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LAMPIRAN

Lampiran 1. Data

kabupaten/kota	Y1	Y2	X1	X2	X3	X4	X5
Kepulauan Selayar	5	8	82.87	75.61	72.1	66.87	14
Bulukumba	3	51	76.16	75.41	101.6	56.51	20
Bantaeng	2	6	92.72	88.34	39.6	29.99	13
Jeneponto	11	65	86.01	67.60	108.4	26.94	19
Takalar	7	25	95.68	70.20	91.3	59.03	15
Gowa	14	43	94.32	80.94	72.4	70.63	26
Sinjai	7	47	95.05	74.31	88.8	62.32	16
Maros	3	20	87.25	76.70	84.5	92.74	14
Pangkep	14	45	85.31	75.24	65.7	51.32	23
Barru	7	20	91.96	94.69	94.6	58.6	12
Bone	8	63	93.94	68.74	80.4	72.54	38
Soppeng	4	26	81.51	75.20	65.7	62.15	17
Wajo	4	27	86.29	70.41	72.4	60.55	23
Sidrap	4	13	79.28	78.54	117.3	39.14	14
Pinrang	6	27	93.94	71.36	86.5	72.5	17
Enrekang	1	27	55.84	47.65	76.5	63.43	14
Luwu	5	41	81	67.78	80.5	52.18	22
Tana Toraja	1	16	86.8	77.01	81.2	24.86	21
Luwu Utara	8	20	81.28	67.70	65.2	32.53	14
Luwu Timur	8	34	97.05	78.69	84	54.81	17
Toraja Utara	7	25	66.29	74.82	64.1	35.82	26
Makassar	10	33	92.71	70.09	45.2	94.16	46
Pare Pare	4	18	86.63	77.04	74.8	61.72	7
Palopo	1	14	95.5	72.26	82.4	76.38	12

Lampiran 2. Uraian Rumus

Fungsi kepadatan peluang bersama Y_1 dan Y_2 dapat dituliskan sebagai berikut:

$$f\left(y_j|\boldsymbol{\beta}_j;\tau;j=1{,}2\right)$$

$$=e^{\frac{1}{\tau}}K_s(z)\left(\frac{2}{\pi\tau}\right)^{\frac{1}{2}}\left(1+2\tau\sum_{j=1}^{2}\mu_j\right)^{-\frac{\left(2\sum_{j=1}^{2}y_j-1\right)}{4}}\prod_{j=1}^{2}\frac{\mu_j{}^{y_j}}{y_j\,!}$$

$$=e^{\frac{1}{\tau}}K_s(z)\left(\frac{2}{\pi\tau}\right)^{\frac{1}{2}}\left(1+2\tau\sum_{j=1}^{2}e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_j}\right)^{-\frac{\left(2\sum_{j=1}^{2}y_j-1\right)}{4}}\prod_{j=1}^{2}\frac{e^{X_i^T\boldsymbol{\beta}_j y_j}}{y_j\,!} \tag{1}$$

dengan $z=\frac{1}{\tau}\left(1+2\tau\sum_{j=1}^{2}\mu_j\right)^{\frac{1}{2}}$ dan $s=y_1+y_2-\frac{1}{2}$ sehingga

$$K_s(z)=K_{y_1+y_2-\frac{1}{2}}\frac{1}{\tau}\left(1+2\tau\sum_{j=1}^{2}\mu_j\right)^{\frac{1}{2}}$$

Fungsi *likelihood* untuk populasi dari Persamaan (1) yaitu:

$$L\left(\boldsymbol{\beta}_j;\tau;j=1{,}2\right)=L(\boldsymbol{\theta})$$

$$=\prod_{i=1}^{n}\left(e^{\frac{1}{\tau}}K_s(z)\left(\frac{2}{\pi\tau}\right)^{\frac{1}{2}}\left(1+2\tau\sum_{j=1}^{2}e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_j}\right)^{\frac{\left(2\sum_{j=1}^{2}y_j-1\right)}{4}}\prod_{j=1}^{2}\frac{e^{X_i^T\boldsymbol{\beta}_j y_{ij}}}{y_{ij}\,!}\right)$$

dan fungsi ln *likelihood* sebagai berikut:

$$l\left(\boldsymbol{\beta}_j;\tau;j=1{,}2\right)=$$

$$=\sum_{i=1}^{n}ln\left(e^{\frac{1}{\tau}}K_s(z)\left(\frac{2}{\pi\tau}\right)^{\frac{1}{2}}\left(1+2\tau\sum_{j=1}^{2}e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_j}\right)^{-\frac{\left(2\sum_{j=1}^{2}y_{ij}-1\right)}{4}}\prod_{j=1}^{2}\frac{e^{X_i^T\boldsymbol{\beta}_j y_{ij}}}{y_{ij}\,!}\right)$$

$$=\frac{n}{\tau}+\sum_{i=1}^{n}ln\,K_s(z)+\frac{n}{2}\ln\left(\frac{2}{\pi\tau}\right)-\sum_{i=1}^{n}\frac{\left(2\sum_{j=1}^{2}y_{ij}-1\right)}{4}\ln\left(1+2\tau\sum_{j=1}^{2}e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_j}\right)+\sum_{i=1}^{n}\sum_{j=1}^{2}y_{ij}\,\boldsymbol{X}_i^T\boldsymbol{\beta}_j-\ln\left(\sum_{i=1}^{n}\sum_{j=1}^{2}y_{ij}!\right)$$

$$=\frac{n}{\tau}-\frac{n}{2}\ln(\tau)+\frac{n}{2}ln\left(\frac{2}{\pi}\right)+\sum_{i=1}^{n}ln\left[K_{s_i}(z_i)\right]+-\sum_{i=1}^{n}\frac{\left(2\sum_{j=1}^{2}y_{ij}-1\right)}{4}ln\left(1+2\tau\sum_{j=1}^{2}e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_j}\right)+\sum_{i=1}^{n}\sum_{j=1}^{2}y_{ij}\boldsymbol{X}_i^T\boldsymbol{\beta}_j-\ln\left(\sum_{i=1}^{n}\sum_{j=1}^{2}y_{ij}!\right)$$

Selanjutnya, fungsi ln *likelihood* diturunkan terhadap $\boldsymbol{\beta}_1$ yaitu:

$$\frac{\partial l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_1} = \frac{\partial}{\partial \boldsymbol{\beta}_1}\left\{\frac{n}{\tau} - \frac{n}{2}\ln(\tau) + \frac{n}{2} ln\left(\frac{2}{\pi}\right) + \sum_{i=1}^{n} ln\left[K_{s_i}(z_i)\right] + \right.$$

$$- \sum_{i=1}^{n} \frac{\left(2\sum_{j=1}^{2} y_{ij} - 1\right)}{4} ln\left(1 + 2\tau \sum_{j=1}^{2} e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_j}\right) + \sum_{i=1}^{n}\sum_{j=1}^{2} y_{ij} \boldsymbol{X}_i^T \boldsymbol{\beta}_j -$$

$$\left. \ln\left(\sum_{i=1}^{n}\sum_{j=1}^{2} y_{ij}!\right) \right\}$$

$$= \sum_{i=1}^{n} \frac{\partial \ln K_{s_i}(z_i)}{\partial \boldsymbol{\beta}_1} + \sum_{i=1}^{n}\left(-\frac{\left(2\sum_{j=1}^{2} y_{ij} - 1\right)}{4}\right) \frac{1}{\left(1 + 2\tau \sum_{j=1}^{2} e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_j}\right)} 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} \boldsymbol{X}_i^T +$$

$$\sum_{i=1}^{n} y_{i1} \boldsymbol{X}_i^T$$

$$= \sum_{i=1}^{n} \frac{1}{K_{s_i}(z_i)} \frac{\partial K_{s_i}(z_i)}{\partial \boldsymbol{\beta}_1} + \sum_{i=1}^{n}\left(-\frac{\left(2\sum_{j=1}^{2} y_{ij} - 1\right)}{4}\right) \frac{1}{\left(1 + 2\tau \sum_{j=1}^{2} e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_j}\right)} 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} \boldsymbol{X}_i^T +$$

$$\sum_{i=1}^{n} y_{i1} \boldsymbol{X}_i^T$$

$$= \sum_{i=1}^{n} \frac{1}{K_{s_i}(z_i)} \frac{\partial K_{s_i}(z_i)}{\partial \boldsymbol{\beta}_1} - \sum_{i=1}^{n} \frac{\left(2\sum_{j=1}^{2} y_j - 1\right)\tau \mu_{i1} \boldsymbol{X}_i^T}{2\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)} + \sum_{i=1}^{n} y_{i1} \boldsymbol{X}_i^T \qquad (2)$$

Berdasarkan tabel integral, diketahui bahwa

$K_{s+1}(z) = K_{s+1}(z) + \frac{2s}{z} K_s(z)$

$z \frac{\partial K_s(z)}{\partial z} = -z K_{s+1}(z) + s K_s(z)$

sehingga

$$\frac{\partial K_{\sum_{j=1}^{2} y_j - \frac{1}{2}}(z_i)}{\partial (z_i)} = \frac{-z_i K_{\sum_{j=1}^{2} y_j - \frac{1}{2}}(z_i) + \left(\sum_{j=1}^{2} y_j - \frac{1}{2}\right) K_{\sum_{j=1}^{2} y_j - \frac{1}{2}}(z_i)}{z_i}$$

