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## LAMPIRAN

### Lampiran 1. Titik Kesetimbangan Model Penyebaran Differi Menggunakan Maple

- > *restart;*
- > *with(linalg) :*
- > *with(plots) : with(linalg) : with(VectorCalculus) :*
- >  $dS1 := A + \eta \cdot R_p - (1 - u_1 \cdot \omega) \cdot \beta \cdot S \cdot (I_s + I_a) - u_1 \cdot \omega \cdot S - \mu \cdot S$   

$$dS1 := \eta R_p + A - (-\omega u_1 + 1) \beta S (I_s + I_a) - u_1 \omega S - \mu S$$
- >  $dL1 := (1 - u_1 \cdot \omega) \cdot \beta \cdot S \cdot (I_s + I_a) - (\delta + \mu) \cdot L$   

$$dL1 := (-\omega u_1 + 1) \beta S (I_s + I_a) - (\delta + \mu) L$$
- >  $dI1 := (1 - n) \cdot \delta \cdot L - (1 + u_2 \cdot \tau) \cdot \theta \cdot I_s - (\mu + \alpha) \cdot I_s$   

$$dI1 := (1 - n) \delta L - (\tau u_2 + 1) \theta I_s - (\mu + \alpha) I_s$$
- >  $dA1 := n \cdot \delta \cdot L - (1 + u_2 \cdot \tau) \cdot \gamma \cdot I_a - \mu \cdot I_a$   

$$dA1 := n \delta L - (\tau u_2 + 1) \gamma I_a - \mu I_a$$
- >
- >  $dR1 := m \cdot (1 + u_2 \cdot \tau) \cdot \theta \cdot I_s + u_1 \cdot \omega \cdot S - (\rho + \mu) \cdot R_f$   

$$dR1 := m (\tau u_2 + 1) \theta I_s + u_1 \omega S - (\rho + \mu) R_f$$
- >  $dR2 := (1 - m) \cdot (1 + u_2 \cdot \tau) \cdot \theta \cdot I_s + (1 + u_2 \cdot \tau) \cdot \gamma \cdot I_a + \rho \cdot R_f - (\eta + \mu) \cdot R_p$   

$$dR2 := (1 - m) (\tau u_2 + 1) \theta I_s + (\tau u_2 + 1) \gamma I_a + \rho R_f - (\eta + \mu) R_p$$
- >  $dN := dS1 + dL1 + dA1 + dI1 + dR1 + dR2$   

$$dN := \eta R_p + A - \mu S - (\delta + \mu) L + n \delta L - \mu I_a + (1 - n) \delta L - (\tau u_2 + 1) \theta I_s - (\mu + \alpha) I_s + m (\tau u_2 + 1) \theta I_s - (\rho + \mu) R_f + (1 - m) (\tau u_2 + 1) \theta I_s + \rho R_f - (\eta + \mu) R_p$$
- > *simplify(dN)*  

$$-L \mu - S \mu - \alpha I_s - \mu I_a - \mu I_s - \mu R_f - \mu R_p + A$$
- >  $fixpoint := solve(\{dS1, dL1, dI1, dA1, dR1, dR2\}, \{S, L, I_s, I_a, R_f, R_p\})$

$$\begin{aligned}
\text{fixpoint} := & \left\{ L=0, S = \frac{(\eta\mu + \eta\rho + \mu^2 + \mu\rho)A}{(\eta\omega u_1 + \mu\omega u_1 + \omega\rho u_1 + \eta\mu + \eta\rho + \mu^2 + \mu\rho)\mu}, I_a=0, I_s=0, R_f \right. \\
= & \left. \frac{A(\eta + \mu)\omega u_1}{(\eta\omega u_1 + \mu\omega u_1 + \omega\rho u_1 + \eta\mu + \eta\rho + \mu^2 + \mu\rho)\mu}, R_p = \frac{A\omega\rho u_1}{(\eta\omega u_1 + \mu\omega u_1 + \omega\rho u_1 + \eta\mu + \eta\rho + \mu^2 + \mu\rho)\mu} \right\}, \left\{ L \right. \\
= & - \left( (A\beta\delta\eta\gamma\mu n\omega\tau^2\theta u_1 u_2^2 + A\beta\delta\eta\gamma n\omega\rho\tau^2\theta u_1 u_2^2 - A\beta\delta\eta\mu n\omega\tau^2\theta^2 u_1 u_2^2 - A\beta\delta\eta n\omega\rho\tau^2\theta^2 u_1 u_2^2 \right. \\
& + A\beta\delta\gamma\mu^2 n\omega\tau^2\theta u_1 u_2^2 + A\beta\delta\gamma\mu n\omega\rho\tau^2\theta u_1 u_2^2 - A\beta\delta\mu^2 n\omega\tau^2\theta^2 u_1 u_2^2 - A\beta\delta\mu n\omega\rho\tau^2\theta^2 u_1 u_2^2 - \delta\eta\gamma\mu\omega\tau^3\theta^2 u_1 u_2^3 \\
& - \delta\gamma\mu^2\omega\tau^3\theta^2 u_1 u_2^3 - \delta\gamma\mu\omega\rho\tau^3\theta^2 u_1 u_2^3 - \eta\gamma\mu^2\omega\tau^3\theta^2 u_1 u_2^3 - \gamma\mu^3\omega\tau^3\theta^2 u_1 u_2^3 - \gamma\mu^2\omega\rho\tau^3\theta^2 u_1 u_2^3 \\
& - A\beta\delta\eta\gamma\mu\omega\tau^2\theta u_1 u_2^2 - A\beta\delta\eta\gamma\omega\rho\tau^2\theta u_1 u_2^2 - A\beta\delta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - A\beta\delta\gamma\mu\omega\rho\tau^2\theta u_1 u_2^2 - \delta\eta\gamma\mu^2\tau^3\theta^2 u_2^3 \\
& - \delta\eta\gamma\mu\rho\tau^3\theta^2 u_2^3 - \delta\gamma\mu^3\tau^3\theta^2 u_2^3 - \delta\gamma\mu^2\rho\tau^3\theta^2 u_2^3 - \eta\gamma\mu^3\tau^3\theta^2 u_2^3 - \eta\gamma\mu^2\rho\tau^3\theta^2 u_2^3 - \gamma\mu^4\tau^3\theta^2 u_2^3 - \gamma\mu^3\rho\tau^3\theta^2 u_2^3 \\
& + A\alpha\beta\delta\eta\gamma\mu n\omega\tau u_1 u_2 + A\alpha\beta\delta\eta\gamma n\omega\rho\tau u_1 u_2 - 2A\alpha\beta\delta\eta\mu n\omega\tau\theta u_1 u_2 - 2A\alpha\beta\delta\eta n\omega\rho\tau\theta u_1 u_2 \\
& + A\alpha\beta\delta\gamma\mu n\omega\rho\tau u_1 u_2 - 2A\alpha\beta\delta\mu^2 n\omega\tau\theta u_1 u_2 - 2A\alpha\beta\delta\mu n\omega\rho\tau\theta u_1 u_2 + A\beta\delta\eta\gamma\mu^2 n\omega\tau u_1 u_2 + A\beta\delta\eta\gamma\mu n\omega\rho\tau u_1 u_2 \\
& + 2A\beta\delta\eta\gamma\mu n\omega\tau\theta u_1 u_2 - A\beta\delta\eta\gamma\mu n\tau^2\theta u_2^2 + 2A\beta\delta\eta\gamma n\omega\rho\tau\theta u_1 u_2 - A\beta\delta\eta\gamma n\rho\tau^2\theta u_2^2 - A\beta\delta\eta\mu^2 n\omega\tau\theta u_1 u_2 \\
& - A\beta\delta\eta\mu n\omega\rho\tau\theta u_1 u_2 - 2A\beta\delta\eta\mu n\omega\tau\theta^2 u_1 u_2 + A\beta\delta\eta\mu n\tau^2\theta^2 u_2^2 - 2A\beta\delta\eta n\omega\rho\tau\theta^2 u_1 u_2 + A\beta\delta\eta n\rho\tau^2\theta^2 u_2^2 \\
& + A\beta\delta\gamma\mu^3 n\omega\tau u_1 u_2 + A\beta\delta\gamma\mu^2 n\omega\rho\tau u_1 u_2 + 2A\beta\delta\gamma\mu^2 n\omega\tau\theta u_1 u_2 - A\beta\delta\gamma\mu^2 n\tau^2\theta u_2^2 + 2A\beta\delta\gamma\mu n\omega\rho\tau\theta u_1 u_2 \\
& - A\beta\delta\gamma\mu n\rho\tau^2\theta u_2^2 - A\beta\delta\mu^3 n\omega\tau\theta u_1 u_2 - A\beta\delta\mu^2 n\omega\rho\tau\theta u_1 u_2 - 2A\beta\delta\mu^2 n\omega\tau\theta^2 u_1 u_2 + A\beta\delta\mu^2 n\tau^2\theta^2 u_2^2 \\
& - 2A\beta\delta\mu n\omega\rho\tau\theta^2 u_1 u_2 + A\beta\delta\mu n\rho\tau^2\theta^2 u_2^2 - 2\alpha\delta\eta\gamma\mu\omega\tau^2\theta u_1 u_2^2 - 2\alpha\delta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 2\alpha\delta\gamma\mu\omega\rho\tau^2\theta u_1 u_2^2 \\
& - 2\alpha\eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 2\alpha\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 2\alpha\gamma\mu^2\omega\rho\tau^2\theta u_1 u_2^2 - 2\delta\eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 3\delta\eta\gamma\mu\omega\tau^2\theta^2 u_1 u_2^2 \\
& - \delta\eta\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - 2\delta\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 2\delta\gamma\mu^2\omega\rho\tau^2\theta u_1 u_2^2 - 3\delta\gamma\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - 3\delta\gamma\mu\omega\rho\tau^2\theta^2 u_1 u_2^2 - \delta\mu^3\omega\tau^2\theta^2 u_1 \\
& u_2^2 - \delta\mu^2\omega\rho\tau^2\theta^2 u_1 u_2^2 - 2\eta\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 3\eta\gamma\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - \eta\mu^3\omega\tau^2\theta^2 u_1 u_2^2 - 2\gamma\mu^4\omega\tau^2\theta u_1 u_2^2 - 2\gamma\mu^3\omega\rho\tau^2\theta u_1 u_2^2 \\
& - 3\gamma\mu^3\omega\tau^2\theta^2 u_1 u_2^2 - 3\gamma\mu^2\omega\rho\tau^2\theta^2 u_1 u_2^2 - \mu^4\omega\tau^2\theta^2 u_1 u_2^2 - \mu^3\omega\rho\tau^2\theta^2 u_1 u_2^2 - A\alpha\beta\delta\eta\gamma\mu\omega\tau u_1 u_2 - A\alpha\beta\delta\eta\gamma\omega\rho\tau u_1 u_2 \\
& - A\alpha\beta\delta\gamma\mu^2\omega\tau u_1 u_2 - A\alpha\beta\delta\gamma\mu\omega\rho\tau u_1 u_2 - A\beta\delta\eta\gamma\mu^2\omega\tau u_1 u_2 - A\beta\delta\eta\gamma\mu\omega\rho\tau u_1 u_2 - 2A\beta\delta\eta\gamma\mu\omega\tau\theta u_1 u_2 \\
& + A\beta\delta\eta\gamma\mu\tau^2\theta u_2^2 - 2A\beta\delta\eta\gamma\omega\rho\tau\theta u_1 u_2 + A\beta\delta\eta\gamma\rho\tau^2\theta u_2^2 - A\beta\delta\eta\mu^2\omega\tau\theta u_1 u_2 - A\beta\delta\eta\mu\rho\tau\theta u_1 u_2 \\
& - A\beta\delta\gamma\mu^3\omega\tau u_1 u_2 - A\beta\delta\gamma\mu^2\omega\rho\tau u_1 u_2 - 2A\beta\delta\gamma\mu^2\omega\tau\theta u_1 u_2 + A\beta\delta\gamma\mu^2\tau^2\theta u_2^2 - 2A\beta\delta\gamma\mu\omega\rho\tau\theta u_1 u_2 \\
& + A\beta\delta\gamma\mu\rho\tau^2\theta u_2^2 - A\beta\delta\mu^3\omega\tau\theta u_1 u_2 - A\beta\delta\mu^2\omega\rho\tau\theta u_1 u_2 - 2\alpha\delta\eta\gamma\mu^2\tau^2\theta u_2^2 - 2\alpha\delta\eta\gamma\mu\rho\tau^2\theta u_2^2 - 2\alpha\delta\gamma\mu^3\tau^2\theta u_2^2 \\
& - 2\alpha\delta\gamma\mu^2\rho\tau^2\theta u_2^2 - 2\alpha\eta\gamma\mu^3\tau^2\theta u_2^2 - 2\alpha\eta\gamma\mu^2\rho\tau^2\theta u_2^2 - 2\alpha\gamma\mu^4\tau^2\theta u_2^2 - 2\alpha\gamma\mu^3\rho\tau^2\theta u_2^2 - 2\delta\eta\gamma\mu^3\tau^2\theta u_2^2
\end{aligned}$$

