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On Total Vertex Irregularity Strength of Hexagonal Cluster Graphs

Nurdin Hinding[1], Hye Kyung Kim[2], Nurtiti Sunusi[3], and Riskawaty Mise[4]

[1]Department of Mathematics, Faculty of Mathematics and Natural Sciences, University of Hasanuddin, 40133, Indonesia.
[2]Department of Mathematics Education, Daegu Catholic University, Gyeongsan 38430, Republic of Korea.
[3]Department of Mathematics, Faculty of Mathematics and Natural Sciences, University of Hasanuddin, 40133, Indonesia.
[4]Department of Mathematics, Maros Muhammadiyah University, Indonesia

nurdin1701@unhas.ac.id, hkkim@cu.ac.kr, ntitisanusi@gmail.com, riskawaty@gmail.com

Abstract

For a simple graph G with a vertex set $V(G)$ and an edge set $E(G)$, a labeling $f: V(G) \cup E(G) \rightarrow \{1,2,\cdots,k\}$ is called a vertex irregular total $k-$labeling of G if for any two different vertices x and y in $V(G)$ we have $wt(x) \neq wt(y)$ where $wt(x) = f(x) + \sum_{u \in V(G)} f(xu)$. The smallest positive integer k such that G has a vertex irregular total $k-$labeling is called the total vertex irregularity strength of G, denoted by $tvs(G)$. In [2], Baca *et al*. studied the lower bound of $tvs(G)$ for any graph G.

In this paper, we determined the exact value of the total vertex irregularity strength of the hexagonal cluster graph on n cluster for $n \geq 2$. Moreover, we show that the total vertex irregularity strength of the hexagonal cluster graph on n cluster is $\left\lceil \frac{3n^2+1}{2} \right\rceil$.

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