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LAMPIRAN A

SIMBOL CHRISTOFFEL, TENSOR RICCI DAN SKALAR RICCI

A.1 Simbol Christoffel

Persamaan untuk mencari simbol Christoffel adalah

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\left(\partial_\nu g_{\mu\sigma} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}\right) \qquad (A.1)$$

1. Untuk $\sigma = \rho = \mu = \nu$

$$\Gamma^{\mu}_{\mu\mu} = \frac{1}{2} g^{\mu\mu}(\partial_\mu g_{\mu\mu} + \partial_\mu g_{\mu\mu} - \partial_\mu g_{\mu\mu})$$

$$\Gamma^{\mu}_{\mu\mu} = \frac{1}{2} g^{\mu\mu}\partial_\mu g_{\mu\mu} \qquad (A.2)$$

2. Untuk $\sigma = \rho \neq \mu = \nu$

$$\Gamma^{\rho}_{\mu\mu} = \frac{1}{2} g^{\rho\rho}(\partial_\mu g_{\mu\rho} + \partial_\mu g_{\rho\mu} - \partial_\rho g_{\mu\mu})$$

$$\Gamma^{\rho}_{\mu\mu} = -\frac{1}{2} g^{\rho\rho}\partial_\rho g_{\mu\mu} \qquad (A.3)$$

3. Untuk $\sigma = \rho = \mu \neq \nu$

$$\Gamma^{\mu}_{\mu\nu} = \frac{1}{2} g^{\mu\mu}(\partial_\nu g_{\mu\mu} + \partial_\mu g_{\mu\nu} - \partial_\mu g_{\mu\nu})$$

$$\Gamma^{\mu}_{\mu\nu} = \Gamma^{\mu}_{\nu\mu} = \frac{1}{2} g^{\mu\mu}\partial_\nu g_{\mu\mu} \qquad (A.4)$$

Komponen- komponen simbol Christoffel yang tidak nol yaitu:

1. Untuk $\sigma = \rho = \mu = \nu$

$$\Gamma^{\mu}_{\mu\mu} = \frac{1}{2} g^{\mu\mu} \partial_{\mu} g_{\mu\mu}$$

$$\Gamma^{1}_{11} = \frac{1}{2} g^{11} \partial_{1} g_{11}$$

$$\Gamma^{1}_{11} = \frac{1}{2} \left(-e^{-2\beta}\right) \partial_{r} (-e^{2\beta})$$

$$\Gamma^{1}_{11} = \frac{1}{2} \left(-e^{-2\beta}\right) 2\beta' \left(-e^{2\beta}\right)$$

$$\Gamma^{1}_{11} = \beta'$$

2. Untuk $\sigma = \rho \neq \mu = \nu$

$$\Gamma^{\rho}_{\mu\mu} = -\frac{1}{2} g^{\rho\rho} \partial_{\rho} g_{\mu\mu}$$

$$\Gamma^{1}_{00} = -\tfrac{1}{2} g^{11} \partial_{1} g_{00}$$

$$\Gamma^{1}_{00} = -\frac{1}{2} \left(e^{-2\beta}\right) \partial_{r} (-e^{2\alpha})$$

$$\Gamma^{1}_{00} = -\frac{1}{2} \left(e^{-2\beta}\right) 2\alpha' (-e^{2\alpha})$$

$$\Gamma^{1}_{00} = \alpha' e^{2\alpha - 2\beta}$$

$$\Gamma^{1}_{22} = -\frac{1}{2} g^{11} \partial_{1} g_{22}$$

$$\Gamma^{1}_{22} = -\frac{1}{2} \left(e^{-2\beta}\right) \partial_{r} (r^{2})$$

$$\Gamma^{1}_{22} = -\frac{1}{2} \left(e^{-2\beta}\right) (2r)$$

$$\Gamma^{1}_{22} = -r e^{-2\beta}$$

$$\Gamma_{33}^{1} = -\frac{1}{2}g^{11}\partial_1 g_{33}$$

$$\Gamma_{33}^{1} = -\frac{1}{2}\left(e^{-2\beta}\right)\partial_r(r^2 \sin^2\theta)$$

$$\Gamma_{33}^{1} = -\frac{1}{2}\left(e^{-2\beta}\right)(2r \sin^2\theta)$$

$$\Gamma_{33}^{1} = -re^{-2\beta}\sin^2\theta$$

$$\Gamma_{33}^{2} = -\frac{1}{2}g^{22}\partial_2 g_{33}$$

$$\Gamma_{33}^{2} = -\frac{1}{2}(r^{-2})\partial_\theta(r^2 \sin^2\theta)$$

$$\Gamma_{33}^{2} = -\sin\theta\cos\theta$$

3. Untuk $\sigma = \rho = \mu \neq \nu$

$$\Gamma_{\mu\nu}^{\mu} = \Gamma_{\nu\mu}^{\mu} = \frac{1}{2}g^{\mu\mu}\partial_\nu g_{\mu\mu}$$

