\eta4\eta2 + \psi\beta^2\eta4\eta3\big)\,(u2 + \beta$$

$$+ \eta3)\Big)\Big/\big(\rho\,\big(\theta\beta^2p\eta2\eta4\eta3 + \theta\beta p\eta2^2\eta4\eta3 + \theta\beta^2p\eta2\eta1u2 + \theta\beta p\eta2^2\eta1u2 + \theta\beta^2p\eta2\eta1\eta3 + \theta\beta p\eta2^2\eta1\eta3 + \theta\psi p\eta2^2\eta4u2 + \theta\psi p\eta2^2\eta4\eta3$$

$$+ \theta\psi p\eta2^2\eta1u2 + \theta\psi p\eta2^2\eta1\eta3 + \theta\psi p\eta2\eta4\beta^2 + \theta\psi p\eta2^2\eta4\beta + \theta\psi p\eta2\beta^2u2 + \theta\psi p\eta2^2\beta u2 + \theta\psi p\eta2\beta^2\eta1 + \theta\psi p\eta2\beta^2\eta3 + \theta\psi p\eta2^2\beta\eta1$$

$$+ \theta\psi p\eta2^2\beta\eta3 + \theta\beta^2p\eta2\eta4u2 + \theta\beta p\eta2^2\eta4u2 + \theta\beta^2p\eta2\eta4\eta1 + 2\psi\beta^3u2 + \psi\beta^3\eta1 + \psi\beta^3\eta2 + 2\psi\beta^3\eta3 + u2^2\beta^2\eta1 + u2^2\beta^2\eta2 + 2u2\beta^3\eta3 + 2\beta^3\eta4u2$$

$$+ \beta^3\eta4\eta1 + \beta^3\eta4\eta2 + 2\beta^3\eta4\eta3 + \beta^2\eta3^2\eta1 + \beta^2\eta3^2\eta2 + \psi u2^2\beta^2 + \psi\beta^3\eta4 + \psi\eta3^2\beta^2 + u2^2\beta^2\eta4 + \beta^2\eta3^2\eta4 + 2\beta^3\eta1u2 + 2\beta^3\eta2u2 + \beta^3\eta1\eta2$$

$$+ 2\beta^3\eta1\eta3 + 2\beta^3\eta2\eta3 + \theta\psi p\eta2\eta4\eta1\eta3 + \theta\psi p\eta2\eta4\beta u2 + \theta\psi p\eta2\eta4\eta1\beta + \theta\psi p\eta2\eta4\beta\eta3 + \theta\psi p\eta2\beta\eta1u2 + \theta\psi p\eta2\beta\eta1\eta3 + \theta\beta p\eta2\eta4\eta1u2$$

$$+ \theta\beta p\eta2\eta4\eta1\eta3 + \theta\psi p\eta2\eta4\eta1u2 + 2\psi\beta\eta1\eta2\eta3 + 2u2\beta\eta1\eta2\eta3 + \theta\beta^3p\eta2u2 + \theta\beta^2p\eta2^2u2 + \theta\beta^3p\eta2\eta1 + \theta\beta^3p\eta2\eta3 + \theta\beta^2p\eta2^2\eta1 + \theta\beta^2p\eta2^2\eta3$$

$$+ 2\psi u2\eta4\eta1\beta + 2\psi u2\eta4\beta\eta2 + 2\psi u2\eta4\beta\eta3 + 2\psi u2\beta\eta1\eta3 + 2\psi u2\eta4\eta1\eta3 + 2\psi u2\eta4\eta2\eta3 + 2\psi u2\eta1\eta2\eta3 + 2\psi u2\beta\eta2\eta3 + 2\psi\beta\eta4\eta1\eta3$$

$$+2\psi\beta\eta4\eta2\eta3+2u2\beta\eta4\eta1\eta3+2u2\beta\eta4\eta2\eta3+\theta\psi p\eta2\beta^3+\theta\psi p\eta2^2\beta^2+\theta\beta^3 p\eta2\eta4+\theta\beta^2 p\eta2^2\eta4+2\psi\beta\eta1\eta2u2+\beta^2+2\beta^4 u2+\beta^4\eta1+\beta^4\eta2$$