$$\begin{aligned}
& -4\alpha\delta\gamma\mu\omega\rho\tau\theta u_1 u_2 - 2\alpha\delta\mu^3\omega\tau\theta u_1 u_2 - 2\alpha\delta\mu^2\omega\rho\tau\theta u_1 u_2 - 2\alpha\eta\gamma\mu^3\omega\tau u_1 u_2 - 4\alpha\eta\gamma\mu^2\omega\tau\theta u_1 u_2 \\
& - 2\alpha\eta\mu^3\omega\tau\theta u_1 u_2 - 2\alpha\gamma\mu^4\omega\tau u_1 u_2 - 2\alpha\gamma\mu^3\omega\rho\tau u_1 u_2 - 4\alpha\gamma\mu^3\omega\tau\theta u_1 u_2 - 4\alpha\gamma\mu^2\omega\rho\tau\theta u_1 u_2 - 2\alpha\mu^4\omega\tau\theta u_1 u_2 \\
& - 2\alpha\mu^3\omega\rho\tau\theta u_1 u_2 - \delta\eta\gamma\mu^3\omega\tau u_1 u_2 - 4\delta\eta\gamma\mu^2\omega\tau\theta u_1 u_2 - 3\delta\eta\gamma\mu\omega\tau\theta^2 u_1 u_2 - 2\delta\eta\mu^3\omega\tau\theta u_1 u_2 - 2\delta\eta\mu^2\omega\tau\theta^2 u_1 u_2 \\
& - \delta\gamma\mu^4\omega\tau u_1 u_2 - \delta\gamma\mu^3\omega\rho\tau u_1 u_2 - 4\delta\gamma\mu^3\omega\tau\theta u_1 u_2 - 4\delta\gamma\mu^2\omega\rho\tau\theta u_1 u_2 - 3\delta\gamma\mu^2\omega\tau\theta^2 u_1 u_2 - 3\delta\gamma\mu\omega\rho\tau\theta^2 u_1 u_2 \\
& - 2\delta\mu^4\omega\tau\theta u_1 u_2 - 2\delta\mu^3\omega\rho\tau\theta u_1 u_2 - 2\delta\mu^3\omega\tau\theta^2 u_1 u_2 - 2\delta\mu^2\omega\rho\tau\theta^2 u_1 u_2 - \eta\gamma\mu^4\omega\tau u_1 u_2 - 4\eta\gamma\mu^3\omega\tau\theta u_1 u_2 \\
& - 3\eta\gamma\mu^2\omega\tau\theta^2 u_1 u_2 - 2\eta\mu^4\omega\tau\theta u_1 u_2 - 2\eta\mu^3\omega\tau\theta^2 u_1 u_2 - \gamma\mu^5\omega\tau u_1 u_2 - \gamma\mu^4\omega\rho\tau u_1 u_2 - 4\gamma\mu^4\omega\tau\theta u_1 u_2 \\
& - 4\gamma\mu^3\omega\rho\tau\theta u_1 u_2 - 3\gamma\mu^3\omega\tau\theta^2 u_1 u_2 - 3\gamma\mu^2\omega\rho\tau\theta^2 u_1 u_2 - 2\mu^5\omega\tau\theta u_1 u_2 - 2\mu^4\omega\rho\tau\theta u_1 u_2 - 2\mu^4\omega\tau\theta^2 u_1 u_2 \\
& - 2\mu^3\omega\rho\tau\theta^2 u_1 u_2 - A\alpha\beta\delta\eta\gamma\mu\omega u_1 + A\alpha\beta\delta\eta\gamma\mu\tau u_2 - A\alpha\beta\delta\eta\gamma\omega\rho u_1 + A\alpha\beta\delta\eta\gamma\rho\tau u_2 - A\alpha\beta\delta\eta\mu^2\omega u_1 \\
& - A\alpha\beta\delta\eta\mu\omega\rho u_1 - A\alpha\beta\delta\gamma\mu^2\omega u_1 + A\alpha\beta\delta\gamma\mu^2\tau u_2 - A\alpha\beta\delta\gamma\mu\omega\rho u_1 + A\alpha\beta\delta\gamma\mu\rho\tau u_2 - A\alpha\beta\delta\mu^3\omega u_1 \\
& - A\alpha\beta\delta\mu^2\omega\rho u_1 - A\beta\delta\eta\gamma\mu^2\omega u_1 + A\beta\delta\eta\gamma\mu^2\tau u_2 - A\beta\delta\eta\gamma\mu\omega\rho u_1 - A\beta\delta\eta\gamma\mu\omega\theta u_1 + A\beta\delta\eta\gamma\mu\rho\tau u_2 \\
& + 2A\beta\delta\eta\gamma\mu\tau\theta u_2 - A\beta\delta\eta\gamma\omega\rho\theta u_1 + 2A\beta\delta\eta\gamma\rho\tau\theta u_2 - A\beta\delta\eta\mu^3\omega u_1 - A\beta\delta\eta\mu^2\omega\rho u_1 - A\beta\delta\eta\mu^2\omega\theta u_1 \\
& + A\beta\delta\eta\mu^2\tau\theta u_2 - A\beta\delta\eta\mu\omega\rho\theta u_1 + A\beta\delta\eta\mu\rho\tau\theta u_2 - A\beta\delta\gamma\mu^3\omega u_1 + A\beta\delta\gamma\mu^3\tau u_2 - A\beta\delta\gamma\mu^2\omega\rho u_1 - A\beta\delta\gamma\mu^2\omega\theta u_1 \\
& + A\beta\delta\gamma\mu^2\rho\tau u_2 + 2A\beta\delta\gamma\mu^2\tau\theta u_2 - A\beta\delta\gamma\mu\omega\rho\theta u_1 + 2A\beta\delta\gamma\mu\rho\tau\theta u_2 - A\beta\delta\mu^4\omega u_1 - A\beta\delta\mu^3\omega\rho u_1 - A\beta\delta\mu^3\omega\theta u_1 \\
& + A\beta\delta\mu^3\tau\theta u_2 - A\beta\delta\mu^2\omega\rho\theta u_1 + A\beta\delta\mu^2\rho\tau\theta u_2 - \alpha^2\delta\eta\gamma\mu^2\tau u_2 - \alpha^2\delta\eta\gamma\mu\rho\tau u_2 - \alpha^2\delta\gamma\mu^3\tau u_2 - \alpha^2\delta\gamma\mu^2\rho\tau u_2 \\
& - \alpha^2\eta\gamma\mu^3\tau u_2 - \alpha^2\eta\gamma\mu^2\rho\tau u_2 - \alpha^2\gamma\mu^4\tau u_2 - \alpha^2\gamma\mu^3\rho\tau u_2 - 2\alpha\delta\eta\gamma\mu^3\tau u_2 - 2\alpha\delta\eta\gamma\mu^2\rho\tau u_2 - 4\alpha\delta\eta\gamma\mu^2\tau\theta u_2 \\
& - 4\alpha\delta\eta\gamma\mu\rho\tau\theta u_2 - 2\alpha\delta\eta\mu^3\tau\theta u_2 - 2\alpha\delta\eta\mu^2\rho\tau\theta u_2 - 2\alpha\delta\gamma\mu^4\tau u_2 - 2\alpha\delta\gamma\mu^3\rho\tau u_2 - 4\alpha\delta\gamma\mu^3\tau\theta u_2
\end{aligned}$$

$$\begin{aligned}
& -\eta\gamma^4\rho - 2\eta\gamma^4\theta - 2\eta\gamma\mu^3\rho\theta - \eta\gamma\mu^3\theta^2 - \eta\gamma\mu^2\rho\theta^2 - \eta\mu^6 - \eta\mu^5\rho - 2\eta\mu^5\theta - 2\eta\mu^4\rho\theta - \eta\mu^4\theta^2 - \eta\mu^3\rho\theta^2 \\
& - \gamma\mu^6 - \gamma\mu^5\rho - 2\gamma\mu^5\theta - 2\gamma\mu^4\rho\theta - \gamma\mu^4\theta^2 - \gamma\mu^3\rho\theta^2 - \mu^7 - \mu^6\rho - 2\mu^6\theta - 2\mu^5\rho\theta - \mu^5\theta^2 - \mu^4\rho\theta^2) (\gamma\tau u_2 + \gamma + \mu) \\
& / (\beta (\delta\eta\gamma^2 m\mu n^2 \omega \tau^3 \theta u_1 u_2^2 - \delta\eta\gamma m\mu n^2 \omega \tau^3 \theta^2 u_1 u_2^2 - 2\delta\eta\gamma^2 m\mu n \omega \tau^3 \theta u_1 u_2^2 + \delta\eta\gamma m\mu n \omega \tau^3 \theta^2 u_1 u_2^2 \\
& - \alpha\delta\eta\gamma m\mu n^2 \omega \tau^2 \theta u_1 u_2^2 + 3\delta\eta\gamma^2 m\mu n^2 \omega \tau^2 \theta u_1 u_2^2 - \delta\eta\gamma^2 m\mu n^2 \tau^3 \theta u_1 u_2^2 + \delta\eta\gamma^2 m\mu \omega \tau^3 \theta u_1 u_2^2 + \delta\eta\gamma m\mu^2 n^2 \omega \tau^2 \theta u_1 u_2^2 \\
& - 3\delta\eta\gamma m\mu n^2 \omega \tau^2 \theta^2 u_1 u_2^2 + \delta\eta\gamma m\mu n^2 \tau^3 \theta^2 u_1 u_2^2 - \delta\eta m\mu^2 n^2 \omega \tau^2 \theta^2 u_1 u_2^2 - \delta\gamma^2 \mu^2 n \omega \tau^3 \theta u_1 u_2^2 - \delta\gamma^2 \mu n \omega \rho \tau^3 \theta u_1 u_2^2 \\
& + \delta\gamma\mu^2 n \omega \tau^3 \theta^2 u_1 u_2^2 + \delta\gamma\mu n \omega \rho \tau^3 \theta^2 u_1 u_2^2 - \eta\gamma^2 \mu^2 n \omega \tau^3 \theta u_1 u_2^2 - \eta\gamma^2 \mu n \omega \rho \tau^3 \theta u_1 u_2^2 + \eta\gamma\mu^2 n \omega \tau^3 \theta^2 u_1 u_2^2 \\
& + \eta\gamma\mu n \omega \rho \tau^3 \theta^2 u_1 u_2^2 - \gamma^2 \mu^3 n \omega \tau^3 \theta u_1 u_2^2 - \gamma^2 \mu^2 n \omega \rho \tau^3 \theta u_1 u_2^2 + \gamma\mu^3 n \omega \tau^3 \theta^2 u_1 u_2^2 + \gamma\mu^2 n \omega \rho \tau^3 \theta^2 u_1 u_2^2 \\
& + \alpha\delta\eta\gamma^2 \mu n^2 \omega \tau^2 u_1 u_2^2 + \alpha\delta\eta\gamma^2 n^2 \omega \rho \tau^2 u_1 u_2^2 + \alpha\delta\eta\gamma m\mu n \omega \tau^2 \theta u_1 u_2^2 - \alpha\delta\eta\gamma\mu n^2 \omega \tau^2 \theta u_1 u_2^2 - \alpha\delta\eta\gamma n^2 \omega \rho \tau^2 \theta u_1 u_2^2 \\
& - \alpha\delta\gamma^2 \mu \rho - 2\alpha\delta\gamma\mu^3 - 2\alpha\delta\gamma\mu^2 \rho - \alpha\delta\mu^4 - \alpha\delta\mu^3 \rho - \alpha\eta\gamma^2 \mu^2 - \alpha\eta\gamma^2 \mu \rho - 2\alpha\eta\gamma\mu^3 - 2\alpha\eta\gamma\mu^2 \rho - \alpha\eta\mu^4 \\
& - \alpha\eta\mu^3 \rho - \alpha\gamma^2 \mu^3 - \alpha\gamma^2 \mu^2 \rho - 2\alpha\gamma\mu^4 - 2\alpha\gamma\mu^3 \rho - \alpha\mu^5 - \alpha\mu^4 \rho - \delta\eta\gamma^2 \mu^2 - \delta\eta\gamma^2 \mu \rho - 2\delta\eta\gamma\mu^3 - 2\delta\eta\gamma\mu^2 \rho \\
& - \delta\eta\mu^4 - \delta\eta\mu^3 \rho - \delta\gamma^2 \mu^3 - \delta\gamma^2 \mu^2 \rho - \delta\gamma^2 \mu^2 \theta - \delta\gamma^2 \mu \rho \theta - 2\delta\gamma\mu^4 - 2\delta\gamma\mu^3 \rho - 2\delta\gamma\mu^3 \theta - 2\delta\gamma\mu^2 \rho \theta - \delta\mu^5 - \delta\mu^4 \rho \\
& - \delta\mu^4 \theta - \delta\mu^3 \rho \theta - \eta\gamma^2 \mu^3 - \eta\gamma^2 \mu^2 \rho - \eta\gamma^2 \mu^2 \theta - \eta\gamma^2 \mu \rho \theta - 2\eta\gamma\mu^4 - 2\eta\gamma\mu^3 \rho - 2\eta\gamma\mu^3 \theta - 2\eta\gamma\mu^2 \rho \theta - \eta\mu^5 \\
& - \eta\mu^4 \rho - \eta\mu^4 \theta - \eta\mu^3 \rho \theta - \gamma^2 \mu^4 - \gamma^2 \mu^3 \rho - \gamma^2 \mu^3 \theta - \gamma^2 \mu^2 \rho \theta - 2\gamma\mu^5 - 2\gamma\mu^4 \rho - 2\gamma\mu^4 \theta - 2\gamma\mu^3 \rho \theta - \mu^6 - \mu^5 \rho - \mu^5 \theta \\
& - \mu^4 \rho \theta) \delta), S = (\delta\gamma\tau^2 \theta u_2^2 + \gamma\mu\tau^2 \theta u_2^2 + \alpha\delta\gamma\tau u_2 + \alpha\gamma\mu\tau u_2 + \delta\gamma\mu\tau u_2 + 2\delta\gamma\tau\theta u_2 + \delta\mu\tau\theta u_2 + \gamma\mu^2 \tau u_2
\end{aligned}$$