Diketahui bahwa $z = \frac{1}{\tau}\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}} = \frac{1}{\tau}\sqrt{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}}$, maka

$$\frac{\partial K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}{\partial (z_i)} = \frac{-\frac{1}{\tau}\sqrt{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}} K_{\sum_{j=1}^{2} y_j - \frac{1}{2}}(z_i) + \left(\sum_{j=1}^{2} y_{ij} - \frac{1}{2}\right) K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}{\frac{1}{\tau}\sqrt{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}}}$$

$$= -K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i) + \frac{\tau\left(\sum_{j=1}^{2} y_{ij} - \frac{1}{2}\right) K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}{\sqrt{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}}} \qquad (3)$$

Berdasarkan Persamaan (2), maka turunan ln $K_{s_i}(z_i)$ terhadap $\boldsymbol{\beta}_1$ yaitu:

$$\frac{\partial \ln K_{s_i}(z_i)}{\partial \boldsymbol{\beta}_1} = \frac{1}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} . \frac{\partial K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}{\partial \boldsymbol{\beta}_1}$$

$$= \frac{1}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} \cdot \frac{\partial K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}{\partial (z_i)} \cdot \frac{\partial (z_i)}{\partial \boldsymbol{\beta}_1}$$

$$= \frac{1}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} \cdot \frac{\partial K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}{\partial (z_i)} \cdot \frac{\partial \left(\frac{1}{\tau}\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}\right)}{\partial \boldsymbol{\beta}_1}$$

$$= \frac{1}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} \left[-K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i) + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right) K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \right] \left[\frac{2\tau\mu_{i1}\boldsymbol{X}_i^T}{2\tau\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} \right]$$

$$= \left[-\frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \right] \frac{\mu_{i1}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} \tag{4}$$

Diketahui $M(y_i) = \frac{1}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \cdot \frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}$, maka berdasarkan Persamaan (4) yaitu:

$$\frac{\partial \ln K_{s_i}(z_i)}{\partial \boldsymbol{\beta}_1} = \left[-\frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \right] \frac{\mu_{i1}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}$$

$$= \left[-\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}} M(y_i) + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \right] \frac{\mu_{i1}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} \tag{5}$$

Substitusi Persamaan (5) ke Persamaan (2), sehingga

$$\frac{\partial l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_1} = \sum_{i=1}^{n} \frac{\partial \ln K_{s_i}(z_i)}{\partial \boldsymbol{\beta}_1} - \sum_{i=1}^{n} \frac{\left(2\sum_{j=1}^{2} y_{ij}-1\right)\tau\mu_{i1}\boldsymbol{X}_i^T}{2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} + \sum_{i=1}^{n} y_{i1}\boldsymbol{X}_i^T$$

$$= \sum_{i=1}^{n} \left\{ \left[-\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}} M(y_i) + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \right] \frac{\mu_{i1}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} + \right.$$

$$\left. - \frac{\left(2\sum_{j=1}^{2} y_{ij}-1\right)\tau\mu_{i1}\boldsymbol{X}_i^T}{2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} + y_{i1}\boldsymbol{X}_i^T \right\}$$

$$= \sum_{i=1}^{n} \left[-\frac{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}} M(y_i)\mu_{i1}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)\mu_{i1}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} - \frac{\left(2\sum_{j=1}^{2} y_{ij}-1\right)\tau\mu_{i1}\boldsymbol{X}_i^T}{2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} + \right.$$

$$\left. y_{i1}\boldsymbol{X}_i^T \right]$$

$$= \sum_{i=1}^{n} \left[-M(y_i)\mu_{i1}\boldsymbol{X}_i^T + y_{i1}\boldsymbol{X}_i^T \right]$$

$$= \sum_{i=1}^{n} [y_{i1} - M(y_i)\mu_{i1}] \boldsymbol{X}_i^T \qquad (6)$$

Turunan kedua fungsi ln *likelihood* terhadap $\boldsymbol{\beta}_1$ dan $\boldsymbol{\beta}_1^T$ yaitu:

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_1^T \partial \boldsymbol{\beta}_1} = \frac{\partial \sum_{i=1}^{n} [y_{i1} - M(\boldsymbol{y}_i)\mu_{i1}] \boldsymbol{X}_i^T}{\partial \boldsymbol{\beta}_1^T}$$

$$= \sum_{i=1}^{n} \left[\frac{\partial (y_{i1} \boldsymbol{X}_i^T)}{\partial \boldsymbol{\beta}_1^T} - \frac{\partial (M(y_i)\mu_{i1} \boldsymbol{X}_i^T)}{\partial \boldsymbol{\beta}_1^T} \right]$$

$$= \sum_{i=1}^{n} \left[- \frac{\partial (M(y_i)\mu_{i1} \boldsymbol{X}_i^T)}{\partial \boldsymbol{\beta}_1^T} \right] \qquad (7)$$

Untuk mendapatkan turunan dari $M(y_i)\mu_{i1}\boldsymbol{X}_i^T$ terhadap $\boldsymbol{\beta}_1^T$ sebagai berikut:

Misalkan:

$u = M(y_i)$ dan $v = \mu_{i1}\boldsymbol{X}_i^T = e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} \boldsymbol{X}_i^T$

$$u' = M'^{(y_i)} = \frac{\partial M(y_i)}{\partial \boldsymbol{\beta}_1^T} = \frac{\partial}{\partial \boldsymbol{\beta}_1^T} \left[\frac{1}{\sqrt{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}}} \cdot \frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} \right]$$

$$= \left[\frac{\partial \left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_1^T} \right] \frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} + \frac{1}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}} \left[\frac{\partial \left(\frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} \right)}{\partial \boldsymbol{\beta}_1^T} \right] \qquad (7)$$

Turunan dari $\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}$ terhadap $\boldsymbol{\beta}_1^T$ sebagai berikut:

$$\frac{\partial \left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_1^T} = \frac{\partial \left(1 + 2\tau \sum_{j=1}^{2} e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_j}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_1^T} = \frac{\partial \left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_1^T}$$

$$= -\frac{1}{2} \left(2\tau \boldsymbol{X}_i e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1}\right) \left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{-\frac{3}{2}}$$

$$= -\tau \mu_{i1} \boldsymbol{X}_i \left(1 + 2\tau \sum_{j=1}^{2} e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_j}\right)^{-\frac{3}{2}} \qquad (9)$$

Turunan pertama $\frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}$ terhadap $\boldsymbol{\beta}_1^T$ sebagai berikut:

$$\frac{\partial \left(\frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} \right)}{\partial \boldsymbol{\beta}_1^T} = \frac{\left[\frac{\partial \left(K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i) \right)}{\partial \boldsymbol{\beta}_1^T} \right] K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i) - K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i) \left[\frac{\partial \left(K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i) \right)}{\partial \boldsymbol{\beta}_1^T} \right]}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}$$

$$= \frac{1}{\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)^2}\left\{\left[\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial \boldsymbol{\beta}_1^T}\right]K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i) + \right.$$

$$\left. -K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\left[\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial \boldsymbol{\beta}_1^T}\right]\right\} \tag{10}$$

Persamaan (10) dapat diselesaikan dengan menggunakan aturan rantai sehingga

$$\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial \boldsymbol{\beta}_1^T} = \frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial (z_i)} \cdot \frac{\partial (z_i)}{\partial \boldsymbol{\beta}_1^T} \tag{11}$$

dengan

$$\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial (z_i)} = \frac{-zK_{\sum_{j=1}^{2} y_{ij}+\frac{3}{2}}(z_i)+\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i}$$

$$= -K_{\sum_{j=1}^{2} y_{ij}+\frac{3}{2}}(z_i) + \frac{\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i} \tag{12}$$

dan

$$K_{\sum_{j=1}^{2} y_{ij}+\frac{3}{2}}(z_i) = K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i) + \frac{2\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i} \tag{13}$$

Substitusi Persamaan (13) ke Persamaan (12), sehingga

$$\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial (z_i)} = -K_{\sum_{j=1}^{2} y_{ij}+\frac{3}{2}}(z_i) + \frac{\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i}$$

$$= -\left[K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i) + \frac{2\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)}{z_i}K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right] + \frac{\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i}$$

$$= -K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i) - \frac{2\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i} + \frac{\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i}$$

$$= -K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i) - \frac{\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i} \tag{14}$$