$$+u2^2\beta^3+2\beta^4\eta3+\psi\beta^4+\beta^4\eta4+\beta^3\eta3^2+\psi\eta3^2\eta4\eta2+\psi\eta3^2\eta1\eta2+\psi u2^2\eta4\eta1+\psi u2^2\eta4\eta2+\psi u2^2\eta1\eta2+2\beta^2\eta1\eta2u2+2\beta^2\eta1\eta2\eta3+2\psi\beta^2\eta1u2$$

$$+2\psi\beta^2\eta2u2+\psi\beta^2\eta1\eta2+2\psi\beta^2\eta1\eta3+2\psi\beta^2\eta2\eta3+u2^2\beta\eta1\eta2+2u2\beta^2\eta1\eta3+2u2\beta^2\eta2\eta3+2\beta^2\eta4\eta1u2+2\beta^2\eta4\eta2u2+2\beta^2\eta4\eta1\eta3$$

$$+2\beta^2\eta4\eta2\eta3+\beta\eta3^2\eta1\eta2+\theta\beta^4 p\eta2+\theta\beta^3 p\eta2^2+\psi u2^2\eta4\beta+2\psi u2\eta4\beta^2+\psi u2^2\beta\eta1+\psi u2^2\beta\eta2+2\psi u2\beta^2\eta3+\psi\beta^2\eta4\eta1+\psi\beta^2\eta4\eta2+2\psi\beta^2\eta4\eta3$$

$$+\psi\eta3^2\eta4\beta+\psi\eta3^2\beta\eta1+\psi\eta3^2\beta\eta2+u2^2\beta\eta4\eta1+u2^2\beta\eta4\eta2+2u2\beta^2\eta4\eta3+\beta\eta3^2\eta4\eta1+\beta\eta3^2\eta4\eta2+\psi\eta3^2\eta4\eta1+\beta\psi p\eta2\eta4\eta1+\psi p^2\eta2^2\eta4\eta1\theta$$

$$+\psi p\eta2\eta4\eta1u2+\psi p\eta2\eta4\eta1\eta3+\beta p^2\eta2^2\eta4\eta1\theta+\beta p\eta2\eta4\eta1u2+\beta p\eta2\eta4\eta1\eta3+\beta^2 p\eta2\eta4\eta1\Big)\Big),\ M=\frac{c\beta}{\beta+\psi},\ R=\left(\beta^5 p\eta2\eta1+\beta^4 p\eta2^2\eta1\right.$$

$$+4\psi u1u2\beta^3\eta1+4\psi u1u2\beta^3\eta2+2\psi u1u2\beta^3\eta3+2\psi u1u2\beta^2\eta1^2+2\psi u1u2\beta^2\eta2^2+3\psi u1\beta^3\eta1\eta2+4\psi u1\beta^3\eta1\eta3+4\psi u1\beta^3\eta2\eta3+\psi u1\beta^2\eta1^2\eta2$$

$$+2\psi u1\beta^2\eta1^2\eta3+\psi u1\beta^2\eta1\eta2^2+2\psi u1\beta^2\eta1\eta3^2+2\psi u1\beta^2\eta2^2\eta3+2\psi u1\beta^2\eta2\eta3^2+\psi u1\beta\eta1^2\eta3^2+\psi u1\beta\eta2^2\eta3^2+\psi u1\eta1^2\eta2\eta3^2+\psi u1\eta1\eta2^2\eta3^2$$

$$-2\psi u2\beta\eta1^2\eta2^2-2\psi u2\eta1^2\eta2^2\eta3-2\psi\beta\eta1^2\eta2^2\eta3-c\rho\beta^3\eta1^2\eta2+\psi\rho u2^2\eta1^2\eta2+\psi\rho\beta^2\eta1^2\eta2+\psi\rho\eta1^2\eta2\eta3^2+2\psi u1u2^2\beta^2\eta1+2\psi u1u2^2\beta^2\eta2$$

$$+\psi u1u2^2\beta\eta1^2+\psi u1u2^2\beta\eta2^2+\psi u1u2^2\eta1^2\eta2+\psi u1u2^2\eta1\eta2^2+p\psi\beta^2\eta1^2\eta2^2-\psi u2^2\beta^2\eta1\eta2-\psi u2^2\beta\eta1\eta2^2-2\psi u2\beta^3\eta1\eta2-2\psi u2\beta^2\eta1\eta2^2$$