$$\begin{aligned}
& -2A\beta\delta\mu n\omega\rho\tau^2 u_1 u_2 + A\beta\delta\mu n\rho\tau^2 u_2^2 - 2\alpha\delta\eta\gamma\mu\omega\tau^2\theta u_1 u_2^2 - 2\alpha\delta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 2\alpha\delta\gamma\mu\omega\rho\tau^2\theta u_1 u_2^2 \\
& - 2\alpha\eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 2\alpha\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 2\alpha\gamma\mu^2\omega\rho\tau^2\theta u_1 u_2^2 - 2\delta\eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 3\delta\eta\gamma\mu\omega\tau^2\theta^2 u_1 u_2^2 \\
& - \delta\eta\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - 2\delta\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 2\delta\gamma\mu^2\omega\rho\tau^2\theta u_1 u_2^2 - 3\delta\gamma\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - 3\delta\gamma\mu\omega\rho\tau^2\theta^2 u_1 u_2^2 - \delta\mu^3\omega\tau^2\theta^2 u_1 \\
& u_2^2 - \delta\mu^2\omega\rho\tau^2\theta^2 u_1 u_2^2 - 2\eta\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 3\eta\gamma\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - \eta\mu^3\omega\tau^2\theta^2 u_1 u_2^2 - 2\gamma\mu^4\omega\tau^2\theta u_1 u_2^2 - 2\gamma\mu^3\omega\rho\tau^2\theta u_1 u_2^2 \\
& - 3\gamma\mu^3\omega\tau^2\theta^2 u_1 u_2^2 - 3\gamma\mu^2\omega\rho\tau^2\theta^2 u_1 u_2^2 - \mu^4\omega\tau^2\theta^2 u_1 u_2^2 - \mu^3\omega\rho\tau^2\theta^2 u_1 u_2^2 - A\alpha\beta\delta\eta\gamma\mu\omega\tau u_1 u_2 - A\alpha\beta\delta\eta\gamma\omega\rho\tau u_1 u_2 \\
& - A\alpha\beta\delta\gamma\mu^2\omega\tau u_1 u_2 - A\alpha\beta\delta\gamma\mu\omega\rho\tau u_1 u_2 - A\beta\delta\eta\gamma\mu^2\omega\tau u_1 u_2 - A\beta\delta\eta\gamma\mu\omega\rho\tau u_1 u_2 - 2A\beta\delta\eta\gamma\mu\omega\tau\theta u_1 u_2 \\
& + A\beta\delta\eta\gamma\mu\tau^2\theta u_2^2 - 2A\beta\delta\eta\gamma\omega\rho\tau\theta u_1 u_2 + A\beta\delta\eta\gamma\rho\tau^2\theta u_2^2 - A\beta\delta\eta\mu^2\omega\tau\theta u_1 u_2 - A\beta\delta\eta\mu\omega\rho\tau\theta u_1 u_2 \\
& - 2A\beta\delta\mu n\omega\rho\tau^2 u_1 u_2 + A\beta\delta\mu n\rho\tau^2 u_2^2 - 2\alpha\delta\eta\gamma\mu\omega\tau^2\theta u_1 u_2^2 - 2\alpha\delta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 2\alpha\delta\gamma\mu\omega\rho\tau^2\theta u_1 u_2^2 \\
& - 2\alpha\eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 2\alpha\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 2\alpha\gamma\mu^2\omega\rho\tau^2\theta u_1 u_2^2 - 2\delta\eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 3\delta\eta\gamma\mu\omega\tau^2\theta^2 u_1 u_2^2 \\
& - \delta\eta\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - 2\delta\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 2\delta\gamma\mu^2\omega\rho\tau^2\theta u_1 u_2^2 - 3\delta\gamma\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - 3\delta\gamma\mu\omega\rho\tau^2\theta^2 u_1 u_2^2 - \delta\mu^3\omega\tau^2\theta^2 u_1 \\
& u_2^2 - \delta\mu^2\omega\rho\tau^2\theta^2 u_1 u_2^2 - 2\eta\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 3\eta\gamma\mu^2\omega\tau^2\theta^2 u_1 u_2^2 - \eta\mu^3\omega\tau^2\theta^2 u_1 u_2^2 - 2\gamma\mu^4\omega\tau^2\theta u_1 u_2^2 - 2\gamma\mu^3\omega\rho\tau^2\theta u_1 u_2^2 \\
& - 3\gamma\mu^3\omega\tau^2\theta^2 u_1 u_2^2 - 3\gamma\mu^2\omega\rho\tau^2\theta^2 u_1 u_2^2 - \mu^4\omega\tau^2\theta^2 u_1 u_2^2 - \mu^3\omega\rho\tau^2\theta^2 u_1 u_2^2 - A\alpha\beta\delta\eta\gamma\mu\omega\tau u_1 u_2 - A\alpha\beta\delta\eta\gamma\omega\rho\tau u_1 u_2 \\
& - A\alpha\beta\delta\gamma\mu^2\omega\tau u_1 u_2 - A\alpha\beta\delta\gamma\mu\omega\rho\tau u_1 u_2 - A\beta\delta\eta\gamma\mu^2\omega\tau u_1 u_2 - A\beta\delta\eta\gamma\mu\omega\rho\tau u_1 u_2 - 2A\beta\delta\eta\gamma\mu\omega\tau\theta u_1 u_2 \\
& + A\beta\delta\eta\gamma\mu\tau^2\theta u_2^2 - 2A\beta\delta\eta\gamma\omega\rho\tau\theta u_1 u_2 + A\beta\delta\eta\gamma\rho\tau^2\theta u_2^2 - A\beta\delta\eta\mu^2\omega\tau\theta u_1 u_2 - A\beta\delta\eta\mu\omega\rho\tau\theta u_1 u_2 \\
& + A\beta\delta\gamma\mu\rho\tau^2\theta u_2^2 - A\beta\delta\mu^3\omega\tau\theta u_1 u_2 - A\beta\delta\mu^2\omega\rho\tau\theta u_1 u_2 - 2\alpha\delta\eta\gamma\mu^2\tau^2\theta u_2^2 - 2\alpha\delta\eta\gamma\mu\rho\tau^2\theta u_2^2 - 2\alpha\delta\gamma\mu^3\tau^2\theta u_2^2 \\
& - 2\alpha\delta\gamma\mu^2\rho\tau^2\theta u_2^2 - 2\alpha\eta\gamma\mu^3\tau^2\theta u_2^2 - 2\alpha\eta\gamma\mu^2\rho\tau^2\theta u_2^2 - 2\alpha\gamma\mu^4\tau^2\theta u_2^2 - 2\alpha\gamma\mu^3\rho\tau^2\theta u_2^2 - 2\delta\eta\gamma\mu^3\tau^2\theta u_2^2 \\
& - 2\delta\eta\gamma\mu^2\rho\tau^2\theta u_2^2 - 3\delta\eta\gamma\mu^2\tau^2\theta^2 u_2^2 - 3\delta\eta\gamma\mu\rho\tau^2\theta^2 u_2^2 - \delta\eta\mu^3\tau^2\theta^2 u_2^2 - \delta\eta\mu^2\rho\tau^2\theta^2 u_2^2 - 2\delta\gamma\mu^4\tau^2\theta u_2^2 - 2\delta\gamma\mu^3\rho\tau^2\theta \\
& u_2^2 - 3\delta\gamma\mu^3\tau^2\theta^2 u_2^2 - 3\delta\gamma\mu^2\rho\tau^2\theta^2 u_2^2 - \delta\mu^4\tau^2\theta^2 u_2^2 - \delta\mu^3\rho\tau^2\theta^2 u_2^2 - 2\eta\gamma\mu^4\tau^2\theta u_2^2 - 2\eta\gamma\mu^3\rho\tau^2\theta u_2^2 - 3\eta\gamma\mu^3\tau^2\theta^2 u_2^2 \\
& - 3\eta\gamma\mu^2\rho\tau^2\theta^2 u_2^2 - \eta\mu^4\tau^2\theta^2 u_2^2 - \eta\mu^3\rho\tau^2\theta^2 u_2^2 - 2\gamma\mu^5\tau^2\theta u_2^2 - 2\gamma\mu^4\rho\tau^2\theta u_2^2 - 3\gamma\mu^4\tau^2\theta^2 u_2^2 - 3\gamma\mu^3\rho\tau^2\theta^2 u_2^2 - \mu^5\tau^2\theta^2 \\
& u_2^2 - \mu^4\rho\tau^2\theta^2 u_2^2 - A\alpha^2\beta\delta\eta\mu n\omega u_1 - A\alpha^2\beta\delta\eta n\omega\rho u_1 - A\alpha^2\beta\delta\mu^2 n\omega u_1 - A\alpha^2\beta\delta\mu n\omega\rho u_1 + A\alpha\beta\delta\eta\gamma\mu n\omega u_1 \\
& - A\alpha\beta\delta\eta\gamma\mu n\tau u_2 + A\alpha\beta\delta\eta\gamma n\omega\rho u_1 - A\alpha\beta\delta\eta\gamma n\rho\tau u_2 - A\alpha\beta\delta\eta\mu^2 n\omega u_1 - A\alpha\beta\delta\eta\mu n\omega\rho u_1 \\
& - 2A\alpha\beta\delta\eta\mu n\omega\theta u_1 + 2A\alpha\beta\delta\eta\mu n\tau\theta u_2 - 2A\alpha\beta\delta\eta n\omega\rho\theta u_1 + 2A\alpha\beta\delta\eta n\rho\tau\theta u_2 + A\alpha\beta\delta\gamma\mu^2 n\omega u_1
\end{aligned}$$

$$\begin{aligned}
& -A\alpha\beta\delta\gamma\mu^2 n\tau u_2 + A\alpha\beta\delta\gamma\mu n\omega\rho u_1 - A\alpha\beta\delta\gamma\mu n\rho\tau u_2 - A\alpha\beta\delta\mu^3 n\omega u_1 - A\alpha\beta\delta\mu^2 n\omega\rho u_1 - 2A\alpha\beta\delta\mu^2 n\omega\theta u_1 \\
& + 2A\alpha\beta\delta\mu^2 n\tau\theta u_2 - 2A\alpha\beta\delta\mu n\omega\rho\theta u_1 + 2A\alpha\beta\delta\mu n\rho\tau\theta u_2 + A\beta\delta\eta\gamma\mu^2 n\omega u_1 - A\beta\delta\eta\gamma\mu^2 n\tau u_2 + A\beta\delta\eta\gamma\mu n\omega\rho u_1 \\
& + A\beta\delta\eta\gamma\mu n\omega\theta u_1 - A\beta\delta\eta\gamma\mu n\rho\tau u_2 - 2A\beta\delta\eta\gamma\mu n\tau\theta u_2 + A\beta\delta\eta\gamma n\rho\theta u_1 - 2A\beta\delta\eta\gamma n\rho\tau\theta u_2 - A\beta\delta\eta\mu^2 n\omega\theta u_1 \\
& + A\beta\delta\eta\mu^2 n\tau\theta u_2 - A\beta\delta\eta\mu n\omega\rho\theta u_1 - A\beta\delta\eta\mu n\omega\theta^2 u_1 + A\beta\delta\eta\mu n\rho\tau\theta u_2 + 2A\beta\delta\eta\mu n\tau\theta^2 u_2 - A\beta\delta\eta n\omega\rho\theta^2 u_1 \\
& + 2A\beta\delta\eta n\rho\tau\theta^2 u_2 + A\beta\delta\gamma\mu^3 n\omega u_1 - A\beta\delta\gamma\mu^3 n\tau u_2 + A\beta\delta\gamma\mu^2 n\omega\rho u_1 + A\beta\delta\gamma\mu^2 n\omega\theta u_1 - A\beta\delta\gamma\mu^2 n\rho\tau u_2 \\
& - \mu^4 \rho\theta) \delta, S = (\delta\gamma\tau^2\theta u_2^2 + \gamma\mu\tau^2\theta u_2^2 + \alpha\delta\gamma\tau u_2 + \alpha\gamma\mu\tau u_2 + \delta\gamma\mu\tau u_2 + 2\delta\gamma\tau\theta u_2 + \delta\mu\tau\theta u_2 + \gamma\mu^2\tau u_2 \\
& + 2\gamma\mu\tau\theta u_2 + \mu^2\tau\theta u_2 + \alpha\delta\gamma + \alpha\delta\mu + \alpha\gamma\mu + \alpha\mu^2 + \delta\gamma\mu + \delta\gamma\theta + \delta\mu^2 + \delta\mu\theta + \gamma\mu^2 + \gamma\mu\theta + \mu^3 + \mu^2\theta) / \\
& (\beta\delta(\gamma n\omega\tau u_1 u_2 - n\omega\theta u_1 u_2 - \gamma\omega\tau u_1 u_2 - \alpha n\omega u_1 + \gamma n\omega u_1 - \gamma n\tau u_2 - n\omega\theta u_1 + n\tau\theta u_2 - \gamma\omega u_1 + \gamma\tau u_2 - \mu\omega u_1 \\
& + \alpha n - \gamma n + n\theta + \gamma + \mu)). I_{\sigma} = -(A\beta\delta\eta\gamma\mu n\omega\tau^2\theta u_1 u_2^2 + A\beta\delta\eta\gamma n\omega\rho\tau^2\theta u_1 u_2^2 - A\beta\delta\eta\mu n\omega\tau^2\theta^2 u_1 u_2^2 \\
& - A\beta\delta\eta n\omega\rho\tau^2\theta^2 u_1 u_2^2 + A\beta\delta\gamma\mu^2 n\omega\tau^2\theta u_1 u_2^2 + A\beta\delta\gamma\mu n\omega\rho\tau^2\theta u_1 u_2^2 - A\beta\delta\mu^2 n\omega\tau^2\theta^2 u_1 u_2^2 - A\beta\delta\mu n\omega\rho\tau^2\theta^2 u_1 u_2^2 \\
& - \delta\eta\gamma\mu\omega\tau^3\theta^2 u_1 u_2^2 - \delta\gamma\mu^2\omega\tau^3\theta^2 u_1 u_2^2 - \delta\gamma\mu\omega\rho\tau^3\theta^2 u_1 u_2^2 - \eta\gamma\mu^2\omega\tau^3\theta^2 u_1 u_2^2 - \gamma\mu^3\omega\tau^3\theta^2 u_1 u_2^2 - \gamma\mu^2\omega\rho\tau^3\theta^2 u_1 u_2^2 \\
& - A\beta\delta\eta\gamma\mu\omega\tau^2\theta u_1 u_2^2 - A\beta\delta\eta\gamma\omega\rho\tau^2\theta u_1 u_2^2 - A\beta\delta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - A\beta\delta\gamma\mu\omega\rho\tau^2\theta u_1 u_2^2 - \delta\eta\gamma\mu^2\tau^3\theta^2 u_2^2 \\
& - \delta\eta\gamma\mu\rho\tau^3\theta^2 u_2^2 - \delta\gamma\mu^3\tau^3\theta^2 u_2^2 - \delta\gamma\mu^2\rho\tau^3\theta^2 u_2^2 - \eta\gamma\mu^3\tau^3\theta^2 u_2^2 - \eta\gamma\mu^2\rho\tau^3\theta^2 u_2^2 - \gamma\mu^4\tau^3\theta^2 u_2^2 - \gamma\mu^3\rho\tau^3\theta^2 u_2^2 \\
& + A\alpha\beta\delta\eta\gamma\mu n\omega\tau u_1 u_2 + A\alpha\beta\delta\eta\gamma n\omega\rho\tau u_1 u_2 - 2A\alpha\beta\delta\eta\mu n\omega\tau\theta u_1 u_2 - 2A\alpha\beta\delta\eta n\omega\rho\tau\theta u_1 u_2 \\
& + A\alpha\beta\delta\gamma\mu^2 n\omega\tau u_1 u_2 + A\alpha\beta\delta\gamma\mu n\omega\rho\tau u_1 u_2 - 2A\alpha\beta\delta\mu^2 n\omega\tau\theta u_1 u_2 - 2A\alpha\beta\delta\mu n\omega\rho\tau\theta u_1 u_2 + A\beta\delta\eta\gamma\mu^2 n\omega\tau u_1 u_2 \\
& + A\beta\delta\eta\gamma\mu n\omega\rho\tau u_1 u_2 + 2A\beta\delta\eta\gamma\mu n\omega\tau\theta u_1 u_2 - A\beta\delta\eta\gamma\mu n\tau^2\theta u_2^2 + 2A\beta\delta\eta n\omega\rho\tau\theta u_1 u_2 - A\beta\delta\eta\gamma n\rho\tau^2\theta u_2^2 \\
& - A\beta\delta\eta\mu^2 n\omega\tau\theta u_1 u_2 - A\beta\delta\eta\mu n\omega\rho\tau\theta u_1 u_2 - 2A\beta\delta\eta\mu n\omega\tau\theta^2 u_1 u_2 + A\beta\delta\eta\mu n\tau^2\theta^2 u_2^2 - 2A\beta\delta\eta n\omega\rho\tau\theta^2 u_1 u_2 \\
& + A\beta\delta\eta n\rho\tau^2\theta^2 u_2^2 + A\beta\delta\gamma\mu^3 n\omega\tau u_1 u_2 + A\beta\delta\gamma\mu^2 n\omega\rho\tau u_1 u_2 + 2A\beta\delta\gamma\mu^2 n\omega\tau\theta u_1 u_2 - A\beta\delta\gamma\mu^2 n\tau^2\theta u_2^2 \\
& + 2A\beta\delta\gamma\mu n\omega\rho\tau\theta u_1 u_2 - A\beta\delta\gamma\mu n\rho\tau^2\theta u_2^2 - A\beta\delta\mu^3 n\omega\tau\theta u_1 u_2 - A\beta\delta\mu^2 n\omega\rho\tau\theta u_1 u_2 - 2A\beta\delta\mu^2 n\omega\tau\theta^2 u_1 u_2 \\
& + A\beta\delta\mu^2 n\tau^2\theta^2 u_2^2 - 2A\beta\delta\mu n\omega\rho\tau\theta^2 u_1 u_2 + A\beta\delta\mu n\rho\tau^2\theta^2 u_2^2 - 2\alpha\delta\eta\gamma\mu\omega\tau^2\theta u_1 u_2^2 - 2\alpha\delta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 \\
& - 2\alpha\delta\gamma\mu\omega\rho\tau^2\theta u_1 u_2^2 - 2\alpha\eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - 2\alpha\gamma\mu^3\omega\tau^2\theta u_1 u_2^2 - 2\alpha\gamma\mu^2\omega\rho\tau^2\theta u_1 u_2^2 - 2\delta\eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 \\
& - \delta\mu^5\rho - 2\delta\mu^5\theta - 2\delta\mu^4\rho\theta - \delta\mu^4\theta^2 - \delta\mu^3\rho\theta^2 - \eta\gamma\mu^5 - \eta\gamma\mu^4\rho - 2\eta\gamma\mu^4\theta - 2\eta\gamma\mu^3\rho\theta - \eta\gamma\mu^3\theta^2 - \eta\gamma\mu^2\rho\theta^2 \\
& - \eta\mu^6 - \eta\mu^5\rho - 2\eta\mu^5\theta - 2\eta\mu^4\rho\theta - \eta\mu^4\theta^2 - \eta\mu^3\rho\theta^2 - \gamma\mu^6 - \gamma\mu^5\rho - 2\gamma\mu^5\theta - 2\gamma\mu^4\rho\theta - \gamma\mu^4\theta^2 - \gamma\mu^3\rho\theta^2 - \mu^7 \\
& - \mu^6\rho - 2\mu^6\theta - 2\mu^5\rho\theta - \mu^5\theta^2 - \mu^4\rho\theta^2) / (\beta(\delta\eta\gamma^2 m\mu n^2\omega\tau^3\theta u_1 u_2^2 - \delta\eta\gamma m\mu n^2\omega\tau^3\theta^2 u_1 u_2^2 \\
& - 2\delta\eta\gamma^2 m\mu n\omega\tau^3\theta u_1 u_2^2 + \delta\eta\gamma m\mu n\omega\tau^3\theta^2 u_1 u_2^2 - \alpha\delta\eta\gamma m\mu n^2\omega\tau^3\theta u_1 u_2^2 + 3\delta\eta\gamma^2 m\mu n^2\omega\tau^2\theta u_1 u_2^2 - \delta\eta\gamma^2 m\mu n^2\tau^3\theta \\
& u_2^2 + \delta\eta\gamma^2 m\mu\omega\tau^3\theta u_1 u_2^2 + \delta\eta\gamma m\mu^2 n^2\omega\tau^2\theta u_1 u_2^2 - 3\delta\eta\gamma m\mu n^2\omega\tau^2\theta^2 u_1 u_2^2 + \delta\eta\gamma m\mu n^2\tau^3\theta^2 u_2^2 - \delta\eta m\mu^2 n^2\omega\tau^2\theta^2 u_1 u_2^2 \\
& - \delta\gamma^2\mu^2 n\omega\tau^3\theta u_1 u_2^2 - \delta\gamma^2\mu n\omega\rho\tau^3\theta u_1 u_2^2 + \delta\gamma\mu^2 n\omega\tau^3\theta^2 u_1 u_2^2 + \delta\gamma\mu n\omega\rho\tau^3\theta^2 u_1 u_2^2 - \eta\gamma^2\mu^2 n\omega\tau^3\theta u_1 u_2^2 \\
& - \eta\gamma^2\mu n\omega\rho\tau^3\theta u_1 u_2^2 + \eta\gamma\mu^2 n\omega\tau^3\theta^2 u_1 u_2^2 + \eta\gamma\mu n\omega\rho\tau^3\theta^2 u_1 u_2^2 - \gamma^2\mu^3 n\omega\tau^3\theta u_1 u_2^2 - \gamma^2\mu^2 n\omega\rho\tau^3\theta u_1 u_2^2 \\
& + \gamma\mu^3 n\omega\tau^3\theta^2 u_1 u_2^2 + \gamma\mu^2 n\omega\rho\tau^3\theta^2 u_1 u_2^2 + \alpha\delta\eta\gamma^2 m\mu n^2\omega\tau^2 u_1 u_2^2 + \alpha\delta\eta\gamma^2 m\mu n^2\omega\rho\tau^2 u_1 u_2^2 + \alpha\delta\eta\gamma m\mu n\omega\tau^2\theta u_1 u_2^2
\end{aligned}$$



