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A MATHEMATICAL STUDY OF TUBERCULOSIS INFECTIONS USING A DETERMINISTIC MODEL IN COMPARISON WITH CONTINUOUS MARKOV CHAIN MODEL

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Abstract: Mathematical model of the transmission of tuberculosis infection was widely studied to capture the transient behavior of the disease transmission. In this study we model the dynamic of the disease by considering an epidemiological model called SEIIR model to capture the deterministic behavior of the disease. We also applied a continuous-time Markov chain model to take into consideration the randomness of the system. In the deterministic model, a disease-free equilibrium point and a basic reproduction number of the model are found which are mainly influenced by the contact rate of susceptible individuals with infective individuals. Other parameters such as progression rate of the latent individuals to be infectious individuals and the treatment rate of latent individuals also influence not only the deterministic model but also the stochastic sample paths. For a certain critical value of the treatment rate of latent class, deterministic and stochastic solutions show different behavior at the final time of observation. A high degree of randomness is observed in the latent and infected class (hospitalize or not-hospitalized). While in the susceptible class, effect of randomness is almost not observed. This suggests the robustness of deterministic model of susceptible class to the stochastic perturbations.

Keywords: tuberculosis; SEIIR epidemic model; basic reproduction number; continuous-time Markov chain;

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1. INTRODUCTION

Tuberculosis is a contagious disease caused by infection with the bacteria Mycobacterium tuberculosis (Mtb). Some of the factors that trigger infection are an unhealthy lifestyle and low human immunity since the bacteria can spread from one person to another through tiny droplets released into the air through coughing and sneezing of people with acute infection. When the human body contains bacteria, the immune system can usually prevent it from getting sick. People with this condition are called latent TB where the bacteria in their bodies are inactive and cause no symptoms. However, without treatment, latent TB can turn into active TB. If not handled immediately, active tuberculosis can be dangerous. Therefore, treatment is needed for people with latent TB and to help control the spread of TB [1], [2]. Furthermore, in order to clear the infection and prevent the development of antibiotic resistance, people with active TB have to take several types of drugs for months. Without drug treatment, Infectious tuberculosis has the potential to become a serious disease because it attacks the lungs and also affects other parts of human body, including kidneys, spine or brain. According to the WHO data, the deadliest infectious killer in the world is still caused by this TB disease. More than 4,000 people lose their lives every day due to the infection of TB. Nearly 30,000 people fall ill from this preventable and curable disease. However, global efforts to fight the TB infection have saved an estimated 58 million lives since 2000 [2].

Due to the population growth, tuberculosis remains a major global health problem in the world [1], [3]. Some diagnostics and novel therapies have been developed to bring great potential to reduce TB burden and mortality. However, limitations on the resources of TB endemic settings remain exist. A theoretical approach is needed to estimate the impact of various interventions on the outcome of interest. An intensive research will also provide an area for developing diagnostic