$$-2\psi\beta^3\eta1\eta2\eta3-2\psi\beta^2\eta1\eta2^2\eta3-\psi\beta^2\eta1\eta2\eta3^2-\psi\beta\eta1\eta2^2\eta3^2+\beta^3 p\eta2\eta1^2\psi-2\eta1^2\eta2\beta^2\psi\eta3-\eta1^2\eta2\beta\psi\eta3^2-\eta1^2\eta2u2^2\psi\beta-2\eta1^2\eta2u2\psi\beta^2$$

$$+\psi u1u2^2\beta^3+2\psi u1u2\beta^4+2\psi u1\beta^4\eta1+2\psi u1\beta^4\eta2+2\psi u1\beta^4\eta3+\psi u1\beta^3\eta1^2+\psi u1\beta^3\eta2^2+\psi u1\beta^3\eta3^2-\psi u2^2\eta1^2\eta2^2-\psi\beta^2\eta1^2\eta2^2-\psi\eta1^2\eta2^2\eta3^2$$

$$-\psi\beta^4\eta1\eta2-\psi\beta^3\eta1\eta2^2-\beta^3\eta1\eta2\eta3^2-\eta1^2\eta2\beta^3\psi+2u1\beta^3\eta2^2\eta3+2u1\beta^3\eta2\eta3^2+u1\beta^2\eta1^2\eta3^2+u1\beta^2\eta2^2\eta3^2+2u1\beta^3\eta1\eta3^2+2u1u2^2\beta^3\eta1$$

$$+2u1u2^2\beta^3\eta2+u1u2^2\beta^2\eta1^2+u1u2^2\beta^2\eta2^2+4u1u2\beta^4\eta1+4u1u2\beta^4\eta2+2u1u2\beta^4\eta3+2u1u2\beta^3\eta1^2+2u1u2\beta^3\eta2^2+3u1\beta^4\eta1\eta2+4u1\beta^4\eta1\eta3$$

$$+4u1\beta^4\eta2\eta3+u1\beta^3\eta1^2\eta2+2u1\beta^3\eta1^2\eta3+u1\beta^3\eta1\eta2^2+\eta1^2\eta2\beta^3\rho+\beta^3 p\eta2^2\eta1^2+\beta^4 p\eta2\eta1^2-2\beta^2\eta1^2\eta2^2\eta3-\beta^2\eta1^2\eta2\eta3^2-\beta^2\eta1\eta2^2\eta3^2$$

$$-\beta\eta1^2\eta2^2\eta3^2-u2^2\beta^3\eta1\eta2-u2^2\beta^2\eta1^2\eta2-u2^2\beta^2\eta1\eta2^2-u2^2\beta\eta1^2\eta2^2-2u2\beta^4\eta1\eta2-2u2\beta^3\eta1^2\eta2-2u2\beta^3\eta1\eta2^2-2u2\beta^2\eta1^2\eta2^2-2\beta^4\eta1\eta2\eta3$$

$$-2\beta^3\eta1^2\eta2\eta3-2\beta^3\eta1\eta2^2\eta3+\psi u1\beta^5+u1u2^2\beta^4+2u1u2\beta^5+2u1\beta^5\eta1+2u1\beta^5\eta2+2u1\beta^5\eta3+u1\beta^4\eta1^2+u1\beta^4\eta2^2+u1\beta^4\eta3^2-\beta^5\eta1\eta2$$

$$-\beta^4\eta1^2\eta2-\beta^4\eta1\eta2^2-\beta^3\eta1^2\eta2^2+u1\beta^6+4\psi u1u2\beta^2\eta2\eta3+2\psi u1u2\beta\eta1^2\eta2+2\psi u1u2\beta\eta1^2\eta3+2\psi u1u2\beta\eta1\eta2^2+2\psi u1u2\beta\eta2^2\eta3$$

$$+2\psi u1u2\eta1^2\eta2\eta3+2\psi u1u2\eta1\eta2^2\eta3+6\psi u1\beta^2\eta1\eta2\eta3+2\psi u1\beta\eta1^2\eta2\eta3+2\psi u1\beta\eta1\eta2^2\eta3+3\psi u1\beta\eta1\eta2\eta3^2+c\rho\beta^3\eta1^2\eta2-c\rho u2^2\beta\eta1^2\eta2$$