$$\begin{aligned}
& -\alpha\delta\gamma^2\mu\rho - 2\alpha\delta\gamma\mu^3 - 2\alpha\delta\gamma\mu^2\rho - \alpha\delta\mu^4 - \alpha\delta\mu^3\rho - \alpha\eta\gamma^2\mu^2 - \alpha\eta\gamma^2\mu\rho - 2\alpha\eta\gamma\mu^3 - 2\alpha\eta\gamma\mu^2\rho - \alpha\eta\mu^4 \\
& -\alpha\eta\mu^3\rho - \alpha\gamma^2\mu^3 - \alpha\gamma^2\mu^2\rho - 2\alpha\gamma\mu^4 - 2\alpha\gamma\mu^3\rho - \alpha\mu^5 - \alpha\mu^4\rho - \delta\eta\gamma^2\mu^2 - \delta\eta\gamma^2\mu\rho - 2\delta\eta\gamma\mu^3 - 2\delta\eta\gamma\mu^2\rho \\
& -\delta\eta\mu^4 - \delta\eta\mu^3\rho - \delta\gamma^2\mu^3 - \delta\gamma^2\mu^2\rho - \delta\gamma^2\mu^2\theta - \delta\gamma^2\mu\rho\theta - 2\delta\gamma\mu^4 - 2\delta\gamma\mu^3\rho - 2\delta\gamma\mu^3\theta - 2\delta\gamma\mu^2\rho\theta - \delta\mu^5 - \delta\mu^4\rho \\
& -\delta\mu^4\theta - \delta\mu^3\rho\theta - \eta\gamma^2\mu^3 - \eta\gamma^2\mu^2\rho - \eta\gamma^2\mu^2\theta - \eta\gamma^2\mu\rho\theta - 2\eta\gamma\mu^4 - 2\eta\gamma\mu^3\rho - 2\eta\gamma\mu^3\theta - 2\eta\gamma\mu^2\rho\theta - \eta\mu^5 \\
& -\eta\mu^4\rho - \eta\mu^4\theta - \eta\mu^3\rho\theta - \gamma^2\mu^4 - \gamma^2\mu^3\rho - \gamma^2\mu^3\theta - \gamma^2\mu^2\rho\theta - 2\gamma\mu^5 - 2\gamma\mu^4\rho - 2\gamma\mu^4\theta - 2\gamma\mu^3\rho\theta - \mu^6 - \mu^5\rho - \mu^5\theta \\
& -\mu^4\rho\theta), I_5 = ((\gamma n\tau u_2 - \gamma\tau u_2 + \gamma n + \mu n - \gamma - \mu)(A\beta\delta\eta\gamma\mu n\omega\tau u_1 u_2 + A\beta\delta\eta\gamma n\omega\rho\tau u_1 u_2 \\
& - A\beta\delta\eta n\omega\rho\tau\theta u_1 u_2 - A\beta\delta\eta n\omega\rho\tau\theta u_1 u_2 + A\beta\delta\gamma\mu^2 n\omega\tau u_1 u_2 + A\beta\delta\gamma\mu n\omega\rho\tau u_1 u_2 - A\beta\delta\mu^2 n\omega\tau\theta u_1 u_2 \\
& - A\beta\delta\mu n\omega\rho\tau\theta u_1 u_2 - \delta\eta\gamma\mu\omega\tau^2\theta u_1 u_2^2 - \delta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - \delta\gamma\mu\omega\rho\tau^2\theta u_1 u_2^2 - \eta\gamma\mu^2\omega\tau^2\theta u_1 u_2^2 - \gamma\mu^3\omega\tau^2\theta u_1 u_2^2 \\
& - \gamma\mu^2\omega\rho\tau^2\theta u_1 u_2^2 - A\beta\delta\eta\gamma\mu\omega\tau u_1 u_2 - A\beta\delta\eta\gamma\omega\rho\tau u_1 u_2 - A\beta\delta\gamma\mu^2\omega\tau u_1 u_2 - A\beta\delta\gamma\mu\omega\rho\tau u_1 u_2 - \delta\eta\gamma\mu^2\tau^2\theta u_1 u_2^2 \\
& - \delta\eta\gamma\mu\rho\tau^2\theta u_1 u_2^2 - \delta\gamma\mu^3\tau^2\theta u_1 u_2^2 - \delta\gamma\mu^2\rho\tau^2\theta u_1 u_2^2 - \eta\gamma\mu^3\tau^2\theta u_1 u_2^2 - \eta\gamma\mu^2\rho\tau^2\theta u_1 u_2^2 - \gamma\mu^4\tau^2\theta u_1 u_2^2 - \gamma\mu^3\rho\tau^2\theta u_1 u_2^2 \\
& - A\alpha\beta\delta\eta n\omega u_1 - A\alpha\beta\delta\eta n\omega\rho u_1 - A\alpha\beta\delta\mu^2 n\omega u_1 - A\alpha\beta\delta\mu n\omega\rho u_1 + A\beta\delta\eta\gamma\mu n\omega u_1 - A\beta\delta\eta\gamma\mu n\tau u_2 \\
& + A\beta\delta\eta\gamma n\omega\rho u_1 - A\beta\delta\eta\gamma n\omega\rho\tau u_2 - A\beta\delta\eta\mu n\omega\theta u_1 + A\beta\delta\eta\mu n\tau\theta u_2 - A\beta\delta\eta n\omega\rho\theta u_1 + A\beta\delta\eta n\omega\rho\tau\theta u_2 \\
& + A\beta\delta\gamma\mu^2 n\omega u_1 - A\beta\delta\gamma\mu^2 n\tau u_2 + A\beta\delta\gamma\mu n\omega\rho u_1 - A\beta\delta\gamma\mu n\omega\rho\tau u_2 - A\beta\delta\mu^2 n\omega\theta u_1 + A\beta\delta\mu^2 n\tau\theta u_2 \\
& - A\beta\delta\mu n\omega\rho\theta u_1 + A\beta\delta\mu n\omega\rho\tau\theta u_2 - \alpha\delta\eta\gamma\mu\omega\tau u_1 u_2 - \alpha\delta\gamma\mu^2\omega\tau u_1 u_2 - \alpha\delta\gamma\mu\omega\rho\tau u_1 u_2 - \alpha\eta\gamma\mu^2\omega\tau u_1 u_2 \\
& - \alpha\gamma\mu^3\omega\tau u_1 u_2 - \alpha\gamma\mu^2\omega\rho\tau u_1 u_2 - \delta\eta\gamma\mu^2\omega\tau u_1 u_2 - 2\delta\eta\gamma\mu\omega\tau\theta u_1 u_2 - \delta\eta\mu^2\omega\tau\theta u_1 u_2 - \delta\gamma\mu^3\omega\tau u_1 u_2 \\
& - \delta\gamma\mu^2\omega\rho\tau u_1 u_2 - 2\delta\gamma\mu^2\omega\tau\theta u_1 u_2 - 2\delta\gamma\mu\omega\rho\tau\theta u_1 u_2 - \delta\mu^3\omega\tau\theta u_1 u_2 - \delta\mu^2\omega\rho\tau\theta u_1 u_2 - \eta\gamma\mu^3\omega\tau u_1 u_2 \\
& - \eta\gamma\mu^3\rho - \eta\gamma\mu^3\theta - \eta\gamma\mu^2\rho\theta - \eta\mu^5 - \eta\mu^4\rho - \eta\mu^4\theta - \eta\mu^3\rho\theta - \gamma\mu^5 - \gamma\mu^4\rho - \gamma\mu^4\theta - \gamma\mu^3\rho\theta - \mu^6 - \mu^5\rho - \mu^5\theta \\
& - \mu^4\rho\theta) / (\beta(\delta\eta\gamma^2 m\mu n^2\omega\tau^3\theta u_1 u_2^3 - \delta\eta\gamma m\mu n^2\omega\tau^3\theta^2 u_1 u_2^3 - 2\delta\eta\gamma^2 m\mu n\omega\tau^3\theta u_1 u_2^3 + \delta\eta\gamma m\mu n\omega\tau^3\theta^2 u_1 u_2^3 \\
& - \alpha\delta\eta\gamma m\mu n^2\omega\tau^2\theta u_1 u_2^3 + 3\delta\eta\gamma^2 m\mu n^2\omega\tau^2\theta u_1 u_2^3 - \delta\eta\gamma^2 m\mu n^2\tau^3\theta u_1 u_2^3 + \delta\eta\gamma^2 m\mu\omega\tau^3\theta u_1 u_2^3 + \delta\eta\gamma m\mu^2 n^2\omega\tau^2\theta u_1 u_2^3 \\
& - 3\delta\eta\gamma m\mu n^2\omega\tau^2\theta^2 u_1 u_2^3 + \delta\eta\gamma m\mu n^2\tau^3\theta^2 u_1 u_2^3 - \delta\eta m\mu^2 n^2\omega\tau^2\theta^2 u_1 u_2^3 - \delta\gamma^2\mu^2 n\omega\tau^3\theta u_1 u_2^3 - \delta\gamma^2\mu n\omega\rho\tau^3\theta u_1 u_2^3 \\
& + \delta\gamma\mu^2 n\omega\tau^3\theta^2 u_1 u_2^3 + \delta\gamma\mu n\omega\rho\tau^3\theta^2 u_1 u_2^3 - \eta\gamma^2\mu^2 n\omega\tau^3\theta u_1 u_2^3 - \eta\gamma^2\mu n\omega\rho\tau^3\theta u_1 u_2^3 + \eta\gamma\mu^2 n\omega\tau^3\theta^2 u_1 u_2^3 \\
& + \eta\gamma\mu n\omega\rho\tau^3\theta^2 u_1 u_2^3 - \gamma^2\mu^3 n\omega\tau^3\theta u_1 u_2^3 - \gamma^2\mu^2 n\omega\rho\tau^3\theta u_1 u_2^3 + \gamma\mu^3 n\omega\tau^3\theta^2 u_1 u_2^3 + \gamma\mu^2 n\omega\rho\tau^3\theta^2 u_1 u_2^3 \\
& + \alpha\delta\eta\gamma^2 m\mu n^2\omega\tau^2 u_1 u_2^3 + \alpha\delta\eta\gamma^2 m\mu n^2\omega\rho\tau^2 u_1 u_2^3 + \alpha\delta\eta\gamma m\mu n\omega\tau^2\theta u_1 u_2^3 - \alpha\delta\eta\gamma m\mu n^2\omega\tau^2\theta u_1 u_2^3 - \alpha\delta\eta\gamma m\mu n^2\omega\rho\tau^2\theta u_1 u_2^3
\end{aligned}$$

