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# LAMPIRAN

## Lampiran 1. Kode Program Simulasi Menggunakan Matlab

```
clear all
close all
clc

%parameter
N0 = 1;
mu1 = 0.485;
b = 0.388;
alpha1 = 0.0555;
alpha2 = 0.2;
b1 = 0.35;
mu2 = 0.07;
beta = 0.76;
gamma1 = 5;
k = 0.3;
c = 0.78;
beta1 = 0.66;
mu3 = 0.03;
theta = 0.085;
D1=0.5;
D2=0.01;
D3=0.9;

% titik kesetimbangan
syms Nm Pm Zm
pers=zeros(3,1);

f_N = pers(1) == (N0 - (mu1*Nm) - (b*Nm*Pm) + (alpha1*Pm) +
(alpha2*Zm));
f_P = pers(2) == ((b1*Nm*Pm) - (mu2*Pm) -
(beta*Zm*(Pm/(gamma1+Pm))) - k*(Pm/(c+Pm)));
f_Z = pers(3) == ((beta1*Zm*(Pm/(gamma1+Pm))) - (mu3*Zm) -
(theta*Zm*(Pm/(gamma1+Pm))));
[Nm,Pm,Zm]=solve(f_N,f_P,f_Z,Nm,Pm,Zm);

Nbar = Nm(2); Pbar = Pm(2); Zbar = Zm (2);

%domain spasial
x= 100;
y= 100;
t= 0;
delta_x=1;
delta_y=1;
delta_t=0.01;
Nx = x/delta_x;
Ny = y/delta_y;
Nt = t/delta_t;

%membuat matriks
```

```
N = zeros(Nx,Ny); P = zeros(Nx,Ny); Z = zeros(Nx,Ny);
Na = zeros (Nx,Ny); Pa = zeros (Nx,Ny); Za = zeros (Nx,Ny);

%Syarat Awal
for i=1:Nx
    for j=1:Ny
         Na(i,j) = Nbar - (2*10^-7)*(i-180)*(i-720)-(6*10^-
7)*(j-90)*(j-210);
         Pa(i,j) = Pbar - (2*10^-7)*(i-180)*(i-720)-(6*10^-
7)*(j-90)*(j-210);
         Za(i,j) = Zbar - (3*10^-5)*(i-450)-(6*10^-5)*(j-135);
    end
end

%implementasi skema FTCS
for n = 1:Nt-1
    for i = 2:Nx-1
        for j = 2:Ny-1
            % Fungsi N, P dan Z
            fN = (N0 - (mu1*Na(i,j)) - (b*Na(i,j)*Pa(i,j)) +
(alpha1*Pa(i,j)) + (alpha2*Za(i,j)));
            fP = ((b1*Na(i,j)*Pa(i,j)) - (mu2*Pa(i,j)) -
((beta*Pa(i,j)*Za(i,j))/(gamma1+Pa(i,j))) -
((k*Pa(i,j))/(c+Pa(i,j))));
            fZ = (((beta1*Pa(i,j)*Za(i,j))/(gamma1+Pa(i,j))) -
(mu3*Za(i,j)) - ((theta*Pa(i,j)*Za(i,j))/(gamma1+Pa(i,j))));
            Nxx = (Na(i+1,j) - 2*Na(i,j) + Na(i-1,j))/delta_x^2;
            Pxx = (Pa(i+1,j) - 2*Pa(i,j) + Pa(i-1,j))/delta_x^2;
            Zxx = (Za(i+1,j) - 2*Za(i,j) + Za(i-1,j))/delta_x^2;
            Nyy = (Na(i,j+1) - 2*Na(i,j) + Na(i,j-1))/delta_y^2;
            Pyy = (Pa(i,j+1) - 2*Pa(i,j) + Pa(i,j-1))/delta_y^2;
            Zyy = (Za(i,j+1) - 2*Za(i,j) + Za(i,j-1))/delta_y^2;
            spatial_N = Nxx + Nyy;
            spatial_P = Pxx + Pyy;
            spatial_Z = Zxx + Zyy;
            N(i,j) = Na(i,j) + delta_t*(fN + (D1*spatial_N));
            P(i,j) = Pa(i,j) + delta_t*(fP + (D2*spatial_P));
            Z(i,j) = Za(i,j) + delta_t*(fZ + (D3*spatial_Z));
        end
        N(i,1)= N(i,2);
        N(i,Ny)= N(i,Ny-1);
        P(i,1)= P(i,2);
        P(i,Ny)= P(i,Ny-1);
        Z(i,1)= Z(i,2);
        Z(i,Ny)= Z(i,Ny-1);
    end
    for j=1:Ny
        N(1,j)=N(2,j);
        N(Nx,j)=N(Nx-1,j);
        P(1,j)= P(2,j);
        P(Nx,j)= P(Nx-1,j);
        Z(1,j)= Z(2,j);
        Z(Nx,j)= Z(Nx-1,j);
```

```
    end
    disp(n);
    Na=N;
    Pa=P;
    Za=Z;
end

figure(1)
contourf(Na,'LineColor','none');
colormap('jet')
title('Nutrisi')
xlabel('x')
ylabel('y')
colorbar

figure(2)
contourf(Pa,'LineColor','none');
colormap('jet')
title('Fitoplankton')
xlabel('x')
ylabel('y')
colorbar

figure(3);
contourf(Za,'LineColor','none');
colormap('jet')
title('Zooplankton')
xlabel('x')
ylabel('y')
colorbar
```