$$-2c\rho u2\beta^2\eta1^2\eta2-2c\rho\beta^2\eta1^2\eta2\eta3-c\rho\beta\eta1^2\eta2\eta3^2-p\psi\rho\beta^2\eta1^2\eta2+p\psi u1u2\eta1^2\eta2^2+p\psi\beta\eta1^2\eta2^2\eta3+p\psi u1\beta^2\eta1^2\eta2+p\psi u1\beta\eta1^2\eta2^2$$

$$+p\psi u1\eta1^2\eta2^2\eta3+2\psi\rho u2\beta\eta1^2\eta2+2\psi\rho u2\eta1^2\eta2\eta3+2\psi\rho\beta\eta1^2\eta2\eta3+3\psi u1u2^2\beta\eta1\eta2+6\psi u1u2\beta^2\eta1\eta2+4\psi u1u2\beta^2\eta1\eta3-2\eta1^2\eta2u2\psi\beta\eta3$$

$$-2\psi u2\beta^2\eta1\eta2\eta3-2\psi u2\beta\eta1\eta2^2\eta3+p\psi u2\beta\eta1^2\eta2^2+\beta^2p\eta2\eta1^2\psi u2+\beta^2p\eta2\eta1^2\psi\eta3+6u1u2\beta^2\eta1\eta2\eta3+2u1u2\beta\eta1^2\eta2\eta3+2u1u2\beta\eta1\eta2^2\eta3$$

$$-\beta^2p^2\eta2^2\eta1^2\rho\theta-\beta^2p\eta2\eta1^2\rho u2-\beta^2p\eta2\eta1^2\rho\eta3+u1pu2\beta^2\eta1^2\eta2+u1pu2\beta\eta1^2\eta2^2+u1p\beta^2\eta1^2\eta2\eta3+u1p\beta\eta1^2\eta2^2\eta3+\eta1^2\eta2^2\beta^2\rho\theta p$$

$$+2\eta1^2\eta2u2\rho\beta\eta3+p\psi u1\beta\eta1^2\eta2\eta3+6\psi u1u2\beta\eta1\eta2\eta3+cp^2\rho\beta^2\eta1^2\eta2^2\theta-cp\rho\beta^2\eta1^2\eta2^2\theta-p^2\psi\rho\beta\eta1^2\eta2^2\theta+cp\rho u2\beta^2\eta1^2\eta2+cp\rho\beta^2\eta1^2\eta2\eta3$$

$$+p\psi\rho u2\eta1^2\eta2^2\theta+p\psi\rho\beta\eta1^2\eta2^2\theta+p\psi\rho\eta1^2\eta2^2\eta3\theta-2c\rho u2\beta\eta1^2\eta2\eta3-p\psi\rho u2\beta\eta1^2\eta2-p\psi\rho\beta\eta1^2\eta2\eta3+p\psi u1u2\beta\eta1^2\eta2+\eta1^2\eta2^2u2\rho\theta p\beta$$

$$+\eta1^2\eta2^2\beta\rho\theta p\eta3+\beta^2p\eta2^2\eta1^2\eta3+\beta^2p\eta2^2\eta1^2u2+\beta^3p\eta2\eta1^2u2+\beta^3p\eta2\eta1^2\eta3-2u2\beta^3\eta1\eta2\eta3-2u2\beta^2\eta1^2\eta2\eta3-2u2\beta^2\eta1\eta2^2\eta3$$

$$-2u2\beta\eta1^2\eta2^2\eta3+u1p\beta^2\eta1^2\eta2^2+3u1u2^2\beta^2\eta1\eta2+u1u2^2\beta\eta1^2\eta2+u1u2^2\beta\eta1\eta2^2-\beta^3p\eta2\eta1^2\rho+6u1u2\beta^3\eta1\eta2+4u1u2\beta^3\eta1\eta3+4u1u2\beta^3\eta2\eta3$$

$$+2u1u2\beta^2\eta1^2\eta2+2u1u2\beta^2\eta1^2\eta3+2u1u2\beta^2\eta1\eta2^2+2u1u2\beta^2\eta2^2\eta3+6u1\beta^3\eta1\eta2\eta3+2u1\beta^2\eta1^2\eta2\eta3+2u1\beta^2\eta1\eta2^2\eta3+3u1\beta^2\eta1\eta2\eta3^2$$