$$\begin{aligned}
& -\gamma^3 \mu^3 n \omega \theta u_1 + 2\gamma^2 \mu^3 n \rho \tau u_2 + 3\gamma^2 \mu^3 n \tau \theta u_2 - \gamma^2 \mu^2 n \omega \rho \theta u_1 + 3\gamma^2 \mu^2 n \rho \tau \theta u_2 - \gamma \mu^5 n \omega u_1 + \gamma \mu^5 n \tau u_2 - \gamma \mu^4 n \omega \rho u_1 \\
& + \gamma \mu^4 n \rho \tau u_2 + \gamma \mu^3 n \omega \theta^2 u_1 - 3\gamma \mu^3 n \tau \theta^2 u_2 + \gamma \mu^2 n \omega \rho \theta^2 u_1 - 3\gamma \mu^2 n \rho \tau \theta^2 u_2 + \mu^5 n \omega \theta u_1 - \mu^5 n \tau \theta u_2 + \mu^4 n \omega \rho \theta u_1 \\
& + \mu^4 n \omega \theta^2 u_1 - \mu^4 n \rho \tau \theta u_2 - 2\mu^4 n \tau \theta^2 u_2 + \mu^3 n \omega \rho \theta^2 u_1 - 2\mu^3 n \rho \tau \theta^2 u_2 + \alpha^2 \delta \eta \gamma \mu n^2 + \alpha^2 \delta \eta \gamma n^2 \rho - \alpha \delta \eta \gamma^2 \mu n^2 \\
& + \alpha \delta \eta \gamma^2 \mu \omega u_1 - 2\alpha \delta \eta \gamma^2 \mu \tau u_2 - \alpha \delta \eta \gamma^2 n^2 \rho + \alpha \delta \eta \gamma^2 \omega \rho u_1 - 2\alpha \delta \eta \gamma^2 \rho \tau u_2 - \alpha \delta \eta \gamma \mu n \theta + \alpha \delta \eta \gamma \mu^2 n^2 \\
& + 2\alpha \delta \eta \gamma \mu^2 \omega u_1 - 2\alpha \delta \eta \gamma \mu^2 \tau u_2 + \alpha \delta \eta \gamma \mu n^2 \rho + \alpha \delta \eta \gamma \mu n^2 \theta + 2\alpha \delta \eta \gamma \mu \omega \rho u_1 - 2\alpha \delta \eta \gamma \mu \rho \tau u_2 + \alpha \delta \eta \gamma n^2 \rho \theta \\
& - \alpha \delta \eta \mu \mu^2 n \theta + \alpha \delta \eta \mu^3 \omega u_1 - \alpha \delta \eta \mu^2 n^2 \theta + \alpha \delta \eta \mu^2 \omega \rho u_1 - \alpha \delta \eta \mu n^2 \rho \theta + \alpha \delta \gamma^2 \mu^2 \omega u_1 - 2\alpha \delta \gamma^2 \mu^2 \tau u_2 \\
& + \alpha \delta \gamma^2 \mu \omega \rho u_1 - 2\alpha \delta \gamma^2 \mu \rho \tau u_2 + 2\alpha \delta \gamma \mu^3 \omega u_1 - 2\alpha \delta \gamma \mu^3 \tau u_2 + 2\alpha \delta \gamma \mu^2 \omega \rho u_1 - 2\alpha \delta \gamma \mu^2 \rho \tau u_2 + \alpha \delta \mu^4 \omega u_1 \\
& + \alpha \delta \mu^3 \omega \rho u_1 + \alpha \eta \gamma^2 \mu^2 \omega u_1 - 2\alpha \eta \gamma^2 \mu^2 \tau u_2 + \alpha \eta \gamma^2 \mu \omega \rho u_1 - 2\alpha \eta \gamma^2 \mu \rho \tau u_2 + 2\alpha \eta \gamma \mu^3 \omega u_1 - 2\alpha \eta \gamma \mu^3 \tau u_2 \\
& + \eta \gamma^2 \mu^2 \omega \theta u_1 - 2\eta \gamma^2 \mu^2 \rho \tau u_2 - 3\eta \gamma^2 \mu^2 \tau \theta u_2 + \eta \gamma^2 \mu \omega \rho \theta u_1 - 3\eta \gamma^2 \mu \rho \tau \theta u_2 + 2\eta \gamma \mu^4 \omega u_1 - 2\eta \gamma \mu^4 \tau u_2 \\
& + 2\eta \gamma \mu^3 \omega \rho u_1 + 2\eta \gamma \mu^3 \omega \theta u_1 - 2\eta \gamma \mu^3 \rho \tau u_2 - 4\eta \gamma \mu^3 \tau \theta u_2 + 2\eta \gamma \mu^2 \omega \rho \theta u_1 - 4\eta \gamma \mu^2 \rho \tau \theta u_2 + \eta \mu^5 \omega u_1 + \eta \mu^4 \omega \rho u_1 \\
& + \eta \mu^4 \omega \theta u_1 - \eta \mu^4 \tau \theta u_2 + \eta \mu^3 \omega \rho \theta u_1 - \eta \mu^3 \rho \tau \theta u_2 + \gamma^2 \mu^4 \omega u_1 - 2\gamma^2 \mu^4 \tau u_2 + \gamma^2 \mu^3 \omega \rho u_1 + \gamma^2 \mu^3 \omega \theta u_1 - 2\gamma^2 \mu^3 \rho \tau u_2 \\
& - 3\gamma^2 \mu^3 \tau \theta u_2 + \gamma^2 \mu^2 \omega \rho \theta u_1 - 3\gamma^2 \mu^2 \rho \tau \theta u_2 + 2\gamma \mu^5 \omega u_1 - 2\gamma \mu^5 \tau u_2 + 2\gamma \mu^4 \omega \rho u_1 + 2\gamma \mu^4 \omega \theta u_1 - 2\gamma \mu^4 \rho \tau u_2 \\
& - 4\gamma \mu^4 \tau \theta u_2 + 2\gamma \mu^3 \omega \rho \theta u_1 - 4\gamma \mu^3 \rho \tau \theta u_2 + \mu^6 \omega u_1 + \mu^5 \omega \rho u_1 + \mu^5 \omega \theta u_1 - \mu^5 \tau \theta u_2 + \mu^4 \omega \rho \theta u_1 - \mu^4 \rho \tau \theta u_2 \\
& - \alpha^2 \delta \eta \gamma \mu n - \alpha^2 \delta \eta \gamma n \rho - \alpha^2 \delta \eta \mu^2 n - \alpha^2 \delta \eta \mu n \rho - \alpha^2 \delta \gamma \mu^2 n - \alpha^2 \delta \gamma \mu n \rho - \alpha^2 \delta \mu^3 n - \alpha^2 \delta \mu^2 n \rho - \alpha^2 \eta \gamma \mu^2 n \\
& - \alpha^2 \eta \gamma \mu n \rho - \alpha^2 \eta \mu^3 n - \alpha^2 \eta \mu^2 n \rho - \alpha^2 \gamma \mu^3 n - \alpha^2 \gamma \mu^2 n \rho - \alpha^2 \mu^4 n - \alpha^2 \mu^3 n \rho + 2\alpha \delta \eta \gamma^2 \mu n + 2\alpha \delta \eta \gamma^2 n \rho \\
& + \alpha \delta \gamma^2 \mu n \rho - 2\alpha \delta \gamma \mu^2 n \theta - 2\alpha \delta \gamma \mu n \rho \theta - \alpha \delta \mu^4 n - \alpha \delta \mu^3 n \rho - 2\alpha \delta \mu^3 n \theta - 2\alpha \delta \mu^2 n \rho \theta + \alpha \eta \gamma^2 \mu^2 n + \alpha \eta \gamma^2 \mu n \rho \\
& - 2\alpha \eta \gamma \mu^2 n \theta - 2\alpha \eta \gamma \mu n \rho \theta - \alpha \eta \mu^4 n - \alpha \eta \mu^3 n \rho - 2\alpha \eta \mu^3 n \theta - 2\alpha \eta \mu^2 n \rho \theta + \alpha \gamma^2 \mu^3 n + \alpha \gamma^2 \mu^2 n \rho - 2\alpha \gamma \mu^3 n \theta \\
& - 2\alpha \gamma \mu^2 n \rho \theta - \alpha \mu^5 n - \alpha \mu^4 n \rho - 2\alpha \mu^4 n \theta - 2\alpha \mu^3 n \rho \theta - \delta \eta \gamma^2 \mu \mu \theta + 2\delta \eta \gamma^2 \mu^2 n + 2\delta \eta \gamma^2 \mu n \rho - 2\delta \eta \gamma \mu \mu^2 \theta \\
& + 2\delta \eta \gamma \mu^3 n + 2\delta \eta \gamma \mu^2 n \rho - 2\delta \eta \gamma \mu^2 n \theta - 2\delta \eta \gamma \mu n \rho \theta - \delta \eta \mu \mu^3 \theta - 2\delta \eta \mu^3 n \theta - 2\delta \eta \mu^2 n \rho \theta + \delta \gamma^2 \mu^3 n + \delta \gamma^2 \mu^2 n \rho \\
& + \delta \gamma^2 \mu^2 n \theta + \delta \gamma^2 \mu n \rho \theta + \delta \gamma \mu^4 n + \delta \gamma \mu^3 n \rho - \delta \gamma \mu^2 n \theta^2 - \delta \gamma \mu n \rho \theta^2 - \delta \mu^4 n \theta - \delta \mu^3 n \rho \theta - \delta \mu^3 n \theta^2 - \delta \mu^2 n \rho \theta^2 \\
& + \eta \gamma^2 \mu^3 n + \eta \gamma^2 \mu^2 n \rho + \eta \gamma^2 \mu^2 n \theta + \eta \gamma^2 \mu n \rho \theta + \eta \gamma \mu^4 n + \eta \gamma \mu^3 n \rho - \eta \gamma \mu^2 n \theta^2 - \eta \gamma \mu n \rho \theta^2 - \eta \mu^4 n \theta - \eta \mu^3 n \rho \theta \\
& - \eta \mu^3 n \theta^2 - \eta \mu^2 n \rho \theta^2 + \gamma^2 \mu^4 n + \gamma^2 \mu^3 n \rho + \gamma^2 \mu^3 n \theta + \gamma^2 \mu^2 n \rho \theta + \gamma \mu^5 n + \gamma \mu^4 n \rho - \gamma \mu^3 n \theta^2 - \gamma \mu^2 n \rho \theta^2 - \mu^5 n \theta \\
& - \mu^4 n \rho \theta - \mu^4 n \theta^2 - \mu^3 n \rho \theta^2 - \alpha \delta \eta \gamma^2 \mu - \alpha \delta \eta \gamma^2 \rho - 2\alpha \delta \eta \gamma \mu^2 - 2\alpha \delta \eta \gamma \mu \rho - \alpha \delta \eta \mu^3 - \alpha \delta \eta \mu^2 \rho - \alpha \delta \gamma^2 \mu^2 \\
& - \alpha \delta \gamma^2 \mu \rho - 2\alpha \delta \gamma \mu^3 - 2\alpha \delta \gamma \mu^2 \rho - \alpha \delta \mu^4 - \alpha \delta \mu^3 \rho - \alpha \eta \gamma^2 \mu^2 - \alpha \eta \gamma^2 \mu \rho - 2\alpha \eta \gamma \mu^3 - 2\alpha \eta \gamma \mu^2 \rho - \alpha \eta \mu^4 \\
& - \alpha \eta \mu^3 \rho - \alpha \gamma^2 \mu^3 - \alpha \gamma^2 \mu^2 \rho - 2\alpha \gamma \mu^4 - 2\alpha \gamma \mu^3 \rho - \alpha \mu^5 - \alpha \mu^4 \rho - \delta \eta \gamma^2 \mu^2 - \delta \eta \gamma^2 \mu \rho - 2\delta \eta \gamma \mu^3 - 2\delta \eta \gamma \mu^2 \rho \\
& - \delta \eta \mu^4 - \delta \eta \mu^3 \rho - \delta \gamma^2 \mu^3 - \delta \gamma^2 \mu^2 \rho - \delta \gamma^2 \mu^2 \theta - \delta \gamma^2 \mu \rho \theta - 2\delta \gamma \mu^4 - 2\delta \gamma \mu^3 \rho - 2\delta \gamma \mu^3 \theta - 2\delta \gamma \mu^2 \rho \theta - \delta \mu^5 - \delta \mu^4 \rho \\
& - \delta \mu^4 \theta - \delta \mu^3 \rho \theta - \eta \gamma^2 \mu^3 - \eta \gamma^2 \mu^2 \rho - \eta \gamma^2 \mu^2 \theta - \eta \gamma^2 \mu \rho \theta - 2\eta \gamma \mu^4 - 2\eta \gamma \mu^3 \rho - 2\eta \gamma \mu^3 \theta - 2\eta \gamma \mu^2 \rho \theta - \eta \mu^5 \\
& - \eta \mu^4 \rho - \eta \mu^4 \theta - \eta \mu^3 \rho \theta - \gamma^2 \mu^4 - \gamma^2 \mu^3 \rho - \gamma^2 \mu^3 \theta - \gamma^2 \mu^2 \rho \theta - 2\gamma \mu^5 - 2\gamma \mu^4 \rho - 2\gamma \mu^4 \theta - 2\gamma \mu^3 \rho \theta - \mu^6 - \mu^5 \rho - \mu^5 \theta \\
& - \mu^4 \rho \theta), \mathcal{R}_j = (\mathcal{A} \beta \delta^2 \eta \gamma^2 m n^2 \omega \tau^3 \theta u_1 u_2^2 - \mathcal{A} \beta \delta^2 \eta \gamma m n^2 \omega \tau^3 \theta^2 u_1 u_2^2 + \mathcal{A} \beta \delta^2 \gamma^2 m n^2 \omega \tau^3 \theta u_1 u_2^2 \\
& - \mathcal{A} \beta \delta^2 \gamma m \mu n^2 \omega \tau^3 \theta^2 u_1 u_2^2 - \delta^2 \gamma^2 m n \omega \tau^4 \theta^2 u_1 u_2^2 - \delta \gamma^2 m \mu^2 n \omega \tau^4 \theta^2 u_1 u_2^2 - 2\mathcal{A} \beta \delta^2 \eta \gamma^2 m n \omega \tau^3 \theta u_1 u_2^2
\end{aligned}$$