$$\Gamma_{01}^{0} = \Gamma_{10}^{0} = \frac{1}{2}g^{00}\partial_1 g_{00}$$

$$\Gamma_{01}^{0} = \Gamma_{10}^{0} = \frac{1}{2}(-e^{2\alpha})\partial_r(-e^{2\alpha})$$

$$\Gamma_{01}^{0} = \Gamma_{10}^{0} = \frac{1}{2}(-e^{2\alpha})2\alpha'(-e^{2\alpha})$$

$$\Gamma_{01}^{0} = \Gamma_{10}^{0} = \alpha'$$

$$\Gamma_{21}^{2} = \Gamma_{12}^{2} = \frac{1}{2}g^{22}\partial_1 g_{22}$$

$$\Gamma_{21}^{2} = \Gamma_{12}^{2} = \frac{1}{2}r^{-2}\partial_r(r^2)$$

$$\Gamma_{21}^{2} = \Gamma_{12}^{2} = \frac{1}{2}r^{-2}(2r) = \frac{1}{r}$$

$$\Gamma^3_{31} = \Gamma^3_{13} = \frac{1}{2} g^{33} \partial_1 g_{33}$$

$$\Gamma^3_{31} = \Gamma^3_{13} = \frac{1}{2} (r^{-2} \sin^{-2}\theta) \partial_r (r^2 \sin^2\theta)$$

$$\Gamma^3_{31} = \Gamma^3_{13} = \frac{1}{2} (r^{-2} \sin^{-2}\theta)(2r \sin^2\theta) = \frac{1}{r}$$

$$\Gamma^3_{32} = \Gamma^3_{23} = \frac{1}{2} g^{33} \partial_2 g_{33}$$

$$\Gamma^3_{32} = \Gamma^3_{23} = \frac{1}{2} (r^{-2} \sin^{-2}\theta) \partial_\theta (r^2 \sin^2\theta)$$

$$\Gamma^3_{32} = \Gamma^3_{23} = \frac{1}{2} (r^{-2} \sin^{-2}\theta)(r^2\, 2 \sin\theta \cos\theta)$$

$$\Gamma^3_{32} = \Gamma^3_{23} = \frac{\cos\theta}{\sin\theta}$$

$$\Gamma^3_{32} = \Gamma^3_{23} = \cot\theta$$

Dengan demikian terdapat 13 komponen simbol Christoffel yang tidak nol,

$$\Gamma^0_{01} = \Gamma^0_{10} = \alpha'$$

$$\Gamma^1_{00} = \alpha' e^{2\alpha - 2\beta}$$

$$\Gamma^1_{11} = \beta'$$

$$\Gamma^1_{22} = -r e^{-2\beta}$$

$$\Gamma^1_{33} = -r e^{-2\beta} \sin^2\theta$$

$$\Gamma^2_{21} = \Gamma^2_{12} = \frac{1}{r}$$

$$\Gamma^2_{33} = -\sin\theta \cos\theta$$

$$\Gamma^3_{31} = \Gamma^3_{13} = \frac{1}{r}$$

$$\Gamma^3_{32} = \Gamma^3_{23} = \cot\theta \qquad (A.5)$$

A.2 Tensor Ricci

Tensor Ricci yang tidak nol dapat pula diturunkan, dimana tensor Ricci didefinisikan sebagai berikut:

$$R_{\mu\nu} = \partial_\nu \Gamma^\sigma_{\mu\sigma} - \partial_\sigma \Gamma^\sigma_{\mu\nu} + \Gamma^\rho_{\mu\nu}\Gamma^\sigma_{\rho\nu} - \Gamma^\rho_{\mu\nu}\Gamma^\sigma_{\rho\sigma} \qquad (B.1)$$