$$+u1\beta\eta1^2\eta2\eta3^2+u1\beta\eta1\eta2^2\eta3^2+u1p\beta^3\eta1^2\eta2+2\eta1^2\eta2\beta^2\rho\eta3+\eta1^2\eta2\beta\rho\eta3^2+\eta1^2\eta2u2^2\rho\beta+2\eta1^2\eta2u2\rho\beta^2+\beta^4p\eta2\eta1u2+\beta^3p\eta2^2\eta1u2$$

$$+\beta^4p\eta2\eta1\eta3+\beta^3p\eta2^2\eta1\eta3+\beta^4\psi p\eta2\eta1+\beta^3\psi p\eta2^2\eta1+u1\beta^4p\eta2\eta1+u1\beta^3p\eta2^2\eta1-cp\rho u2\beta\eta1^2\eta2^2\theta-cp\rho\beta\eta1^2\eta2^2\eta3\theta+u1\psi p\eta2\beta^2\eta1u2$$

$$+u1\psi p\eta2^2\beta\eta1u2+u1\psi p\eta2\beta^2\eta1\eta3+u1\psi p\eta2^2\beta\eta1\eta3+\beta^3\psi p\eta2\eta1u2+\beta^2\psi p\eta2^2\eta1u2+\beta^3\psi p\eta2\eta1\eta3+\beta^2\psi p\eta2^2\eta1\eta3+u1\beta^3p\eta2\eta1u2$$

$$+u1\beta^2p\eta2^2\eta1u2+u1\beta^3p\eta2\eta1\eta3+u1\beta^2p\eta2^2\eta1\eta3+u1\psi p\eta2\beta^3\eta1+u1\psi p\eta2^2\beta^2\eta1\Big)\Big/\Big((p\eta2\theta+u2+\beta+\eta3)\eta1\left(p\psi\eta1\eta2\eta4+p\beta\eta1\eta2\eta4+\psi u2\beta^2\right.$$

$$+\psi u2\beta\eta1+\psi u2\beta\eta2+\psi u2\beta\eta4+\psi u2\eta1\eta2+\psi u2\eta1\eta4+\psi u2\eta2\eta4+\psi\beta^3+\psi\beta^2\eta1+\psi\beta^2\eta2+\psi\beta^2\eta3+\psi\beta^2\eta4+\psi\beta\eta1\eta2+\psi\beta\eta1\eta3+\psi\beta\eta1\eta4$$

$$+\psi\beta\eta2\eta3+\psi\beta\eta2\eta4+\psi\beta\eta3\eta4+\psi\eta1\eta2\eta3+\psi\eta1\eta3\eta4+\psi\eta2\eta3\eta4+u2\beta^3+u2\beta^2\eta1+u2\beta^2\eta2+u2\beta^2\eta4+u2\beta\eta1\eta2+u2\beta\eta1\eta4+u2\beta\eta2\eta4+\beta^4$$

$$\left.+\beta^3\eta1+\beta^3\eta2+\beta^3\eta3+\beta^3\eta4+\beta^2\eta1\eta2+\beta^2\eta1\eta3+\beta^2\eta1\eta4+\beta^2\eta2\eta3+\beta^2\eta2\eta4+\beta^2\eta3\eta4+\beta\eta1\eta2\eta3+\beta\eta1\eta3\eta4+\beta\eta2\eta3\eta4\right)\rho\Big),S$$

$$=\frac{u2\beta^2+u2\beta\eta1+u2\beta\eta2+u2\eta1\eta2+\beta^3+\beta^2\eta1+\beta^2\eta2+\beta^2\eta3+\beta\eta1\eta2+\beta\eta1\eta3+\beta\eta2\eta3+\eta1\eta2\eta3}{\rho\,\eta1\left(p\eta2\theta+u2+\beta+\eta3\right)}\Bigg\|$$