$$\begin{aligned}
& + A\beta\delta^2\eta\gamma mn\omega\tau^3\theta^2 u_1 u_2^2 - 2A\beta\delta^2\gamma^2 m\mu n\omega\tau^3\theta u_1 u_2^2 + A\beta\delta^2\gamma m\mu n\omega\tau^3\theta^2 u_1 u_2^2 - \delta^2\eta\gamma^2 m\mu n\tau^4\theta^2 u_1^4 \\
& - \delta^2\gamma^2 m\mu^2 n\tau^4\theta^2 u_2^4 + \delta^2\gamma^2 m\mu\omega\tau^4\theta^2 u_1 u_2^4 - \delta\eta\gamma^2 m\mu^2 n\tau^4\theta^2 u_2^4 - \delta\gamma^2 m\mu^3 n\tau^4\theta^2 u_2^4 + \delta\gamma^2 m\mu^2\omega\tau^4\theta^2 u_1 u_2^4 \\
& - A\alpha\beta\delta^2\eta\gamma m\mu^2\omega\tau^2\theta u_1 u_2^2 - A\alpha\beta\delta^2\gamma m\mu^2\omega\tau^2\theta u_1 u_2^2 + 3A\beta\delta^2\eta\gamma^2 m\mu^2\omega\tau^2\theta u_1 u_2^2 - A\beta\delta^2\eta\gamma^2 m\mu^2\tau^3\theta u_1^3 \\
& + A\beta\delta^2\eta\gamma^2 m\omega\tau^3\theta u_1 u_2^3 + A\beta\delta^2\eta\gamma m\mu^2\omega\tau^3\theta u_1 u_2^3 - 3A\beta\delta^2\eta\gamma m\mu^2\omega\tau^3\theta^2 u_1 u_2^3 + A\beta\delta^2\eta\gamma m\mu^2\tau^3\theta^2 u_1^3 \\
& - A\beta\delta^2\eta m\mu^2\omega\tau^3\theta^2 u_1 u_2^3 + 3A\beta\delta^2\gamma^2 m\mu^2\omega\tau^3\theta u_1 u_2^3 - A\beta\delta^2\gamma^2 m\mu^2\tau^3\theta u_2^3 + A\beta\delta^2\gamma^2 m\mu\omega\tau^3\theta u_1 u_2^3 \\
& + A\beta\delta^2\gamma m\mu^2\mu^2\omega\tau^3\theta u_1 u_2^3 - 3A\beta\delta^2\gamma m\mu^2\omega\tau^3\theta^2 u_1 u_2^3 + A\beta\delta^2\gamma m\mu^2\tau^3\theta u_2^3 - A\beta\delta^2 m\mu^2\mu^2\omega\tau^3\theta^2 u_1 u_2^3 \\
& - \alpha\delta^2\gamma^2 m\mu n\omega\tau^3\theta u_1 u_2^3 - \alpha\delta^2\gamma^2 m\mu^2 n\omega\tau^3\theta u_1 u_2^3 + \delta^2\eta\gamma^2 m\mu^4\tau^4\theta^2 u_2^4 - \delta^2\gamma^2 m\mu^2 n\omega\tau^3\theta u_1 u_2^3 + \delta^2\gamma^2 m\mu^2\tau^4\theta^2 u_2^4 \\
& - 4\delta^2\gamma^2 m\mu n\omega\tau^3\theta^2 u_1 u_2^3 - \delta^2\gamma^2 m\omega\tau^4\theta^2 u_1 u_2^4 - 2\delta^2\gamma m\mu^2 n\omega\tau^3\theta^2 u_1 u_2^3 + \delta\eta\gamma^2 m\mu^2\tau^4\theta^2 u_2^4 - \delta\eta\gamma^2 m\omega\tau^4\theta^2 u_1 u_2^4 \\
& - \delta\gamma^2 m\mu^3 n\omega\tau^3\theta u_1 u_2^3 + \delta\gamma^2 m\mu^3\tau^4\theta^2 u_2^4 - 4\delta\gamma^2 m\mu^2 n\omega\tau^3\theta^2 u_1 u_2^3 - 2\delta\gamma^2 m\mu^2\omega\tau^4\theta^2 u_1 u_2^4 - 2\delta\gamma m\mu^3 n\omega\tau^3\theta^2 u_1 u_2^3 \\
& - \eta\gamma^2 m\mu^2\omega\tau^4\theta^2 u_1 u_2^4 - \gamma^2 m\mu^3\omega\tau^4\theta^2 u_1 u_2^4 + A\alpha\beta\delta^2\eta\gamma mn\omega\tau^2\theta u_1 u_2^2 + A\alpha\beta\delta^2\gamma m\mu n\omega\tau^2\theta u_1 u_2^2 \\
& - 6A\beta\delta^2\eta\gamma^2 m n\omega\tau^2\theta u_1 u_2^2 + 2A\beta\delta^2\eta\gamma^2 m n\tau^3\theta u_2^3 - 3A\beta\delta^2\eta\gamma m\mu n\omega\tau^2\theta u_1 u_2^2 + 3A\beta\delta^2\eta\gamma mn\omega\tau^2\theta^2 u_1 u_2^2 \\
& - A\beta\delta^2\eta\gamma m n\tau^3\theta^2 u_2^3 + A\beta\delta^2\eta m\mu n\omega\tau^2\theta^2 u_1 u_2^2 - 6A\beta\delta^2\gamma^2 m\mu n\omega\tau^2\theta u_1 u_2^2 + 2A\beta\delta^2\gamma^2 m\mu n\tau^3\theta u_2^3 \\
& - 3A\beta\delta^2\gamma m\mu^2 n\omega\tau^2\theta u_1 u_2^2 + 3A\beta\delta^2\gamma m\mu n\omega\tau^2\theta^2 u_1 u_2^2 - A\beta\delta^2\gamma m\mu n\tau^3\theta^2 u_2^3 + A\beta\delta^2 m\mu^2 n\omega\tau^2\theta^2 u_1 u_2^2 \\
& - \alpha\delta^2\eta\gamma^2 m\mu n\tau^3\theta u_2^3 + \alpha\delta^2\eta\gamma^2 n\omega\tau^3\theta u_1 u_2^3 - \alpha\delta^2\gamma^2 m\mu^2 n\tau^3\theta u_2^3 + \alpha\delta^2\gamma^2 m\mu\omega\tau^3\theta u_1 u_2^3 - \alpha\delta\eta\gamma^2 m\mu^2 n\tau^3\theta u_2^3 \\
& + \alpha\delta\eta\gamma^2 m n\omega\tau^3\theta u_1 u_2^3 - \alpha\delta\gamma^2 m\mu^3 n\tau^3\theta u_2^3 + \alpha\delta\gamma^2 m\mu^2\omega\tau^3\theta u_1 u_2^3 - \delta^2\eta\gamma^2 m\mu^2 n\tau^3\theta u_2^3 - 4\delta^2\eta\gamma^2 m\mu n\tau^3\theta^2 u_2^3 \\
& + \delta^2\gamma^2 m\mu^2\omega\tau^3\theta u_1 u_2^3 + 4\delta^2\gamma^2 m\mu\omega\tau^3\theta^2 u_1 u_2^3 - 2\delta^2\gamma m\mu^3 n\tau^3\theta^2 u_2^3 + 2\delta^2\gamma m\mu^2\omega\tau^3\theta^2 u_1 u_2^3 - \delta\eta\gamma^2 m\mu^3 n\tau^3\theta u_2^3 \\
& - 4\delta\eta\gamma^2 m\mu^2 n\tau^3\theta^2 u_2^3 + \delta\eta\gamma^2 m\mu^2 n\omega\tau^3\theta u_1 u_2^3 - 2\delta\eta\gamma m\mu^3 n\tau^3\theta^2 u_2^3 - \delta\eta\gamma m\mu^2 n\omega\tau^3\theta^2 u_1 u_2^3 - \delta\gamma^2 m\mu^4 n\tau^3\theta u_2^3 \\
& - 4\delta\gamma^2 m\mu^3 n\tau^3\theta^2 u_2^3 + \delta\gamma^2 m\mu^3\omega\tau^3\theta u_1 u_2^3 + 4\delta\gamma^2 m\mu^2\omega\tau^3\theta^2 u_1 u_2^3 - 2\delta\gamma m\mu^4 n\tau^3\theta^2 u_2^3 + 2\delta\gamma m\mu^3\omega\tau^3\theta^2 u_1 u_2^3 \\
& - 2A\alpha\beta\delta^2\eta\gamma m\mu^2\omega\tau\theta u_1 u_2 + A\alpha\beta\delta^2\eta\gamma m\mu^2\tau^2\theta u_2^2 - A\alpha\beta\delta^2\eta m\mu\mu^2\omega\tau\theta u_1 u_2 - 2A\alpha\beta\delta^2\gamma m\mu^2\omega\tau\theta u_1 u_2 \\
& + A\alpha\beta\delta^2\gamma m\mu^2\tau^2\theta u_2^2 - A\alpha\beta\delta^2 m\mu^2\mu^2\omega\tau\theta u_1 u_2 + 3A\beta\delta^2\eta\gamma^2 m\mu^2\omega\tau\theta u_1 u_2 - 3A\beta\delta^2\eta\gamma^2 m\mu^2\tau^2\theta u_2^2 \\
& + 3A\beta\delta^2\eta\gamma^2 m\omega\tau^2\theta u_1 u_2^2 - A\beta\delta^2\eta\gamma^2 m\tau^3\theta u_2^3 + 2A\beta\delta^2\eta\gamma m\mu^2\omega\tau\theta u_1 u_2 - A\beta\delta^2\eta\gamma m\mu^2\tau^2\theta u_2^2 \\
& + 2A\beta\delta^2\eta\gamma m\mu\omega\tau^2\theta u_1 u_2^2 - 3A\beta\delta^2\eta\gamma m\mu^2\omega\tau\theta^2 u_1 u_2 + 3A\beta\delta^2\eta\gamma m\mu^2\tau^2\theta^2 u_2^2 - 2A\beta\delta^2\eta m\mu^2\omega\tau\theta^2 u_1 u_2 \\
& - \delta\eta\gamma^2 m\mu^3 n\theta - \delta\eta\gamma^2 m\mu^2 n\theta^2 + \delta\eta\gamma^2 m^3 n\omega u_1 + \delta\eta\gamma^2 m^2 n\omega\theta u_1 - 2\delta\eta\gamma m\mu^4 n\theta - 2\delta\eta\gamma m\mu^3 n\theta^2 + \delta\eta\gamma m^4 n\omega u_1 \\
& - \delta\eta\gamma m^2 n\omega\theta^2 u_1 - \delta\eta m\mu^5 n\theta - \delta\eta m\mu^4 n\theta^2 - \delta\eta m^4 n\omega\theta u_1 - \delta\eta m^3 n\omega\theta^2 u_1 - \delta\gamma^2 m\mu^4 n\theta - \delta\gamma^2 m\mu^3 n\theta^2 \\
& + \delta\gamma^2 m\mu^3\omega\theta u_1 + \delta\gamma^2 m\mu^2\omega\theta^2 u_1 - 2\delta\gamma m\mu^5 n\theta - 2\delta\gamma m\mu^4 n\theta^2 + 2\delta\gamma m\mu^4\omega\theta u_1 + 2\delta\gamma m\mu^3\omega\theta^2 u_1 - \delta m\mu^6 n\theta \\
& - \delta m\mu^5 n\theta^2 + \delta m\mu^5\omega\theta u_1 + \delta m\mu^4\omega\theta^2 u_1 - A\beta\delta^2\eta\gamma^2 m\theta - 2A\beta\delta^2\eta\gamma m\mu\theta - A\beta\delta^2\eta m\mu^2\theta - A\beta\delta^2\gamma^2 m\mu\theta \\
& - 2A\beta\delta^2\gamma m\mu^2\theta - A\beta\delta^2 m\mu^3\theta - \alpha^2\delta^2\eta\gamma^2\omega u_1 - 2\alpha^2\delta^2\eta\gamma\mu\omega u_1 - \alpha^2\delta^2\eta\mu^2\omega u_1 - \alpha^2\delta^2\gamma^2\mu\omega u_1 - 2\alpha^2\delta^2\gamma\mu^2\omega u_1 \\
& - \alpha^2\delta^2\mu^3\omega u_1 - 2\alpha^2\delta\eta\gamma^2\mu\omega u_1 - 4\alpha^2\delta\eta\gamma\mu^2\omega u_1 - 2\alpha^2\delta\eta\mu^3\omega u_1 - 2\alpha^2\delta\gamma^2\mu^2\omega u_1 - 4\alpha^2\delta\gamma\mu^3\omega u_1 - 2\alpha^2\delta\mu^4\omega u_1 \\
& - \alpha^2\eta\gamma^2\mu^2\omega u_1 - 2\alpha^2\eta\gamma\mu^3\omega u_1 - \alpha^2\eta\mu^4\omega u_1 - \alpha^2\gamma^2\mu^3\omega u_1 - 2\alpha^2\gamma\mu^4\omega u_1 - \alpha^2\mu^5\omega u_1 + \alpha\delta^2\eta\gamma^2 m\mu\theta \\
& - 2\alpha\delta^2\eta\gamma^2\mu\omega u_1 - \alpha\delta^2\eta\gamma^2\omega\theta u_1 + 2\alpha\delta^2\eta\gamma m\mu^2\theta - 4\alpha\delta^2\eta\gamma\mu^2\omega u_1 - 2\alpha\delta^2\eta\gamma\mu\omega\theta u_1 + \alpha\delta^2\eta m\mu^3\theta
\end{aligned}$$