Turunan z_i terhadap $\boldsymbol{\beta}_1^T$ sebagai berikut:

$$\frac{\partial (z_i)}{\partial \boldsymbol{\beta}_1^T} = \frac{1}{\tau}\,\frac{\partial\left(1+2\tau e^{X_i^T\beta_1}+2\tau e^{X_i^T\beta_2}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_1^T}$$

$$= \frac{1}{\tau}\,\frac{1}{2}\left(1 + 2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\right)^{-\frac{1}{2}}\left(2\tau\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}\right)$$

$$= \frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}}{\sqrt{1+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}}}$$

$$= \frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \qquad (15)$$

Substitusi Persamaan (14) dan (15) ke Persamaan (11), sehingga

$$\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial\boldsymbol{\beta}_1^T} =$$

$$= \frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial(z_i)}\cdot\frac{\partial(z_i)}{\partial\boldsymbol{\beta}_1^T}$$

$$= \left[-K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i) - \frac{\left(\sum_{j=1}^{2}y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{z_i}\right]\frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \qquad (16)$$

Persamaan (10) dapat diselesaikan dengan menggunakan aturan rantai sehingga

$$\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial\boldsymbol{\beta}_1^T} = \frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial(z_i)}\cdot\frac{\partial(z_i)}{\partial\boldsymbol{\beta}_1^T} \qquad (17)$$

dengan

$$\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial(z_i)} = \frac{-zK_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)+\left(\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}{z_i}$$

$$= -K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i) + \frac{\left(\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}{z_i} \qquad (18)$$

Substitusi Persamaan (18) dan (15) ke Persamaan (17), sehingga

$$\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial\boldsymbol{\beta}_1^T} =$$

$$= \frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial(z_i)}\cdot\frac{\partial(z_i)}{\partial\boldsymbol{\beta}_1^T}$$

$$= \left[-K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i) + \frac{\left(\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}{z_i}\right]\frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \qquad (19)$$

Substitusi Persamaan (16) dan (19) ke Persamaan (10), sehingga

$$\frac{\partial\left(\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right)}{\partial\boldsymbol{\beta}_1^T}=\frac{1}{\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)^2}\left\{\left[\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial\boldsymbol{\beta}_1^T}\right]K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)+\right.$$

$$\left.-K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\left[\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial\boldsymbol{\beta}_1^T}\right]\right\}$$

$$=\frac{1}{\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)^2}\left\{\frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\left[-K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)+\right.\right.$$

$$\left.-\frac{\left(\sum_{j=1}^{2}y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{z_i}\right]-$$

$$\frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\left[-K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)+\right.$$

$$\left.\left.+\frac{\left(\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}{z_i}\right]\right\}$$

$$=\frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}}{\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)^2\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\left\{-\left[K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right]^2+\right.$$

$$-\frac{\left(\sum_{j=1}^{2}y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}{z_i}+\left[K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\right]^2-$$

$$\left.-\frac{\left(\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{z_i}\right\}$$

$$=\frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\left\{-1-\frac{\left(\sum_{j=1}^{2}y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{z_iK_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}+\right.$$

$$\left.+\left[\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right]^2-\frac{\left(\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{z_iK_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right\}$$

$$= \frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \left\{ -1 - \frac{\tau\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right) M(y_i)\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} + \right.$$

$$\left. M^2(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right) - \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right) M(y_i)\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \right\}$$

$$= -\frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \left\{ 1 + \tau\left(\sum_{j=1}^{2} y_{ij} + \frac{1}{2}\right) M(y_i) - M^2(y_i)\left(1 + \right.\right.$$

$$\left.\left. 2\tau\sum_{j=1}^{2}\mu_{ij}\right) + \tau\left(\sum_{j=1}^{2} y_{ij} - \frac{1}{2}\right) M(y_i) \right\}$$

$$= -\frac{\boldsymbol{X}_i \mu_{i1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \left\{ 1 + \tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij}\right) - M^2(y_i)\left(1 + \right.\right.$$

$$\left.\left. 2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\} \tag{20}$$

Lalu, substitusi Persamaan (9) dan (20) ke Persamaan (8), sehingga

$$\boldsymbol{u}_i' = \left[\frac{\partial\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}{\partial\boldsymbol{\beta}_1^T}\right] \frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} + \frac{1}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} \left[\frac{\partial\left(\frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}\right)}{\partial\boldsymbol{\beta}_1^T}\right]$$

$$= -\tau\mu_{i1}\boldsymbol{X}_i\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{-\frac{3}{2}} M(\boldsymbol{y}_i)\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}} + \frac{1}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} -$$

$$\left[-\frac{\boldsymbol{X}_i \mu_{i1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \left\{ 1 + \tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij}\right) - M^2(\boldsymbol{y}_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}\right]$$

$$= -\frac{\tau\mu_{i1}\boldsymbol{X}_i M(y_i)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} - \frac{\boldsymbol{X}_i\mu_{i1}}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} \left\{ 1 + \tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij}\right) - M^2(y_i)\left(1 + \right.\right.$$

$$\left.\left. 2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}$$

$$= -\frac{\tau\mu_{i1}\boldsymbol{X}_i M(y_i)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} - \frac{(\boldsymbol{X}_i\mu_{i1}) + \tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij}\right) - M^2(y_i)\boldsymbol{X}_i\mu_{i1}\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}}$$

$$= -\frac{\mu_{i1}\boldsymbol{X}_i}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} \left\{ \tau M(y_i) + 1 + \tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij} + 1\right) - M^2(y_i)\right\}$$

$$= -\frac{\mu_{i1}\boldsymbol{X}_i\left\{1+\tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij}+1\right)\right\}}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} - M^2(y_i)\boldsymbol{X}_i\mu_{i1} \tag{21}$$

Turunan pertama dari $e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}\boldsymbol{X}_i^T$ terhadap $\boldsymbol{\beta}_1^T$ sebagai berikut:

$$\boldsymbol{v}_i' = \frac{\partial\left(e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}\boldsymbol{X}_i^T\right)}{\partial\boldsymbol{\beta}_1^T} = \boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}\boldsymbol{X}_i^T = \mu_{i1}\boldsymbol{X}_i\boldsymbol{X}_i^T$$

Turunan pertama dari $M(y_i)\mu_{i1}\boldsymbol{X}_i^T$ terhadap $\boldsymbol{\beta}_1^T$ sebagai berikut:

$$\frac{\partial\left(M(\boldsymbol{y}_i)\mu_{i1}\boldsymbol{X}_i^T\right)}{\partial\boldsymbol{\beta}_1^T} = \boldsymbol{u}_i'\boldsymbol{v}_i + u_i\boldsymbol{v}_i'$$

$$= \left\{-\frac{\mu_{i1}\boldsymbol{X}_i\left\{1+\tau M(y_i)\left(2\sum_{j=1}^{2}y_{ij}+1\right)\right\}}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} + \mu_{i1}\boldsymbol{X}_i M^2(y_i)\right\}\mu_{i1}\boldsymbol{X}_i^T + M(y_i)\mu_{i1}\boldsymbol{X}_i\boldsymbol{X}_i^T$$

$$= \mu_{i1}\left\{-\frac{\mu_{i1}\left\{1+\tau M(y_i)\left(2\sum_{j=1}^{2}y_{ij}+1\right)\right\}}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} + \mu_{i1}M^2(y_i)\right\}\boldsymbol{X}_i\boldsymbol{X}_i^T \qquad (22)$$

Substitusi Persamaan (22) ke Persamaan (7)

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\boldsymbol{\beta}_1^T\partial\boldsymbol{\beta}_1} = \sum_{i=1}^{n}\left[\frac{\partial\left(y_{i1}\boldsymbol{X}_i^T\right)}{\partial\boldsymbol{\beta}_1^T} - \frac{\partial\left(M(y_i)\mu_{i1}\boldsymbol{X}_i^T\right)}{\partial\boldsymbol{\beta}_1^T}\right]$$

$$= \sum_{i=1}^{n}\left[0 - \left[-\frac{\mu_{i1}\left\{1+\tau M(y_i)\left(2\sum_{j=1}^{2}y_{ij}+1\right)\right\}}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} + \mu_{i1}\boldsymbol{X}_i M^2(y_i) + M(y_i)\boldsymbol{X}_i\right]\mu_{i1}\boldsymbol{X}_i^T\right]$$

$$= -\sum_{i=1}^{n}\mu_{i1}\left[M(y_i) - \frac{\mu_{i1}\left\{1+\tau M(y_i)\left(2\sum_{j=1}^{2}y_{ij}+1\right)\right\}}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} + \mu_{i1}\boldsymbol{X}_i M^2(y_i)\right]\boldsymbol{X}_i\boldsymbol{X}_i^T \qquad (23)$$