Lampiran 2. Susunan program perubahan individu terhadap waktu dengan parameter vaksinasi dengan $u_1 = 0.09$

```
clear all;
clc
alpha=input('Masukkan nilai alpha yang diinginkan = ');

% dengan nilai-nilai parameter
v=0.3; bheta=0.0121;
rho=0.5;omega=0.0015;theta=0.5;eta1=0.6;eta2=0.8;
eta3=0.025;eta4=0.6;P=0.1;u1=0.09;u2=0;
%dengan nilai awal
M(1)=0.396;
S(1)=0.98;
E(1)=0.05;
I(1)=0.05;
C(1)=0.08;
R(1)=0.7;
%dengan mengambil selang
h=0.5;
t=0:h:60;
Nt=length(t);
    w(1)=alpha;
    for i=2:Nt,
        w(i)=(1-(1+alpha)/i)*w(i-1);
    end;
    for n=2:Nt,
        x1=0; x2=0;x3=0;x4=0;x5=0;x6=0;
        for j=2:n
            x1=x1+w(j-1)*M(n+1-j);
            x2=x2+w(j-1)*S(n+1-j);
            x3=x3+w(j-1)*E(n+1-j);
            x4=x4+w(j-1)*I(n+1-j);
            x5=x5+w(j-1)*C(n+1-j);
            x6=x6+w(j-1)*R(n+1-j);
        end

        G1=(n+1)^(-alpha);
        G=G1/gamma(1-alpha);

        M(n)=G*M(1)+x1+h^alpha*(v*bheta)-
(bheta+omega)*M(n-1);
        S(n)=G*S(1)+x2+h^alpha*(1-v)*bheta+(omega*M(n-
heta*S(n-1))-rho*(I(n-1)+theta*C(n-1))*S(n-1)-
n-1))+(eta4*R(n-1));
        E(n)=G*E(1)+x3+h^alpha*(rho*(I(n-1)+theta*C(n-
n-1))-(eta1+bheta)*E(n-1);
```

```
        I(n)=G*I(1)+x4+h^alpha*(eta1*E(n-1))-
((P*+eta2)*I(n-1))-(bheta*I(n-1))-(1-P)*eta2*I(n-1);
        C(n)=G*C(1)+x5+h^alpha*(P*eta2*I(n-1))-
(bheta*C(n-1))-((u2+eta3)*C(n-1));
        R(n)=G*R(1)+x6+h^alpha*(u2+eta3)*C(n-1)+(1-
P)*eta2*I(n-1)+(u1*S(n-1))-(eta4+bheta)*R(n-1);
    end
figure (1)
plot(t,M,'b',t,S,'g',t,E,'m',t,I,'c',t,C,'k',t,R,'y','L
ineWidth',2.5);
xlabel('Time(t)');
ylabel('Population')
title('\bf\it{Grunwald-Letnikov Method}');
legend('M(Immunized)','S(Susceptible)','E(Exposed)','I(
Infection)','C(Carrier)','R(Recovered)',1);
grid on;
```

Lampiran 3. Susunan program perubahan individu terhadap waktu dengan parameter pengobatan dengan $u_2 = 0.02$

```
clear all;
clc
alpha=input('Masukkan nilai alpha yang diinginkan = ');

% dengan nilai-nilai parameter
v=0.3; bheta=0.0121;
rho=0.5;omega=0.0015;theta=0.5;eta1=0.6;eta2=0.8;
eta3=0.025;eta4=0.6;P=0.1;u1=0;u2=0.02;
%dengan nilai awal
M(1)=0.396;
S(1)=0.98;
E(1)=0.05;
I(1)=0.05;
C(1)=0.08;
R(1)=0.7;
%dengan mengambil selang
h=0.5;
t=0:h:60;
Nt=length(t);
    w(1)=alpha;
    for i=2:Nt,
        w(i)=(1-(1+alpha)/i)*w(i-1);
    end;
    for n=2:Nt,
        x1=0; x2=0;x3=0;x4=0;x5=0;x6=0;
        for j=2:n
            x1=x1+w(j-1)*M(n+1-j);
            x2=x2+w(j-1)*S(n+1-j);
            x3=x3+w(j-1)*E(n+1-j);
            x4=x4+w(j-1)*I(n+1-j);
            x5=x5+w(j-1)*C(n+1-j);
            x6=x6+w(j-1)*R(n+1-j);
        end

        G1=(n+1)^(-alpha);
        G=G1/gamma(1-alpha);

        M(n)=G*M(1)+x1+h^alpha*(v*bheta)-
(bheta+omega)*M(n-1);
        S(n)=G*S(1)+x2+h^alpha*(1-v)*bheta+(omega*M(n-
heta*S(n-1))-rho*(I(n-1)+theta*C(n-1))*S(n-1)-
n-1))+(eta4*R(n-1));
        E(n)=G*E(1)+x3+h^alpha*(rho*(I(n-1)+theta*C(n-
n-1))-(eta1+bheta)*E(n-1);
```

```
        I(n)=G*I(1)+x4+h^alpha*(eta1*E(n-1))-
((P*+eta2)*I(n-1))-(bheta*I(n-1))-(1-P)*eta2*I(n-1);
        C(n)=G*C(1)+x5+h^alpha*(P*eta2*I(n-1))-
(bheta*C(n-1))-((u2+eta3)*C(n-1));
        R(n)=G*R(1)+x6+h^alpha*(u2+eta3)*C(n-1)+(1-
P)*eta2*I(n-1)+(u1*S(n-1))-(eta4+bheta)*R(n-1);
    end
figure (1)
plot(t,M,'b',t,S,'g',t,E,'m',t,I,'c',t,C,'k',t,R,'y','L
ineWidth',2.5);
xlabel('Time(t)');
ylabel('Population')
title('\bf\it{Grunwald-Letnikov Method}');
legend('M(Immunized)','S(Susceptible)','E(Exposed)','I(
Infection)','C(Carrier)','R(Recovered)',1);
grid on;
```