$$\begin{aligned}
& + 2\alpha\mu^5 \omega \rho u_1 + 2\alpha\mu^4 \omega \rho \theta u_1 + \delta^2 \gamma^2 m \mu^3 \theta + \delta^2 \gamma^2 m \mu^2 \theta^2 - \delta^2 \gamma^2 \mu^4 n - \delta^2 \gamma^2 \mu^3 n \rho - \delta^2 \gamma^2 \mu^3 n \theta - \delta^2 \gamma^2 \mu^2 n \rho \theta \\
& + \delta^2 \gamma^2 \mu^2 \omega \rho u_1 - \delta^2 \gamma^2 \mu^2 \omega \theta u_1 + \delta^2 \gamma^2 \mu \omega \rho \theta u_1 - \delta^2 \gamma^2 \mu \omega \theta^2 u_1 + 2\delta^2 \gamma m \mu^4 \theta + 2\delta^2 \gamma m \mu^3 \theta^2 - \delta^2 \gamma \mu^5 n - \delta^2 \gamma \mu^4 n \rho \\
& + \delta^2 \gamma \mu^3 n \theta^2 + 2\delta^2 \gamma \mu^3 \omega \rho u_1 - 2\delta^2 \gamma \mu^3 \omega \theta u_1 + \delta^2 \gamma \mu^2 n \rho \theta^2 + 2\delta^2 \gamma \mu^2 \omega \rho \theta u_1 - 2\delta^2 \gamma \mu^2 \omega \theta^2 u_1 + \delta^2 m \mu^5 \theta + \delta^2 m \mu^4 \theta^2 \\
& + \delta^2 \mu^5 n \theta + \delta^2 \mu^4 n \rho \theta + \delta^2 \mu^4 n \theta^2 + \delta^2 \mu^4 \omega \rho u_1 - \delta^2 \mu^4 \omega \theta u_1 + \delta^2 \mu^3 n \rho \theta^2 + \delta^2 \mu^3 \omega \rho \theta u_1 - \delta^2 \mu^3 \omega \theta^2 u_1 + \delta \gamma^2 m \mu^4 \theta \\
& + \delta \gamma^2 m \mu^3 \theta^2 - \delta \gamma^2 \mu^5 n - \delta \gamma^2 \mu^4 n \rho - \delta \gamma^2 \mu^4 n \theta - \delta \gamma^2 \mu^3 n \rho \theta + 2\delta \gamma^2 \mu^3 \omega \rho u_1 - \delta \gamma^2 \mu^3 \omega \theta u_1 + 3\delta \gamma^2 \mu^2 \omega \rho \theta u_1 \\
& - \delta \gamma^2 \mu^2 \omega \theta^2 u_1 + \delta \gamma^2 \mu \omega \rho \theta^2 u_1 + 2\delta \gamma m \mu^5 \theta + 2\delta \gamma m \mu^4 \theta^2 - \delta \gamma \mu^6 n - \delta \gamma \mu^5 n \rho + \delta \gamma \mu^4 n \theta^2 + 4\delta \gamma \mu^4 \omega \rho u_1 \\
& - 2\delta \gamma \mu^4 \omega \theta u_1 + \delta \gamma \mu^3 n \rho \theta^2 + 6\delta \gamma \mu^3 \omega \rho \theta u_1 - 2\delta \gamma \mu^3 \omega \theta^2 u_1 + 2\delta \gamma \mu^2 \omega \rho \theta^2 u_1 + \delta m \mu^6 \theta + \delta m \mu^5 \theta^2 + \delta \mu^6 n \theta \\
& + \delta \mu^5 n \rho \theta + \delta \mu^5 n \theta^2 + 2\delta \mu^5 \omega \rho u_1 - \delta \mu^5 \omega \theta u_1 + \delta \mu^4 n \rho \theta^2 + 3\delta \mu^4 \omega \rho \theta u_1 - \delta \mu^4 \omega \theta^2 u_1 + \delta \mu^3 \omega \rho \theta^2 u_1 + \gamma^2 \mu^4 \omega \rho u_1 \\
& + 2\gamma^2 \mu^3 \omega \rho \theta u_1 + \gamma^2 \mu^2 \omega \rho \theta^2 u_1 + 2\gamma \mu^5 \omega \rho u_1 + 4\gamma \mu^4 \omega \rho \theta u_1 + 2\gamma \mu^3 \omega \rho \theta^2 u_1 + \mu^6 \omega \rho u_1 + 2\mu^5 \omega \rho \theta u_1 + \mu^4 \omega \rho \theta^2 u_1 \\
& + A\beta \delta^2 \gamma^2 \mu \theta + A\beta \delta^2 \gamma^2 \rho \theta + 2A\beta \delta^2 \gamma \mu^2 \theta + 2A\beta \delta^2 \gamma \mu \rho \theta + A\beta \delta^2 \mu^3 \theta + A\beta \delta^2 \mu^2 \rho \theta - \alpha \delta^2 \gamma^2 \mu^2 \theta - \alpha \delta^2 \gamma^2 \mu \rho \theta \\
& - 2\alpha \delta^2 \gamma \mu^3 \theta - 2\alpha \delta^2 \gamma \mu^2 \rho \theta - \alpha \delta^2 \mu^4 \theta - \alpha \delta^2 \mu^3 \rho \theta - \alpha \delta \gamma^2 \mu^3 \theta - \alpha \delta \gamma^2 \mu^2 \rho \theta - 2\alpha \delta \gamma \mu^4 \theta - 2\alpha \delta \gamma \mu^3 \rho \theta - \alpha \delta \mu^5 \theta \\
& - \alpha \delta \mu^4 \rho \theta - \delta^2 \gamma^2 \mu^3 \theta - \delta^2 \gamma^2 \mu^2 \rho \theta - \delta^2 \gamma^2 \mu^2 \theta^2 - \delta^2 \gamma^2 \mu \rho \theta^2 - 2\delta^2 \gamma \mu^4 \theta - 2\delta^2 \gamma \mu^3 \rho \theta - 2\delta^2 \gamma \mu^3 \theta^2 - 2\delta^2 \gamma \mu^2 \rho \theta^2 \\
& - \delta^2 \mu^5 \theta - \delta^2 \mu^4 \rho \theta - \delta^2 \mu^4 \theta^2 - \delta^2 \mu^3 \rho \theta^2 - \delta \gamma^2 \mu^4 \theta - \delta \gamma^2 \mu^3 \rho \theta - \delta \gamma^2 \mu^3 \theta^2 - \delta \gamma^2 \mu^2 \rho \theta^2 - 2\delta \gamma \mu^5 \theta - 2\delta \gamma \mu^4 \rho \theta \\
& - 2\delta \gamma \mu^4 \theta^2 - 2\delta \gamma \mu^3 \rho \theta^2 - \delta \mu^6 \theta - \delta \mu^5 \rho \theta - \delta \mu^5 \theta^2 - \delta \mu^4 \rho \theta^2) / ((\delta \eta \gamma^2 m \mu n^2 \omega^2 \theta u_1 u_2^2 - \delta \eta \gamma m \mu n^2 \omega^2 \theta^2 u_1 u_2^2 \\
& - 2\alpha \gamma \mu \rho \tau u_2 + 2\alpha \gamma \mu \omega u_1 - 2\alpha \gamma \mu \tau u_2 + 2\alpha \gamma \mu \omega \rho u_1 - 2\alpha \gamma \mu \rho \tau u_2 + \alpha \mu \omega u_1 + \alpha \mu \omega \rho u_1 + 2\delta \eta \gamma m \mu n \theta \\
& - \delta \eta \gamma^2 \mu^2 n^2 + \delta \eta \gamma^2 \mu^2 \omega u_1 - 2\delta \eta \gamma^2 \mu^2 \tau u_2 - \delta \eta \gamma^2 \mu n^2 \rho + \delta \eta \gamma^2 \mu \omega \rho u_1 - 2\delta \eta \gamma^2 \mu \rho \tau u_2 + 3\delta \eta \gamma m \mu^2 n \theta \\
& - \delta \eta \gamma m \mu n \theta^2 + 2\delta \eta \gamma \mu^3 \omega u_1 - 2\delta \eta \gamma \mu^3 \tau u_2 + 2\delta \eta \gamma \mu^2 n^2 \theta + 2\delta \eta \gamma \mu^2 \omega \rho u_1 - 2\delta \eta \gamma \mu^2 \rho \tau u_2 + 2\delta \eta \gamma \mu n^2 \rho \theta \\
& + \delta \eta m \mu^3 n \theta - \delta \eta m \mu^2 n \theta^2 + \delta \eta \mu^4 \omega u_1 + \delta \eta \mu^3 \omega \rho u_1 - \delta \eta \mu^2 n^2 \theta^2 - \delta \eta \mu n^2 \rho \theta^2 + \delta \gamma^2 \mu^3 \omega u_1 - 2\delta \gamma^2 \mu^3 \tau u_2 \\
& + \delta \gamma^2 \mu^2 \omega \rho u_1 + \delta \gamma^2 \mu^2 \omega \theta u_1 - 2\delta \gamma^2 \mu^2 \rho \tau u_2 - 3\delta \gamma^2 \mu^2 \tau \theta u_2 + \delta \gamma^2 \mu \omega \rho \theta u_1 - 3\delta \gamma^2 \mu \rho \tau \theta u_2 + 2\delta \gamma \mu^4 \omega u_1 \\
& - 2\delta \gamma \mu^4 \tau u_2 + 2\delta \gamma \mu^3 \omega \rho u_1 + 2\delta \gamma \mu^3 \omega \theta u_1 - 2\delta \gamma \mu^3 \rho \tau u_2 - 4\delta \gamma \mu^3 \tau \theta u_2 + 2\delta \gamma \mu^2 \omega \rho \theta u_1 - 4\delta \gamma \mu^2 \rho \tau \theta u_2 + \delta \mu^5 \omega u_1 \\
& + \delta \mu^4 \omega \rho u_1 + \delta \mu^4 \omega \theta u_1 - \delta \mu^4 \tau \theta u_2 + \delta \mu^3 \omega \rho \theta u_1 - \delta \mu^3 \rho \tau \theta u_2 + \eta \gamma^2 \mu^3 \omega u_1 - 2\eta \gamma^2 \mu^3 \tau u_2 + \eta \gamma^2 \mu^2 \omega \rho u_1 \\
& + \eta \gamma^2 \mu^2 \omega \theta u_1 - 2\eta \gamma^2 \mu^2 \rho \tau u_2 - 3\eta \gamma^2 \mu^2 \tau \theta u_2 + \eta \gamma^2 \mu \omega \rho \theta u_1 - 3\eta \gamma^2 \mu \rho \tau \theta u_2 + 2\eta \gamma \mu^4 \omega u_1 - 2\eta \gamma \mu^4 \tau u_2 \\
& + 2\eta \gamma \mu^3 \omega \rho u_1 + 2\eta \gamma \mu^3 \omega \theta u_1 - 2\eta \gamma \mu^3 \rho \tau u_2 - 4\eta \gamma \mu^3 \tau \theta u_2 + 2\eta \gamma \mu^2 \omega \rho \theta u_1 - 4\eta \gamma \mu^2 \rho \tau \theta u_2 + \eta \mu^5 \omega u_1 + \eta \mu^4 \omega \rho u_1 \\
& + \eta \mu^4 \omega \theta u_1 - \eta \mu^4 \tau \theta u_2 + \eta \mu^3 \omega \rho \theta u_1 - \eta \mu^3 \rho \tau \theta u_2 + \gamma^2 \mu^4 \omega u_1 - 2\gamma^2 \mu^4 \tau u_2 + \gamma^2 \mu^3 \omega \rho u_1 + \gamma^2 \mu^3 \omega \theta u_1 - 2\gamma^2 \mu^3 \rho \tau u_2 \\
& - 3\gamma^2 \mu^3 \tau \theta u_2 + \gamma^2 \mu^2 \omega \rho \theta u_1 - 3\gamma^2 \mu^2 \rho \tau \theta u_2 + 2\gamma \mu^5 \omega u_1 - 2\gamma \mu^5 \tau u_2 + 2\gamma \mu^4 \omega \rho u_1 + 2\gamma \mu^4 \omega \theta u_1 - 2\gamma \mu^4 \rho \tau u_2 \\
& - 4\gamma \mu^4 \tau \theta u_2 + 2\gamma \mu^3 \omega \rho \theta u_1 - 4\gamma \mu^3 \rho \tau \theta u_2 + \mu^6 \omega u_1 + \mu^5 \omega \rho u_1 + \mu^5 \omega \theta u_1 - \mu^5 \tau \theta u_2 + \mu^4 \omega \rho \theta u_1 - \mu^4 \rho \tau \theta u_2 \\
& - \alpha^2 \delta \eta \gamma \mu n - \alpha^2 \delta \eta \gamma n \rho - \alpha^2 \delta \eta \mu^2 n - \alpha^2 \delta \eta \mu n \rho - \alpha^2 \delta \gamma \mu^2 n - \alpha^2 \delta \gamma \mu n \rho - \alpha^2 \delta \mu^3 n - \alpha^2 \delta \mu^2 n \rho - \alpha^2 \eta \gamma \mu^2 n \\
& - \alpha^2 \eta \gamma \mu n \rho - \alpha^2 \eta \mu^3 n - \alpha^2 \eta \mu^2 n \rho - \alpha^2 \gamma \mu^3 n - \alpha^2 \gamma \mu^2 n \rho - \alpha^2 \mu^4 n - \alpha^2 \mu^3 n \rho + 2\alpha \delta \eta \gamma^2 \mu n + 2\alpha \delta \eta \gamma^2 n \rho \\
& + \alpha \delta \eta \gamma \mu^2 n + \alpha \delta \eta \gamma \mu n \rho - \alpha \delta \eta \gamma \mu n \theta - \alpha \delta \eta \gamma n \rho \theta - \alpha \delta \eta \mu^3 n - \alpha \delta \eta \mu^2 n \rho - \alpha \delta \eta \mu^2 n \theta - \alpha \delta \eta \mu n \rho \theta + \alpha \delta \gamma^2 \mu^2 n \\
& - 2\alpha \eta \gamma \mu^2 n \theta - 2\alpha \eta \gamma \mu n \rho \theta - \alpha \eta \mu^4 n - \alpha \eta \mu^3 n \rho - 2\alpha \eta \mu^3 n \theta - 2\alpha \eta \mu^2 n \rho \theta + \alpha \gamma^2 \mu^3 n + \alpha \gamma^2 \mu^2 n \rho - 2\alpha \gamma \mu^3 n \theta \\
& - 2\alpha \gamma \mu^2 n \rho \theta - \alpha \mu^5 n - \alpha \mu^4 n \rho - 2\alpha \mu^4 n \theta - 2\alpha \mu^3 n \rho \theta - \delta \eta \gamma^2 m \mu \theta + 2\delta \eta \gamma^2 \mu^2 n + 2\delta \eta \gamma^2 \mu n \rho - 2\delta \eta \gamma m \mu^2 \theta \\
& + 2\delta \eta \gamma \mu^3 n + 2\delta \eta \gamma \mu^2 n \rho - 2\delta \eta \gamma \mu^2 n \theta - 2\delta \eta \gamma \mu n \rho \theta - \delta \eta m \mu^3 \theta - 2\delta \eta \mu^3 n \theta - 2\delta \eta \mu^2 n \rho \theta + \delta \gamma^2 \mu^3 n + \delta \gamma^2 \mu^2 n \rho \\
& + \delta \gamma^2 \mu^2 n \theta + \delta \gamma^2 \mu n \rho \theta + \delta \gamma \mu^4 n + \delta \gamma \mu^3 n \rho - \delta \gamma \mu^2 n \theta^2 - \delta \gamma \mu n \rho \theta^2 - \delta \mu^4 n \theta - \delta \mu^3 n \rho \theta - \delta \mu^3 n \theta^2 - \delta \mu^2 n \rho \theta^2 \\
& + \eta \gamma^2 \mu^3 n + \eta \gamma^2 \mu^2 n \rho + \eta \gamma^2 \mu^2 n \theta + \eta \gamma^2 \mu n \rho \theta + \eta \gamma \mu^4 n + \eta \gamma \mu^3 n \rho - \eta \gamma \mu^2 n \theta^2 - \eta \gamma \mu n \rho \theta^2 - \eta \mu^4 n \theta - \eta \mu^3 n \rho \theta \\
& - \eta \mu^3 n \theta^2 - \eta \mu^2 n \rho \theta^2 + \gamma^2 \mu^4 n + \gamma^2 \mu^3 n \rho + \gamma^2 \mu^3 n \theta + \gamma^2 \mu^2 n \rho \theta + \gamma \mu^5 n + \gamma \mu^4 n \rho - \gamma \mu^3 n \theta^2 - \gamma \mu^2 n \rho \theta^2 - \mu^5 n \theta \\
& - \mu^4 n \rho \theta - \mu^4 n \theta^2 - \mu^3 n \rho \theta^2 - \alpha \delta \eta \gamma^2 \mu - \alpha \delta \eta \gamma^2 \rho - 2\alpha \delta \eta \gamma \mu^2 - 2\alpha \delta \eta \gamma \mu \rho - \alpha \delta \eta \mu^3 - \alpha \delta \eta \mu^2 \rho - \alpha \delta \gamma^2 \mu^2 \\
& - \alpha \delta \gamma^2 \mu \rho - 2\alpha \delta \gamma \mu^3 - 2\alpha \delta \gamma \mu^2 \rho - \alpha \delta \mu^4 - \alpha \delta \mu^3 \rho - \alpha \eta \gamma^2 \mu^2 - \alpha \eta \gamma^2 \mu \rho - 2\alpha \eta \gamma \mu^3 - 2\alpha \eta \gamma \mu^2 \rho - \alpha \eta \mu^4 \\
& - \alpha \eta \mu^3 \rho - \alpha \gamma^2 \mu^3 - \alpha \gamma^2 \mu^2 \rho - 2\alpha \gamma \mu^4 - 2\alpha \gamma \mu^3 \rho - \alpha \mu^5 - \alpha \mu^4 \rho - \delta \eta \gamma^2 \mu^2 - \delta \eta \gamma^2 \mu \rho - 2\delta \eta \gamma \mu^3 - 2\delta \eta \gamma \mu^2 \rho \\
& - \delta \eta \mu^4 - \delta \eta \mu^3 \rho - \delta \gamma^2 \mu^3 - \delta \gamma^2 \mu^2 \rho - \delta \gamma^2 \mu^2 \theta - \delta \gamma^2 \mu \rho \theta - 2\delta \gamma \mu^4 - 2\delta \gamma \mu^3 \rho - 2\delta \gamma \mu^3 \theta - 2\delta \gamma \mu^2 \rho \theta - \delta \mu^5 - \delta \mu^4 \rho \\
& - \delta \mu^4 \theta - \delta \mu^3 \rho \theta - \eta \gamma^2 \mu^3 - \eta \gamma^2 \mu^2 \rho - \eta \gamma^2 \mu^2 \theta - \eta \gamma^2 \mu \rho \theta - 2\eta \gamma \mu^4 - 2\eta \gamma \mu^3 \rho - 2\eta \gamma \mu^3 \theta - 2\eta \gamma \mu^2 \rho \theta - \eta \mu^5 \\
& - \eta \mu^4 \rho - \eta \mu^4 \theta - \eta \mu^3 \rho \theta - \gamma^2 \mu^4 - \gamma^2 \mu^3 \rho - \gamma^2 \mu^3 \theta - \gamma^2 \mu^2 \rho \theta - 2\gamma \mu^5 - 2\gamma \mu^4 \rho - 2\gamma \mu^4 \theta - 2\gamma \mu^3 \rho \theta - \mu^6 - \mu^5 \rho - \mu^5 \theta \\
& - \mu^4 \rho \theta) \delta \beta) \}
\end{aligned}$$