Turunan pertama fungsi ln *likelihood* terhadap $\boldsymbol{\beta}_2$ yaitu:

$$\frac{\partial l(\boldsymbol{\theta})}{\partial\boldsymbol{\beta}_2} = \sum_{i=1}^{n}\frac{1}{K_{s_i}(z_i)}\frac{\partial K_{s_i}(z_i)}{\partial\boldsymbol{\beta}_2} + \sum_{i=1}^{n}\left(-\frac{\left(2\sum_{j=1}^{2}y_{ij}-1\right)}{4}\right)\frac{1}{\left(1+2\tau\sum_{j=1}^{2}e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_j}\right)}2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\boldsymbol{X}_i^T + \sum_{i=1}^{n}y_{i2}\boldsymbol{X}_i^T$$

$$= \sum_{i=1}^{n}\frac{\partial\ln K_{s_i}(z_i)}{\partial\boldsymbol{\beta}_2} - \sum_{i=1}^{n}\frac{\left(2\sum_{j=1}^{2}y_j-1\right)\tau\mu_{i2}\boldsymbol{X}_i^T}{2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} + \sum_{i=1}^{n}y_{i2}\boldsymbol{X}_i^T \qquad (24)$$

Turunan pertama ln $K_{s_i}(z_i)$ terhadap $\boldsymbol{\beta}_2$ yaitu:

$$\frac{\partial\ln K_{s_i}(z_i)}{\partial\boldsymbol{\beta}_2} = \frac{1}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}.\frac{\partial K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}{\partial\boldsymbol{\beta}_2}$$

$$= \frac{1}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}.\frac{\partial K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}{\partial(z_i)}.\frac{\partial(z_i)}{\partial\boldsymbol{\beta}_2}$$

$$= \frac{1}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} \cdot \frac{\partial K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}{\partial(z_i)} \cdot \frac{\partial\left(\frac{1}{\tau}\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}\right)}{\partial\boldsymbol{\beta}_2}$$

$$= \frac{1}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} \Bigg[-K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i) +$$

$$\frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\Bigg]\left[\frac{2\tau\mu_{i2}\boldsymbol{X}_i^T}{2\tau\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\right]$$

$$= \left[-\frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\right]\frac{\mu_{i2}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}$$

Berdasarkan Persamaan (2.4) maka diketahui $M(y_i) = \frac{1}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \cdot \frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}$,

maka

$$\frac{\partial \ln K_{s_i}(z_i)}{\partial\boldsymbol{\beta}_2} = \left[-\frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)} + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\right]\frac{\mu_{i2}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}$$

$$= \left[-\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}M(y_i) + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\right]\frac{\mu_{i2}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}$$

$$= -\mu_{i2}\boldsymbol{X}_i^T M(y_i) + \frac{\tau\mu_{i2}\boldsymbol{X}_i^T\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} \qquad (25)$$

Substitusi Persamaan (25) ke Persamaan (24), sehingga

$$\frac{\partial l(\boldsymbol{\theta})}{\partial\boldsymbol{\beta}_2} = \sum_{i=1}^{n}\frac{\partial \ln K_{s_i}(z_i)}{\partial\boldsymbol{\beta}_2} - \sum_{i=1}^{n}\frac{\left(2\sum_{j=1}^{2} y_{ij}-1\right)\tau\mu_{i2}\boldsymbol{X}_i^T}{2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} + \sum_{i=1}^{n} y_{i2}\boldsymbol{X}_i^T$$

$$= \sum_{i=1}^{n}\left\{-\mu_{i2}\boldsymbol{X}_i^T M(y_i) + \frac{\tau\mu_{i2}\boldsymbol{X}_i^T\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} - \frac{\left(2\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)\tau\mu_{i2}\boldsymbol{X}_i^T}{2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} + y_{i2}\boldsymbol{X}_i^T\right\}$$

$$= \sum_{i=1}^{n}\left[-\mu_{i2}\boldsymbol{X}_i^T M(y_i) + \frac{\tau\mu_{i2}\boldsymbol{X}_i^T\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} - \frac{\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)\tau\mu_{i2}\boldsymbol{X}_i^T}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} + y_{i2}\boldsymbol{X}_i^T\right]$$

$$= \sum_{i=1}^{n}\left[-M(y_i)\mu_{i2}\boldsymbol{X}_i^T + y_{i2}\boldsymbol{X}_i^T\right]$$

$$= \sum_{i=1}^{n}\left[y_{i2} - M(y_i)\mu_{i2}\right]\boldsymbol{X}_i^T \qquad (26)$$

Turunan kedua fungsi ln *likelihood* terhadap $\boldsymbol{\beta}_2$ dan $\boldsymbol{\beta}_1^T$ yaitu:

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\boldsymbol{\beta}_1^T\partial\boldsymbol{\beta}_2} = \frac{\partial\sum_{i=1}^{n}[y_{i2}-M(y_i)\mu_{i2}]\boldsymbol{X}_i^T}{\partial\boldsymbol{\beta}_1^T}$$

$$= \sum_{i=1}^{n} \left[0 - \mu_{i2} \frac{\partial M(y_i)}{\partial \boldsymbol{\beta}_1^T} \right] \boldsymbol{X}_i^T \tag{27}$$

Substitusi Persamaan (27) dengan Persamaan (21).

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_1^T \partial \boldsymbol{\beta}_2} = -\sum_{i=1}^{n} \mu_{i2} \left[\frac{\partial M(y_i)}{\partial \boldsymbol{\beta}_1^T} \right] \boldsymbol{X}_i^T$$

$$= -\sum_{i=1}^{n} \mu_{i2} \left\{ \frac{\mu_{i1} \boldsymbol{X}_i \left\{ 1 + \tau M(y_i) \left(2 \sum_{j=1}^{2} y_{ij} + 1 \right) \right\}}{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}} + \mu_{i1} \boldsymbol{X}_i M^2(y_i) \right\} \boldsymbol{X}_i^T$$

$$= -\sum_{i=1}^{n} \mu_{i1} \mu_{i2} \left\{ \frac{\left\{ 1 + \tau M(y_i) \left(2 \sum_{j=1}^{2} y_{ij} + 1 \right) \right\}}{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}} - M^2(y_i) \right\} \boldsymbol{X}_i \boldsymbol{X}_i^T \tag{28}$$

Turunan pertama fungsi ln *likelihood* terhadap τ yaitu:

$$\frac{\partial l(\boldsymbol{\theta})}{\partial \tau} = \frac{\partial}{\partial \tau} \left[\frac{n}{\tau} - \frac{n}{2} \ln(\tau) + \frac{n}{2} ln\left(\frac{2}{\pi}\right) + \sum_{i=1}^{n} ln\left[K_{s_i}(z_i)\right] + \right.$$

$$- \sum_{i=1}^{n} \frac{\left(2 \sum_{j=1}^{2} y_{ij} - 1\right)}{4} ln\left(1 + 2\tau \sum_{j=1}^{2} e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_j}\right) + + \sum_{i=1}^{n} \sum_{j=1}^{2} y_{ij} \boldsymbol{X}_i^T \boldsymbol{\beta}_j +$$

$$\left. - ln\left(\sum_{i=1}^{n} \sum_{j=1}^{2} y_{ij}!\right) \right]$$

$$= -\frac{n}{\tau^2} - \frac{n}{2\tau} + \sum_{i=1}^{n} \frac{\partial ln\left[K_{s_i}(z_i)\right]}{\partial \tau} + \sum_{i=1}^{n} \frac{\left(\frac{1}{2} - \sum_{j=1}^{2} y_{ij}\right) \sum_{j=1}^{2} \mu_{ij}}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)} \tag{29}$$

Turunan pertama ln $K_{s_i}(z_i)$ terhadap τ yaitu:

$$\frac{\partial \ln K_{s_i}(z_i)}{\partial \tau} = \frac{1}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} \cdot \frac{\partial K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}{\partial \tau}$$

$$= \frac{1}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} \cdot \left[\frac{\partial K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}{\partial (z_i)} \cdot \frac{\partial (z_i)}{\partial \tau} \right]$$

$$= \frac{1}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} \left[-K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i) + \right.$$

$$\left. + \frac{\tau\left(\frac{1}{2} - \sum_{j=1}^{2} y_{ij}\right) K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}{\sqrt{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}}} \right] \left[\frac{\partial \left(\left(\frac{1}{\tau} \sqrt{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}} \right) \right) \tau^{-1}}{\partial \tau} \right]$$

$$= \frac{1}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} \left[-K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i) + \right.$$

$$\left. + \frac{\tau\left(\frac{1}{2} - \sum_{j=1}^{2} y_{ij}\right) K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}{\sqrt{1 + 2\tau \sum_{j=1}^{2} \mu_{ij}}} \right] \frac{1}{\tau^2} \left[\frac{2\tau \sum_{j=1}^{2} \mu_{ij}}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}} \left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}} \right]$$

$$= -\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\left[-\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}-\frac{\tau\left(\frac{1}{2}-\sum_{j=1}^{2}y_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\right]$$

$$= \frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\left[M(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}+\frac{\tau\left(\frac{1}{2}-\sum_{j=1}^{2}y_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\right]$$

$$= \frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left(\frac{1}{2}-\sum_{j=1}^{2}y_{ij}\right)}{\tau\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}+\frac{M(y_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2} \qquad (30)$$

Substitusi Persaman (30) ke Persamaan (29).