Lampiran 4. Susunan program perubahan individu terhadap waktu dengan parameter vaksinasi dan pengobatan dengan $u_1 = 0.01$ dan $u_2 = 0.01$.

```
clear all;
clc
alpha=input('Masukkan nilai alpha yang diinginkan = ');

% dengan nilai-nilai parameter
v=0.3; bheta=0.0121;
rho=0.5;omega=0.0015;theta=0.5;eta1=0.6;eta2=0.8;
eta3=0.025;eta4=0.6;P=0.1;u1=0.09;u2=0.02;
%dengan nilai awal
M(1)=0.396;
S(1)=0.98;
E(1)=0.05;
I(1)=0.05;
C(1)=0.08;
R(1)=0.7;
%dengan mengambil selang
h=0.5;
t=0:h:60;
Nt=length(t);
    w(1)=alpha;
    for i=2:Nt,
        w(i)=(1-(1+alpha)/i)*w(i-1);
    end;
    for n=2:Nt,
        x1=0; x2=0;x3=0;x4=0;x5=0;x6=0;
        for j=2:n
            x1=x1+w(j-1)*M(n+1-j);
            x2=x2+w(j-1)*S(n+1-j);
            x3=x3+w(j-1)*E(n+1-j);
            x4=x4+w(j-1)*I(n+1-j);
            x5=x5+w(j-1)*C(n+1-j);
            x6=x6+w(j-1)*R(n+1-j);
        end

        G1=(n+1)^(-alpha);
        G=G1/gamma(1-alpha);

        M(n)=G*M(1)+x1+h^alpha*(v*bheta)-
+omega)*M(n-1);
        S(n)=G*S(1)+x2+h^alpha*(1-v)*bheta+(omega*M(n-
heta*S(n-1))-rho*(I(n-1)+theta*C(n-1))*S(n-1)-
n-1))+(eta4*R(n-1));
```

```
        E(n)=G*E(1)+x3+h^alpha*(rho*(I(n-1)+theta*C(n-
1))*S(n-1))-(eta1+bheta)*E(n-1);
        I(n)=G*I(1)+x4+h^alpha*(eta1*E(n-1))-
((P*+eta2)*I(n-1))-(bheta*I(n-1))-(1-P)*eta2*I(n-1);
        C(n)=G*C(1)+x5+h^alpha*(P*eta2*I(n-1))-
(bheta*C(n-1))-((u2+eta3)*C(n-1));
        R(n)=G*R(1)+x6+h^alpha*(u2+eta3)*C(n-1)+(1-
P)*eta2*I(n-1)+(u1*S(n-1))-(eta4+bheta)*R(n-1);
    end
figure (1)
plot(t,M,'b',t,S,'g',t,E,'m',t,I,'c',t,C,'k',t,R,'y','L
ineWidth',2.5);
xlabel('Time(t)');
ylabel('Population')
title('\bf\it{Grunwald-Letnikov Method}');
legend('M(Immunized)','S(Susceptible)','E(Exposed)','I(
Infection)','C(Carrier)','R(Recovered)',1);
grid on;
```


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