## Lampiran 2. Syntax Simulasi Numerik Model Kontrol Optimal

Adapun *syntax* dalam aplikasi Matlab2015a yang digunakan untuk mencari solusi numerik model penyebaran penyakit difteri dibagi menjadi beberapa bentuk m file sebagai berikut:

### MAIN.m

```
clear all;
clc;
global C1 C2 C3 C4 C5 A beta eta rho v w teta mu delta alpha gamma
omega tau
% Nilai parameter model
A=200;
beta=0.0000097;
eta=0.0001826;
%0.0002739726;
%;
rho=0.0001826;
%0.0002739726;
%0.0001826;
v=0.9;
w=0.8;
teta=0.071428;
mu=0.002;
delta=0.143;
alpha=0.0054;
gamma=0.00555; %asumsi jika orang tanpa gejala tak diobati bisa
jadi dia jadi kronis sampai 6 bulan
%efektivitas kontrol
omega=1;
%sigma=0.5;
tau=1;

%nilai bobot dalam fungsi objektif
C1=1;
C2=1;
C3=1;
C4=1;
C5=1;

% Nilai awal state
x1awal=50000;
x2awal=100;
x3awal=25;
x4awal=75;
x5awal=1000;
x6awal=1000;
x0=[x1awal;x2awal;x3awal;x4awal;x5awal;x6awal];

nx=6; %jumlahnya tergantung dari jumlah variabel atau kompartemen
lambdaT=zeros(nx,1);%Nilai akhir Costate (syarat transversalitas)
%Interval waktu
Ntime=1000; %partisi semakin banyak partisi maka semakin mulus
grafik yang diperoleh
tf=60; %lamanya waktu pengamatan

ti=linspace(0,tf,Ntime);
%Batas kontrol
B1=0; %nilai kontrol terendah
```

```

B2=1; %nila kontrol tertinggi
nv=2; %jumlah tergantung pada jumlah kontrol yang diberikan dalam
model
BB=B1.*ones(nv,Ntime);
BA=B2.*ones(nv,Ntime);
%Parameter Sweep
test=-1;
deltaa=0.0001;
k=0;
%tebakan awal untuk fungsi kontrol u1, u2 dan biasanya tebakan
yang
%digunakan yaitu 0
u=0*ones(nv,Ntime);
%pemberian kontrol konstan pada
uk1=0.2*ones(nv,Ntime);
uk2=0.8*ones(nv,Ntime);
%solusi sistem tanpa kontrol (u1=0 u2=0)
options = odeset('AbsTol',1e-2,'RelTol',1e-2);
xnon=ode45(@(t,x) pers_state(t, x, u, ti),[0 tf],x0,options);
xnon=deval(xnon,ti);
%solusi sistem dengan kontrol konstan
xk1=ode45(@(t,x) pers_state(t, x, uk1, ti),[0 tf],x0,options);
xk1=deval(xk1,ti);
xk2=ode45(@(t,x) pers_state(t, x, uk2, ti),[0 tf],x0,options);
xk2=deval(xk2,ti);

%Awal Metode Sweep
x=zeros(nx,Ntime);
lambda=zeros(nx,Ntime);
while(test<0)
k=k+1;
oldx=x;
oldp=lambda;
oldu=u;
%Forward Runge Kutta dengan nila awal yang diketahui yaitu 0
x=deval(ode45(@(t,x) pers_state(t, x, u, ti), [0 tf], x0),ti);
%Backward Runge Kutta %yg digunakan nilai akhir lambdaT=0
lambda=deval(ode45(@(t,p) pers_costate(t, p, x, u, ti),[tf 0],
lambdaT),ti);
%menghitung nilai u dari syarat optimal sistem
u1=kontrol(x,lambda); %menggunakan u dH/du=0
%membuat u berada dalam interval yang diharapkan
u1=batas_kontrol(u1,BB,BA);
%mengupdate nilai u dalam metode sweep menggunakan kombinasi
konveks
u=0.5*(u1+oldu); %uji Konvergensi u yang pertama
%uji kekonvergensi jika telah memenuhi syarat konvergensi maka
iterasi
%akan dihentikan
temp1=deltaa*sum(abs(u))-sum(abs(oldu-u));
temp2=deltaa*sum(abs(x))-sum(abs(oldx-x));
temp3=deltaa*sum(abs(lambda))-sum(abs(oldp-lambda));
test=min(temp1,min(temp2,temp3)); %Buku Lenhart Hal:55
%menghitung nilai fungsi tujuan menggunakan u akhir
J(k)=fungsi_objektif(x,u,ti);
disp(['it: ',num2str(k),' ,Test: ', num2str(test)])
end
%menghitung nilai fungsi tujuan menggunakan u optimal
[m,n]=size(J);

```

```

Ju=fungsi_objektif(x,u,ti);

figure (1)
%subplot (121)
plot(ti,xnon(1,:), 'r-',ti, x(1,:), 'b-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('S(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal')
axis('tight')
grid on
figure (2)
%subplot (122)
plot(ti,xnon(1,:), 'r-',ti, x(1,:), 'b-',ti, xk1(1,:), 'g-',ti,
xk2(1,:), 'm-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('S(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal', 'Konstan 1', 'Konstan 1')
axis('tight')
grid on

figure (3)
subplot (321)
plot(ti,xnon(2,:), 'r-', 'LineWidth',1.5)
xlabel('Waktu (hari) ')
ylabel('L(t)')
legend('Tanpa Kontrol')
axis('tight')
grid on
subplot (322)
plot(ti, x(2,:), 'b-', 'LineWidth',1.5)
xlabel('Waktu (hari) ')
ylabel('L(t)')
legend( 'Kontrol Optimal')
axis('tight')
grid on
figure (4)
%subplot (222)
plot(ti,xnon(2,:), 'r-',ti, x(2,:), 'b-',ti, xk1(2,:), 'g-',ti,
xk2(2,:), 'm-', 'LineWidth',1.5)
xlabel('Waktu ')
ylabel('L(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal', 'Konstan 1', 'Konstan 1')
axis('tight')
grid on

figure (5)
subplot (521)
plot(ti,xnon(3,:), 'r-', 'LineWidth',1.5)
xlabel('Waktu (hari) ')
ylabel('I_s(t)')
legend('Tanpa Kontrol')
axis('tight')
grid on
subplot (522)
plot(ti, x(3,:), 'b-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('I_s(t)')
legend( 'Kontrol Optimal')

```



```

axis('tight')
grid on
figure (6)
%subplot (321)
plot(ti,xnon(3,:), 'r-',ti, x(3,:), 'b-',ti, xk1(3,:), 'g-',ti,
xk2(3,:), 'm-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('I_s(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal', 'Konstan 1', 'Konstan 2')
axis('tight')
grid on

figure (7)
subplot (721)
plot(ti,xnon(4,:), 'r-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('I_a(t)')
legend('Tanpa Kontrol')
axis('tight')
grid on
subplot (722)
plot(ti, x(4,:), 'b-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('I_a(t)')
legend( 'Kontrol Optimal')
axis('tight')
grid on
figure (8)
%subplot (422)
plot(ti,xnon(4,:), 'r-',ti, x(4,:), 'b-',ti, xk1(4,:), 'g-',ti,
xk2(4,:), 'm-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('I_a(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal', 'Konstan 1', 'Konstan 2')
axis('tight')
grid on

figure (9)
%subplot (521)
plot(ti,xnon(5,:), 'r-',ti, x(5,:), 'b-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('R_f(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal')
axis ('tight')
grid on
figure (10)
%subplot (522)
plot(ti,xnon(5,:), 'r-',ti, x(5,:), 'b-',ti, xk1(5,:), 'g-',ti,
xk2(5,:), 'm-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('R_f(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal', 'Konstan 1', 'Konstan 2')
axis ('tight')
grid on
figure (11)
%subplot (621)
plot(ti,xnon(6,:), 'r-',ti, x(6,:), 'b-', 'LineWidth',1.5)
xlabel('Waktu (hari)')

```

```

ylabel('R_p(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal')
axis('tight')
grid on
figure(12)
%subplot(621)
plot(ti,xnon(6,:), 'r-',ti, x(6,:), 'b-',ti, xk1(6,:), 'g-',ti,
xk2(6,:), 'm-', 'LineWidth',1.5)
xlabel('Waktu (hari)')
ylabel('R_p(t)')
legend('Tanpa Kontrol', 'Kontrol Optimal', 'Konstan 1', 'Konstan 2')
axis('tight')
grid on

%Fungsi Kontrol
figure(13)
plot(ti,u(1,:), 'b-',ti,u(2,:), 'm-', 'LineWidth',1.5)
legend('kontrol u_1 (Vaksinasi)', 'kontrol u_2 (Pengobatan)')
xlabel('Waktu (hari)')
ylabel('u_1(t), u_2(t)')
title('Fungsi Kontrol')
axis('tight')
grid on

```

### **pers\_state.m**

```

function dx=pers_state(t, x, u, ti) %ti adalah inputan %t,x,u
adalah variabel
global A beta eta rho v w teta mu delta alpha gamma omega tau
%x1=S(t),x2=L(t),x3=Is(t),x4=Is(t),x5=Rf(t), dan x6=Rp(t)
x1=x(1);
x2=x(2);
x3=x(3);
x4=x(4);
x5=x(5);
x6=x(6);
u1=u(1,:);
u1=interp1(ti,u1',t);
u2=u(2,:);
u2=interp1(ti,u2',t);

dx=zeros(6,1);
dx(1)=A+eta.*x6-(1.-u1*omega)*beta.*x1.*(x3+x4)-(u1*omega+mu).*x1;
dx(2)=(1.-u1*omega)*beta.*x1.*(x3+x4)-(delta+mu).*x2;
dx(3)=(1-w)*delta.*x2-(1+u2*tau)*teta.*x3-(mu+alpha).*x3;
dx(4)=w.*delta.*x2-(1+u2*tau)*gamma.*x4-mu.*x4;
dx(5)=v*(1+u2*tau)*teta.*x3+u1.*x1-(rho+mu).*x5;
dx(6)=(1-v)*(1+u2*tau)*teta.*x3+(1+u2*tau)*gamma.*x4+rho.*x5-
(eta+mu).*x6;
end

```

### **pers\_costate.m**

```

function dlambd=pers_costate(t, lambda, x, u, ti)
global beta eta rho v teta w mu delta alpha gamma C1 C2 C3 omega
tau
%x1=S(t),x2=L(t),x3=Is(t),x4=Is(t),x5=Rf(t), dan x6=Rp(t)
x = interp1(ti,x',t);
x1 = x(1);
x2 = x(2);

```

```

x3 = x(3);
x4 = x(4);
x5 = x(5);
x6 = x(6);
u1 = u(1,:);
u2 = u(2,:);

u1 = interp1(ti,u1',t);
u2 = interp1(ti,u2',t);

lambda1=lambda(1,:);
lambda2=lambda(2,:);
lambda3=lambda(3,:);
lambda4=lambda(4,:);
lambda5=lambda(5,:);
lambda6=lambda(6,:);

dlambda=zeros(6,1);
dlambda(1)=(lambda1-lambda2)*(1-u1*omega)*beta.*(x3+x4)+(lambda1-
lambda2)*u1*omega+lambda1*mu;
dlambda(2)=-C1+(lambda2-lambda3*(1-w)-lambda4*w)*delta+lambda2*mu;
dlambda(3)=-C2+(lambda1-lambda2)*(1-u1*omega)*beta.*x1+(lambda3-
lambda5*v-lambda6*(1-v))*(1+u2*tau)*teta+lambda3*(alpha+mu);
dlambda(4)=-C3+(lambda1-lambda2)*(1-u1*omega)*beta.*x1+(lambda4-
lambda6)*(1+u2*tau)*gamma+lambda4*mu;
dlambda(5)=(lambda5-lambda6)*rho+lambda5*mu;
dlambda(6)=(lambda6-lambda1)*eta+lambda6*mu;
end

```

### batas\_kontrol.m

```

function s=batas_kontrol(s,BB,BA)
% untuk batas bawah
ns_tmp=s;
I=ns_tmp<BB;
ns_tmp(I)=BB(I);
% untuk batas atas
J=ns_tmp>BA;
ns_tmp(J)=BA(J);
% Update u
s=ns_tmp;
end

```

### kontrol.m

```

function u= kontrol(x,lambda)
global C4 C5 v beta gamma teta omega tau
lambda1 = lambda(1,:);
lambda2 = lambda(2,:);
lambda3 = lambda(3,:);
lambda4 = lambda(4,:);
lambda5 = lambda(5,:);
lambda6 = lambda(6,:);
x1 = x(1,:);
x2 = x(2,:);
x3 = x(3,:);
x4 = x(4,:);
x5 = x(5,:);
x6 = x(6,:);
%fungsi kontrol:

```

```
u1=( (lambda2-lambda1)*omega*beta.*x1.*(x3+x4)+(lambda1-  
lambda5)*omega.*x1)/C4;  
u2=( (lambda3-lambda5*v-lambda6*(1-v))*tau*teta.*x3+(lambda4-  
lambda6)*tau*gamma.*x4)/C5;  
u=[u1;u2];  
end
```

### **fungsi\_objektif.m**

```
function J= fungsi_objektif(x,u,ti)  
global C1 C2 C3 C4 C5  
x2=x(2,:);  
x3=x(3,:);  
x4=x(4,:);  
x5=x(2,:);  
x6=x(2,:);  
u1=u(1,:);  
u2=u(2,:);  
obj=C1.*x2+C2.*x3+C3.*x4+(C4/2).*u1.^2+(C5/2).*u2.^2;  
J=trapz(ti,obj);  
end
```