$$\frac{\partial l(\boldsymbol{\theta})}{\partial \tau} = -\frac{n}{\tau^2}-\frac{n}{2\tau}+\sum_{i=1}^{n}\frac{\partial ln\left[K_{s_i}(z_i)\right]}{\partial\tau}+\sum_{i=1}^{n}\frac{\left(\frac{1}{2}\sum_{j=1}^{2}y_{ij}-1\right)\sum_{j=1}^{2}\mu_{ij}}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}$$

$$= -\frac{n}{\tau^2}-\frac{n}{2\tau}+\sum_{i=1}^{n}\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left(\frac{1}{2}-\sum_{j=1}^{2}y_{ij}\right)}{\tau\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}+\sum_{i=1}^{n}\frac{M(y_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2}+$$

$$+\sum_{i=1}^{n}\frac{\left(\frac{1}{2}\sum_{j=1}^{2}y_{ij}-1\right)\sum_{j=1}^{2}\mu_{ij}}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}$$

$$= -\frac{n}{\tau^2}-\frac{n}{2\tau}+\sum_{i=1}^{n}\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left(\frac{1}{2}-\sum_{j=1}^{2}y_{ij}\right)}{\tau\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}+\sum_{i=1}^{n}\frac{M(y_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2}+$$

$$+\sum_{i=1}^{n}\frac{\left(\frac{1}{2}\sum_{j=1}^{2}y_{ij}-1\right)\sum_{j=1}^{2}\mu_{ij}}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}$$

$$= -\frac{n}{2\tau}+\sum_{i=1}^{n}\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left(\frac{1}{2}-\sum_{j=1}^{2}y_{ij}\right)}{\tau\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}+\sum_{i=1}^{n}\frac{M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2}$$

$$= \frac{M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)-1}{\tau^2}-\sum_{i=1}^{n}\sum_{j=1}^{2}\frac{y_{ij}}{\tau} \qquad (31)$$

Turunan kedua $\frac{\partial l(\boldsymbol{\theta})}{\partial\tau}$ terhadap $\boldsymbol{\beta}_1$ yaitu:

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\boldsymbol{\beta}_1\partial\tau} = \frac{\partial}{\partial\boldsymbol{\beta}_1}\left[\frac{\partial l(\boldsymbol{\theta})}{\partial\tau}\right]$$

$$= \frac{\partial}{\partial\boldsymbol{\beta}_1}\left[\sum_{i=1}^{n}\frac{M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)-1}{\tau^2}-\sum_{i=1}^{n}\sum_{j=1}^{2}\frac{y_{ij}}{\tau}\right]$$

$$= \frac{1}{\tau^2}\sum_{i=1}^{n}\left\{\left[\frac{\partial\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}{\partial\boldsymbol{\beta}_1}\right]\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}+\right.$$

$$\left.+\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}.\frac{\partial}{\partial\boldsymbol{\beta}_1}\left[\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right]\right\}$$

Turunan pertama $\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}$ terhadap $\boldsymbol{\beta}_1$ yaitu:

$$\frac{\partial\left[\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\right]}{\partial\boldsymbol{\beta}_1}=\frac{1}{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left[\frac{\partial\left(1+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\right)}{\partial\boldsymbol{\beta}_1}\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}-\left(1+\right.\right.$$

$$\left.\left.+\tau\sum_{j=1}^{2}\mu_{ij}\right)\frac{\partial\left(1+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\right)^{\frac{1}{2}}}{\partial\boldsymbol{\beta}_1}\right]$$

$$=\frac{1}{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left[\tau\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}-\left(1+\right.\right.$$

$$\left.\left.+\tau\sum_{j=1}^{2}\mu_{ij}\right)\frac{1}{2}\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)^{-\frac{1}{2}}2\tau\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}\right]$$

$$=\frac{\tau\boldsymbol{X}_i\mu_{i1}}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}-\frac{\tau\boldsymbol{X}_i\mu_{i1}\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}$$

Berdasarkan Persamaan (20), turunan kedua $\frac{\partial l(\boldsymbol{\theta})}{\partial\tau}$ terhadap $\boldsymbol{\beta}_1$ yaitu:

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\boldsymbol{\beta}_1\partial\tau}=\frac{1}{\tau^2}\sum_{i=1}^{n}\left\{\left[\frac{\partial\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}{\partial\boldsymbol{\beta}_1}\right]\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}+\right.$$

$$\left.+\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\cdot\frac{\partial}{\partial\boldsymbol{\beta}_1}\left[\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right]\right\}$$

$$=\frac{1}{\tau^2}\sum_{i=1}^{n}\left\{\left[\frac{\tau\boldsymbol{X}_i\mu_{i1}}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}-\frac{\tau\boldsymbol{X}_i\mu_{i1}\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\right]M(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}+\right.$$

$$\left.+\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\cdot\frac{\tau\boldsymbol{X}_i\mu_{i1}\left(1+\tau M(y_i)\left(2\sum_{j=1}^{2}y_{ij}\right)-M^2(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\right\}$$

$$=\frac{1}{\tau^2}\sum_{i=1}^{n}\mu_{i1}\left\{\tau M(y_i)-\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left(1+\tau M(\boldsymbol{y}_i)\left(1+2\sum_{j=1}^{2}y_{ij}\right)\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}+M^2(y_i)\left(1+\right.\right.$$

$$\left.\left.+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}\boldsymbol{X}_i \qquad (32)$$

Berdasarkan Persamaan (26), turunan kedua fungsi ln *likelihood* terhadap $\boldsymbol{\beta}_2$ yaitu:

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_2^T \partial \boldsymbol{\beta}_2} = \frac{\partial \sum_{i=1}^{n} [y_{i2} - M(\boldsymbol{y}_i)\mu_{i2}] \boldsymbol{X}_i^T}{\partial \boldsymbol{\beta}_1^T}$$

$$= -\sum_{i=1}^{n} \left[\frac{\partial \mu_{i2} M(\boldsymbol{y}_i)}{\partial \boldsymbol{\beta}_2^T}\right] \boldsymbol{X}_i^T \tag{33}$$

Turunan $\mu_{i2} M(y_i)$ terhadap $\boldsymbol{\beta}_2^T$ sebagai berikut:

$$\frac{\partial \mu_{i2} M(\boldsymbol{y}_i)}{\partial \boldsymbol{\beta}_2^T} = \frac{\partial}{\partial \boldsymbol{\beta}_2^T} \left[\frac{\mu_{i2}}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}} \cdot \frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}\right]$$

$$= \frac{\partial}{\partial \boldsymbol{\beta}_2^T} \left[\frac{e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}} \cdot \frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}\right]$$

$$= \left[\frac{\partial \left\{ e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2} \left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{\frac{1}{2}} \right\}}{\partial \boldsymbol{\beta}_2^T}\right] \frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)} +$$

$$+ \frac{e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}}{\left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{\frac{1}{2}}} \frac{\partial}{\partial \boldsymbol{\beta}_2^T} \left[\frac{K_{\sum_{j=1}^{2} y_{ij} + \frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij} - \frac{1}{2}}(z_i)}\right] \tag{34}$$

Turunan $\frac{e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}}{\left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{\frac{1}{2}}}$ terhadap $\boldsymbol{\beta}_2^T$ sebagai berikut:

$$\frac{\partial \left[\frac{e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}}{\left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{\frac{1}{2}}}\right]}{\partial \boldsymbol{\beta}_2^T} = \left[\boldsymbol{X}_i e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2} \left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{\frac{1}{2}} - \right.$$

$$\left. e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} \frac{\boldsymbol{X}_i \tau \mu_{i2}}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{3}{2}}}\right] \frac{1}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)}$$

$$= \boldsymbol{X}_i \mu_{i2} \left[\frac{1}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}} - \frac{\tau \mu_{i2}}{\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{3}{2}}}\right] \tag{35}$$

Turunan z_i terhadap $\boldsymbol{\beta}_2^T$ yaitu:

$$\frac{\partial (z_i)}{\partial \boldsymbol{\beta}_2^T} = \frac{1}{\tau} \frac{\partial \left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_2^T}$$

$$= \frac{1}{\tau} \frac{1}{2} \left(1 + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)^{-\frac{1}{2}} \left(2\tau \boldsymbol{X}_i e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}\right)$$

$$= \frac{\boldsymbol{X}_i e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}}{\sqrt{1+2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_1}+2\tau e^{\boldsymbol{X}_i^T \boldsymbol{\beta}_2}}}$$

$$= \frac{\boldsymbol{X}_i \mu_{i2}}{\sqrt{1+2\tau \sum_{j=1}^{2} \mu_{ij}}} \tag{36}$$

Substitusi Persaman (14) ke Persamaan (36).

$$\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial \boldsymbol{\beta}_2^T} = \frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial (z_i)} \cdot \frac{\partial (z_i)}{\partial \boldsymbol{\beta}_2^T}$$

$$= \left[-K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i) - \frac{\left(\sum_{j=1}^{2} y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{z_i}\right] \frac{\boldsymbol{X}_i \mu_{i2}}{\sqrt{1+2\tau \sum_{j=1}^{2} \mu_{ij}}} \tag{37}$$

Substitusi Persaman (18) ke Persamaan (36).

$$\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial \boldsymbol{\beta}_2^T} = \frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial (z_i)} \cdot \frac{\partial (z_i)}{\partial \boldsymbol{\beta}_2^T}$$

$$= \left[-K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i) + \frac{\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}{z_i}\right] \frac{\boldsymbol{X}_i \mu_{i2}}{\sqrt{1+2\tau \sum_{j=1}^{2} \mu_{ij}}} \tag{38}$$

Substitusi Persamaan (37) dan (38) dengan proses yang sama pada Persamaan (4.20) ke Persamaan berikut

$$\frac{\partial\left(\frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}\right)}{\partial \boldsymbol{\beta}_2^T} = \frac{1}{\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)^2} \left\{\left[\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial \boldsymbol{\beta}_1^T}\right] K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i) + \right.$$

$$\left. -K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i) \left[\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial \boldsymbol{\beta}_1^T}\right]\right\}$$

$$= -\frac{\boldsymbol{X}_i \mu_{i2}}{\sqrt{1+2\tau \sum_{j=1}^{2} \mu_{ij}}} \left\{1 + \tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij}\right) - M^2(y_i)\left(1 + \right.\right.$$

$$\left.\left. +2\tau \sum_{j=1}^{2} \mu_{ij}\right)\right\} \tag{39}$$

Berdasarkan Persamaan (35) dan (39), Persamaan (34) menjadi

$$\frac{\partial \mu_{i2} M(\boldsymbol{y}_i)}{\partial \boldsymbol{\beta}_2^T} = \left[\frac{\partial\left\{e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\left(1+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\right)^{\frac{1}{2}}\right\}}{\partial \boldsymbol{\beta}_2^T}\right]\frac{K_{\sum_{j=1}^2 y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^2 y_{ij}-\frac{1}{2}}(z_i)} +$$

$$+\frac{e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}}{\left(1+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\right)^{\frac{1}{2}}}\frac{\partial}{\partial \boldsymbol{\beta}_2^T}\left[\frac{K_{\sum_{j=1}^2 y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^2 y_{ij}-\frac{1}{2}}(z_i)}\right]$$

$$= \boldsymbol{X}_i\mu_{i2}\left[\frac{1}{\left(1+2\tau\sum_{j=1}^2\mu_{ij}\right)^{\frac{1}{2}}} - \frac{\tau\mu_{i2}}{\left(1+2\tau\sum_{j=1}^2\mu_{ij}\right)^{\frac{3}{2}}}\right]M(y_i)\left(1+2\tau\sum_{j=1}^2\mu_{ij}\right)^{\frac{1}{2}} +$$

$$+\frac{e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}}{\left(1+2\tau\sum_{j=1}^2\mu_{ij}\right)^{\frac{1}{2}}}\left\{\frac{\boldsymbol{X}_i\mu_{i2}\left(1+2\tau M(\boldsymbol{y}_i)\left(\sum_{j=1}^2 y_{ij}\right)-M^2(\boldsymbol{y}_i)\left(1+2\tau\sum_{j=1}^2\mu_{ij}\right)\right)}{\left(1+2\tau\sum_{j=1}^2\mu_{ij}\right)^{\frac{1}{2}}}\right\}$$

$$= \boldsymbol{X}_i\mu_{i2}M(y_i)\left[1-\frac{\tau\mu_{i2}}{1+2\tau\sum_{j=1}^2\mu_{ij}}\right] +$$

$$-\frac{\mu_{i2}\boldsymbol{X}_i\mu_{i2}\left(1+2\tau M(\boldsymbol{y}_i)\left(\sum_{j=1}^2 y_{ij}\right)-M^2(\boldsymbol{y}_i)\left(1+2\tau\sum_{j=1}^2\mu_{ij}\right)\right)}{1+2\tau\sum_{j=1}^2\mu_{ij}}$$

$$= \boldsymbol{X}_i\mu_{i2}\left\{M(y_i) - \frac{\mu_{i2}}{1+2\tau\sum_{j=1}^2\mu_{ij}}\left(1+\tau M(\boldsymbol{y}_i)\left(2\sum_{j=1}^2 y_{ij}\right) -\right.\right.$$

$$\left.\left. M^2(y_i)\left(1+2\tau\sum_{j=1}^2\mu_{ij}\right)\right)\right\}$$

$$= \mu_{i2}\left\{M(y_i) - \frac{\mu_{i2}\left(1+\tau M(\boldsymbol{y}_i)\left(2\sum_{j=1}^2 y_{ij}\right)\right)}{1+2\tau\sum_{j=1}^2\mu_{ij}} + \mu_{i2}M^2(y_i)\right\}\boldsymbol{X}_i \qquad (40)$$

Substitusi Persaman (40) ke Persamaan (33).

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_2^T \partial \boldsymbol{\beta}_2} = -\sum_{i=1}^n\left[\frac{\partial \mu_{i2}M(\boldsymbol{y}_i)}{\partial \boldsymbol{\beta}_2^T}\right]\boldsymbol{X}_i^T$$

$$= -\sum_{i=1}^n \mu_{i2}\left\{M(y_i) - \frac{\mu_{i2}\left(1+\tau M(\boldsymbol{y}_i)\left(2\sum_{j=1}^2 y_{ij}\right)\right)}{1+2\tau\sum_{j=1}^2\mu_{ij}} + \mu_{i2}M^2(y_i)\right\}\boldsymbol{X}_i\boldsymbol{X}_i^T \quad (41)$$

Turunan kedua $\frac{\partial l(\boldsymbol{\theta})}{\partial \tau}$ terhadap $\boldsymbol{\beta}_2$ yaitu:

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}_2 \partial \tau} = \frac{\partial}{\partial \boldsymbol{\beta}_2}\left[\frac{\partial l(\boldsymbol{\theta})}{\partial \tau}\right]$$

$$= \frac{\partial}{\partial \boldsymbol{\beta}_2}\left[\frac{M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^2\mu_{ij}\right)-1}{\tau^2} - \sum_{i=1}^n\sum_{j=1}^2\frac{y_{ij}}{\tau}\right]$$

$$= \frac{1}{\tau^2}\sum_{i=1}^n\frac{\partial\left[M(y_i)\left(1+\tau\sum_{j=1}^2\mu_{ij}\right)\right]}{\partial \boldsymbol{\beta}_2} \qquad (42)$$

Berdasarkan Persamaan (39), turunan dari $M(y_i)$ terhadap $\boldsymbol{\beta}_2$ sebagai berikut:

$$\frac{\partial M(\boldsymbol{y}_i)}{\partial \boldsymbol{\beta}_2} = \frac{\partial}{\partial \boldsymbol{\beta}_2}\left[\frac{1}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}} \cdot \frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right]$$

$$= \left[\frac{\partial\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_2}\right]\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)} + \frac{1}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\left[\frac{\partial\left(\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right)}{\partial \boldsymbol{\beta}_2}\right] \quad (43)$$

Turunan dari $\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{-\frac{1}{2}}$ terhadap $\boldsymbol{\beta}_2$ sebagai berikut:

$$\frac{\partial\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_2} = \frac{\partial\left(1+2\tau\sum_{j=1}^{2}e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_j}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_2} = \frac{\partial\left(1+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_2}$$

$$= -\frac{1}{2}\left(2\tau\boldsymbol{X}_i e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1}\right)\left(1+2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_1} + 2\tau e^{\boldsymbol{X}_i^T\boldsymbol{\beta}_2}\right)^{-\frac{3}{2}}$$

$$= -\tau\mu_{i2}\boldsymbol{X}_i\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{-\frac{3}{2}} \quad (44)$$

Substitusi Persamaan (2) dan (44) ke Persamaan (43) dengan proses yang sama pada Persamaan (4.21)

$$\frac{\partial M(\boldsymbol{y}_i)}{\partial \boldsymbol{\beta}_2} = \left[\frac{\partial\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}{\partial \boldsymbol{\beta}_1^T}\right]\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)} + \frac{1}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\left[\frac{\partial\left(\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right)}{\partial \boldsymbol{\beta}_1^T}\right]$$

$$= -\tau\mu_{i1}\boldsymbol{X}_i\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{-\frac{3}{2}}M(\boldsymbol{y}_i)\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}} + \frac{1}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} - \left[-\frac{\boldsymbol{X}_i\mu_{i1}}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\left\{1+\tau M(y_i)\left(2\sum_{j=1}^{2}y_{ij}\right) - M^2(\boldsymbol{y}_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}\right]$$

$$= -\frac{\tau\mu_{i1}\boldsymbol{X}_i M(y_i)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} - \frac{\boldsymbol{X}_i\mu_{i1}}{1+2\tau\sum_{j=1}^{2}\mu_{ij}}\left\{1+\tau M(y_i)\left(2\sum_{j=1}^{2}y_{ij}\right) - M^2(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}$$

$$= -\frac{\tau\mu_{i1}\boldsymbol{X}_i M(y_i)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}} - \frac{(\boldsymbol{X}_i\mu_{i1})+\tau M(y_i)\left(2\sum_{j=1}^{2}y_{ij}\right)-M^2(y_i)\boldsymbol{X}_i\mu_{i1}\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}}$$

$$= -\frac{\mu_{i1}\boldsymbol{X}_i}{1+2\tau\sum_{j=1}^{2}\mu_{ij}}\left\{\tau M(y_i) + 1 + \tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij} + 1\right) - M^2(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}$$

$$= -\frac{\mu_{i2}\boldsymbol{X}_i}{1+2\tau\sum_{j=1}^{2}\mu_{ij}}\left[1 + \left(1 + 2\sum_{j=1}^{2} y_{ij}\right)\tau M(y_i) - M^2(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right] \quad (45)$$

Berdasarkan Persamaan (42), turunan dari $M(y_i)\left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right)$ terhadap $\boldsymbol{\beta}_2$ sebagai berikut:

$$\frac{\partial\left[M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right]}{\partial\boldsymbol{\beta}_2} = \frac{\partial M(\boldsymbol{y}_i)}{\partial\boldsymbol{\beta}_2}\left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right) + M(y_i)\frac{\partial\left(1+\tau e^{X_i^T\boldsymbol{\beta}_1}+\tau e^{X_i^T\boldsymbol{\beta}_2}\right)^{\frac{1}{2}}}{\partial\boldsymbol{\beta}_2}$$

$$= -\frac{\mu_{i2}\boldsymbol{X}_i\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}}\left[1 + \left(1 + 2\sum_{j=1}^{2} y_{ij}\right)\tau M(\boldsymbol{y}_i) - M^2(y_i)\left(1 + +2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right] + M(y_i)\tau\boldsymbol{X}_i\mu_{i2} \quad (46)$$

Substitusi Persamaan (44) ke Persamaan (40).

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\boldsymbol{\beta}_2\partial\tau} = \frac{1}{\tau^2}\frac{\partial}{\partial\boldsymbol{\beta}_2}\left\{\sum_{i=1}^{n} M(y_i)\left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}$$

$$= \frac{1}{\tau^2}\sum_{i=1}^{n}\frac{\partial}{\partial\boldsymbol{\beta}_2}\left\{M(y_i)\left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}$$

$$= \frac{1}{\tau^2}\sum_{i=1}^{n} -\frac{\mu_{i2}\boldsymbol{X}_i\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{1+2\tau\sum_{j=1}^{2}\mu_{ij}}\left[1 + \left(1 + 2\sum_{j=1}^{2} y_{ij}\right)\tau M(\boldsymbol{y}_i) - M^2(y_i)\left(1 + +2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right] + M(y_i)\tau\boldsymbol{X}_i\mu_{i2}$$

$$= \frac{1}{\tau^2}\sum_{i=1}^{n}\mu_{i2}\boldsymbol{X}_i\left\{\tau M(y_i) - \frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left(1+\tau M(y_i)\left(2\sum_{j=1}^{2} y_{ij}\right)\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}} + M^2(y_i)\left(1 + +\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\} \quad (47)$$

Berdasarkan Persamaan (31), turunan kedua dari $l(\boldsymbol{\theta})$ terhadap τ sebagai berikut:

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\tau^2} = \frac{\partial}{\partial\tau}\left[\frac{\partial l(\boldsymbol{\theta})}{\partial\tau}\right]$$

$$= \frac{\partial}{\partial\tau}\left[\sum_{i=1}^{n}\frac{M(y_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)-1}{\tau^2} - \sum_{i=1}^{n}\sum_{j=1}^{2}\frac{y_{ij}}{\tau}\right]$$

$$= \sum_{i=1}^{n}\left[\frac{\partial\left\{\tau^{-2}M(y_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}}{\partial\tau} - \frac{\partial\tau^{-2}}{\partial\tau} - \sum_{j=1}^{2}\frac{\partial\left(\frac{y_{ij}}{\tau}\right)}{\partial\tau}\right]$$

$$= \sum_{i=1}^{n}\left[\frac{\partial\left\{\tau^{-2}M(y_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}}{\partial\tau} - \frac{2}{\tau^3} - \sum_{j=1}^{2}\frac{y_{ij}}{\tau^2}\right] \qquad (48)$$

Turunan dari $\frac{\tau^{-2}M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2}$ terhadap τ sebagai berikut:

$$\frac{\partial\left\{\tau^{-2}M(y_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}}{\partial\tau} = \frac{1}{\tau^4}\left[\frac{\partial\left\{M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}}{\partial\tau}\tau^2 - 2\tau M(y_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right] \qquad (47)$$

dengan

$$\frac{\partial M(\boldsymbol{y}_i)}{\partial\tau} = \frac{\partial}{\partial\tau}\left[\frac{1}{\sqrt{1+2\tau\sum_{j=1}^{2}\mu_{ij}}}\cdot\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right]$$

$$= \frac{\partial\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}{\partial\tau}\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)} + \frac{1}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\frac{\partial}{\partial\tau}\left[\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right] \qquad (48)$$

Turunan $\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}$ terhadap τ sebagai berikut:

$$\frac{\partial}{\partial\tau}\left[\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right] = \frac{1}{\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)^2}\left\{\left[\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial\tau}\right]K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i) + \right.$$

$$\left. -K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\left[\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial\tau}\right]\right\} \qquad (49)$$

Berdasarkan Persamaan (14), turunan $K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)$ terhadap τ sebagai berikut:

$$\frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial\tau} = \frac{\partial\left(K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial(z_i)}\cdot\frac{\partial(z_i)}{\partial\tau}$$

$$= \left[-K_{\sum_{j=1}^{2}y_{ij}+\frac{3}{2}}(z_i) + \frac{\left(\sum_{j=1}^{2}y_{ij}+\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{(z_i)}\right]\left[\frac{\tau\sum_{j=1}^{2}\mu_{ij}-\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\right]$$

$$= \left[-K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i) + \frac{\tau\left(\sum_{j=1}^{2}y_{ij}+\frac{1}{2}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\right]\left[\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\right]$$

$$= \left[K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}} + \tau \left(\sum_{j=1}^{2} y_{ij} + \frac{1}{2}\right) K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right] \left[\frac{\left(1+\tau \sum_{j=1}^{2} \mu_{ij}\right)}{\tau^2\left(1+2\tau \sum_{j=1}^{2} \mu_{ij}\right)}\right] \tag{50}$$

Berdasarkan Persamaan (18), turunan $K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)$ terhadap τ sebagai berikut:

$$\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial \tau} = \frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial (z_i)} \cdot \frac{\partial (z_i)}{\partial \tau}$$

$$\left[-K_{\sum_{j=1}^{2} y_{ij}-\frac{3}{2}}(z_i) + \frac{\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}{(z_i)}\right] \left[\frac{\tau \sum_{j=1}^{2} \mu_{ij}-\left(1+2\tau \sum_{j=1}^{2} \mu_{ij}\right)}{\left(1+2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}}\right]$$

$$= \left[-K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i) + \frac{\tau\left(\sum_{j=1}^{2} y_{ij}-\frac{1}{2}\right)}{\left(1+2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}} K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right] \left[\frac{\left(1+\tau \sum_{j=1}^{2} \mu_{ij}\right)}{\tau^2\left(1+2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}}\right]$$

$$= \left[K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}} + \tau \left(\sum_{j=1}^{2} y_{ij} - \frac{1}{2}\right) K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right] \left[\frac{\left(1+\tau \sum_{j=1}^{2} \mu_{ij}\right)}{\tau^2\left(1+2\tau \sum_{j=1}^{2} \mu_{ij}\right)}\right] \tag{51}$$

Substitusi Persamaan (50) dan (51) ke Persamaan (49)

$$\frac{\partial}{\partial \tau}\left[\frac{K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}\right] = \frac{1}{\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)^2} \left\{\left[\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right)}{\partial \tau}\right] K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i) + \right.$$

$$\left. -K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\left[\frac{\partial\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)}{\partial \tau}\right]\right\}$$

$$= \frac{1}{\left(K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\right)^2} \left\{\frac{\left(1+\tau \sum_{j=1}^{2} \mu_{ij}\right)K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)}{\tau^2\left(1+2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}}} \times \right.$$

$$\left[K_{\sum_{j=1}^{2} y_{ij}-\frac{1}{2}}(z_i)\left(1 + 2\tau \sum_{j=1}^{2} \mu_{ij}\right)^{\frac{1}{2}} + \tau \left(\sum_{j=1}^{2} y_{ij} + \frac{1}{2}\right) K_{\sum_{j=1}^{2} y_{ij}+\frac{1}{2}}(z_i)\right] +$$

$$\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left[K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}+\right.$$

$$\left.\tau\left(\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\right)K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)\right]\Bigg\}$$

$$=\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\Bigg\{\Bigg[\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}+\tau\Bigg(\sum_{j=1}^{2}y_{ij}+$$

$$\frac{1}{2}\Bigg)\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\Bigg]+\left[\left(\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right)^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}+\right.$$

$$\left.\tau\left(\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\right)\right]\Bigg\}$$

$$=\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left[\left(1-\left(\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right)^2\right)\Bigg(1+\right.$$

$$2\tau\sum_{j=1}^{2}\mu_{ij}\Bigg)^{\frac{1}{2}}+\Bigg(\frac{1}{2}\tau+\tau\sum_{j=1}^{2}y_{ij}-\frac{1}{2}\tau+$$

$$\left.\tau\sum_{j=1}^{2}y_{ij}\Bigg)\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right]$$

$$=\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\Bigg[\left(1-M^2(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right)\Bigg(1+$$

$$2\tau\sum_{j=1}^{2}\mu_{ij}\Bigg)^{\frac{1}{2}}+2\tau M(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}\left(\sum_{j=1}^{2}y_{ij}\right)\Bigg]$$

$$=\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\Bigg[\left(1-M^2(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right)+$$

$$2\tau M(y_i)\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}\left(\sum_{j=1}^{2}y_{ij}\right)\Bigg] \qquad (52)$$

Substitusi Persamaan (52) ke Persamaan (48)

$$\frac{\partial M(\boldsymbol{y}_i)}{\partial\tau}=\frac{\partial\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}{\partial\tau}\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}+\frac{1}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}}\frac{\partial}{\partial\tau}\left[\frac{K_{\sum_{j=1}^{2}y_{ij}+\frac{1}{2}}(z_i)}{K_{\sum_{j=1}^{2}y_{ij}-\frac{1}{2}}(z_i)}\right]$$

$$= \frac{M(y_i)\left(\sum_{j=1}^{2}\mu_{ij}\right)}{\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} + \frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left[\left(1 - M^2(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right) + 2\tau M(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}\left(\sum_{j=1}^{2}y_{ij}\right)\right]$$

$$= -\frac{1}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left[\left(\tau^2 M(y_i)\left(\sum_{j=1}^{2}\mu_{ij}\right) - \left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right)\right) \times \left(1 - M^2(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right) + 2\tau M(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}\left(\sum_{j=1}^{2}y_{ij}\right)\right] \quad (53)$$

Maka

$$\frac{\partial\left\{M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}}{\partial\tau} = \frac{\partial\{M(\boldsymbol{y}_i)\}}{\partial\tau}\left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right) + M(\boldsymbol{y}_i)\left(\sum_{j=1}^{2}\mu_{ij}\right)$$

$$= -\frac{1}{\tau^2\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left[\left(\tau^2 M(y_i)\left(\sum_{j=1}^{2}\mu_{ij}\right) - \left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right)\right) \times \left(1 - M^2(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right)\right) + 2\tau M(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right)^{\frac{1}{2}}\left(\sum_{j=1}^{2}y_{ij}\right)\right]\left(M(\boldsymbol{y}_i)\left(\sum_{j=1}^{2}\mu_{ij}\right)\right) \quad (54)$$

Substitusi Persamaan (54) ke Persamaan (49)

$$\frac{\partial\left\{M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\tau^{-2}\right\}}{\partial\tau} = \frac{1}{\tau^4}\left[\frac{\partial\left\{M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}}{\partial\tau}\tau^2 - 2\tau M(y_i)\left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right)\right]$$

$$= \frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^4\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left\{\left(\tau^2 M(y_i)\left(\sum_{j=1}^{2}\mu_{ij}\right) - \left(1 + \tau\sum_{j=1}^{2}\mu_{ij}\right)\right) \times \left[1 - M^2(y_i)\left(1 + 2\tau\sum_{j=1}^{2}\mu_{ij}\right) + 2\tau M(y_i)\left(\sum_{j=1}^{2}y_{ij}\right)\right]\right\} \quad (55)$$

Sehingga, Persamaan (48) dapat dituliskan sebagai berikut:

$$\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\tau^2} = \sum_{i=1}^{n}\left[\frac{\partial\left\{\tau^{-2}M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\right\}}{\partial\tau} - \frac{2}{\tau^3} - \sum_{j=1}^{2}\frac{y_{ij}}{\tau^2}\right]$$

$$= \sum_{i=1}^{n}\left[-\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^4\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)}\left\{\tau^2 M(y_i)\left(\sum_{j=1}^{2}\mu_{ij}\right) - \left(1 + +\tau\sum_{j=1}^{2}\mu_{ij}\right)\left[1 - -M^2(y_i)\left(1 + +2\tau\sum_{j=1}^{2}\mu_{ij}\right) + +2\tau M(y_i)\left(\sum_{j=1}^{2}y_{ij}\right)\right]\right\} + \frac{M(\boldsymbol{y}_i)\left(\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2} + -\frac{2M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^3} + \frac{2}{\tau^3} + \frac{\sum_{j=1}^{2}y_{ij}}{\tau^2}\right]$$

$$= \sum_{i=1}^{n} \left[-\frac{\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^4\left(1+2\tau\sum_{j=1}^{2}\mu_{ij}\right)} \left\{ \tau^2 M(y_i)\left(\sum_{j=1}^{2}\mu_{ij}\right) - \left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)\left[1 + \right.\right.\right.$$

$$\left.\left. -M^2(y_i)\left(1 + +2\tau\sum_{j=1}^{2}\mu_{ij}\right) + +2\tau M(y_i)\left(\sum_{j=1}^{2}y_{ij}\right)\right]\right\} +$$

$$\left. + \frac{2-2M(\boldsymbol{y}_i)\left(1+\tau\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^3} + \frac{\sum_{j=1}^{2}y_{ij}+M(\boldsymbol{y}_i)\left(\sum_{j=1}^{2}\mu_{ij}\right)}{\tau^2} \right] \quad (56)$$

Lampiran 3. Syarat Cukup Matriks Hessian

```
eigen() decomposition
$values
 [1]  -6.015199e+09  -1.628250e+08  -1.839478e+07  -1.48633
2e+07  -5.766393e+06  -6.574336e+02
 [7] -6.616730e+02 -6.166081e+04 -5.789795e+06 -1.493720e+0
7 -1.848785e+07 -1.631739e+08
[13] -6.032073e+09
```

Lampiran 4. Uji Korelasi Variabel Respon

Correlations			
		Y1	Y2
Y1	Pearson Correlation	1	.569**
	Sig. (2-tailed)		.004
	N	24	24
Y2	Pearson Correlation	.569**	1
	Sig. (2-tailed)	.004	
	N	24	24

**. Correlation is significant at the 0.01 level (2-tailed).

Lampiran 5. Uji Multikolinearitas

Coefficients[a]								
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	-7.570	8.931		-.848	.408		
	X1	.102	.094	.272	1.078	.295	.609	1.641
	X2	.015	.108	.035	.141	.889	.627	1.595
	X3	.016	.044	.078	.369	.716	.878	1.138
	X4	-.031	.043	-.156	-.710	.487	.803	1.246
	X5	.218	.095	.505	2.284	.035	.792	1.263

Lampiran 6. Uji Overdispersi

Jumlah Kematian Ibu

Goodness of Fit[a]			
	Value	df	Value/df
Deviance	37.464	18	2.081
Scaled Deviance	37.464	18	
Pearson Chi-Square	34.761	18	1.931
Scaled Pearson Chi-Square	34.761	18	
Log Likelihood[b]	-60.087		
Akaike's Information Criterion (AIC)	132.173		
Finite Sample Corrected AIC (AICC)	137.114		
Bayesian Information Criterion (BIC)	139.242		
Consistent AIC (CAIC)	145.242		

Jumlah Kematian Neonatal

Goodness of Fit[a]			
	Value	df	Value/df
Deviance	91.786	18	5.099
Scaled Deviance	91.786	18	
Pearson Chi-Square	87.190	18	4.844
Scaled Pearson Chi-Square	87.190	18	
Log Likelihood[b]	-106.899		
Akaike's Information Criterion (AIC)	225.798		
Finite Sample Corrected AIC (AICC)	230.739		
Bayesian Information Criterion (BIC)	232.866		
Consistent AIC (CAIC)	238.866		

Lampiran 7. Estimasi Parameter

```
                   Estimate   Std. Error            W P\002value
 (Interceptb1) -1.051248240 8.414798e-06 1.560713e+10          0
 b1.x1          0.021594012 4.277614e-10 2.548376e+15          0
 b1.x2          0.007880292 7.440037e-10 1.121849e+14          0
 b1.x3          0.002461835 9.927252e-11 6.149781e+14          0
 b1.x4         -0.008324038 1.244488e-10 4.473903e+15          0
 b1.x5          0.033607641 3.671665e-10 8.378183e+15          0
 (Interceptb2)  2.186023900 1.330082e-06 2.701177e+12          0
 b2.x1          0.008234968 8.202398e-11 1.007957e+16          0
 b2.x2         -0.018665658 1.500915e-10 1.546587e+16          0
 b2.x3          0.015803060 3.046210e-11 2.691304e+17          0
 b2.x4         -0.002623545 3.059320e-11 7.354063e+15          0
 b2.x5          0.037137669 7.340982e-11 2.559299e+17          0
 t             10.063269228 1.621776e-05 3.850317e+11          0
```

```
> AIC
[1] 162.463
```