1. Untuk $\mu = \nu = 0$

$$R_{00} = \partial_0 \Gamma^\sigma_{00} - \partial_\sigma \Gamma^\sigma_{00} + \Gamma^\rho_{0\sigma}\Gamma^\sigma_{\rho 0} - \Gamma^\rho_{00}\Gamma^\sigma_{\rho\sigma}$$

$$R_{00} = -\partial_1 \Gamma^1_{00} + \Gamma^1_{00}\Gamma^\sigma_{10} + \Gamma^0_{0\sigma}\Gamma^\sigma_{00} - \Gamma^1_{00}\Gamma^\sigma_{1\sigma}$$

$$R_{00} = -\partial_1 \Gamma^1_{00} + \Gamma^1_{00}\Gamma^0_{10} + \Gamma^0_{01}\Gamma^1_{00} - \Gamma^1_{00}(\Gamma^0_{10}+\Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13})$$

$$R_{00} = -\partial_1 \Gamma^1_{00} + \Gamma^1_{00}\Gamma^0_{10} + \Gamma^0_{01}\Gamma^1_{00} - \Gamma^1_{00}\Gamma^0_{10} - \Gamma^1_{00}\Gamma^1_{11} - \Gamma^1_{00}\Gamma^2_{12} - \Gamma^1_{00}\Gamma^3_{13}$$

$$R_{00} = -\partial_r\left(\alpha' e^{2\alpha-2\beta}\right) + \alpha'(\alpha' e^{2\alpha-2\beta}) - (\alpha' e^{2\alpha-2\beta}\beta') - \left(\alpha' e^{2\alpha-2\beta}\frac{1}{r}\right) - \left(\alpha' e^{2\alpha-2\beta}\frac{1}{r}\right)$$

$$R_{00} = -\left(\alpha'' e^{2\alpha-2\beta}\right) - \alpha'(2\alpha' - 2\beta') + \alpha'(\alpha' e^{2\alpha-2\beta}) - \alpha'\beta' e^{2\alpha-2\beta} - \frac{2\alpha'}{r}e^{2\alpha-2\beta}$$

$$R_{00} = e^{2\alpha-2\beta}\left(-\alpha'' - 2\alpha'^2 + 2\alpha'\beta' + \alpha'^2 - \alpha'\beta' - \frac{2\alpha'}{r}\right)$$

$$R_{00} = \left(-\alpha'' + \alpha'\beta' - \alpha'^2 - \frac{2\alpha'}{r}\right)e^{2\alpha-2\beta}$$

2. Untuk $\mu = \nu = 1$

$$R_{11} = \partial_1 \Gamma^\sigma_{1\sigma} - \partial_\sigma \Gamma^\sigma_{11} + \Gamma^\rho_{1\sigma}\Gamma^\sigma_{\rho 1} - \Gamma^\rho_{11}\Gamma^\sigma_{\rho\sigma}$$

$$R_{11} = (\partial_1 \Gamma^0_{10} + \partial_1 \Gamma^1_{11} + \partial_1 \Gamma^2_{12} + \partial_1 \Gamma^3_{13}) - \partial_1 \Gamma^1_{11} + (\Gamma^0_{1\sigma}\Gamma^\sigma_{01} + \Gamma^1_{1\sigma}\Gamma^\sigma_{11} + \Gamma^2_{1\sigma}\Gamma^\sigma_{21} + \Gamma^3_{1\sigma}\Gamma^\sigma_{31}) - \Gamma^1_{11}\Gamma^\sigma_{1\sigma}$$

$$R_{11} = (\partial_1\Gamma^0_{10} + \partial_1\Gamma^1_{11} + \partial_1\Gamma^2_{12} + \partial_1\Gamma^3_{13}) - \partial_1\Gamma^1_{11} + (\Gamma^0_{10}\Gamma^0_{01} + \Gamma^1_{11}\Gamma^1_{11} + \Gamma^2_{12}\Gamma^2_{21} + \Gamma^3_{13}\Gamma^3_{31}) - \Gamma^1_{11}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13})$$

$$R_{11} = (\partial_1\Gamma^0_{10} + \partial_1\Gamma^1_{11} + \partial_1\Gamma^2_{12} + \partial_1\Gamma^3_{13}) - \partial_1\Gamma^1_{11} + (\Gamma^0_{10}\Gamma^0_{01} + \Gamma^1_{11}\Gamma^1_{11} + \Gamma^2_{12}\Gamma^2_{21} + \Gamma^3_{13}\Gamma^3_{31}) - (\Gamma^1_{11}\Gamma^0_{10} + \Gamma^1_{11}\Gamma^1_{11} + \Gamma^1_{11}\Gamma^2_{12} + \Gamma^1_{11}\Gamma^3_{13})$$

$$R_{11} = \partial_r(\alpha') + \partial_r\left(\frac{1}{r}\right) + \partial_r\left(\frac{1}{r}\right) + \alpha'(\alpha') + \frac{1}{r}\left(\frac{1}{r}\right) + \frac{1}{r}\left(\frac{1}{r}\right) - \left(\beta'(\alpha') + \beta'\left(\frac{1}{r}\right) + \beta'\left(\frac{1}{r}\right)\right)$$

$$R_{11} = \alpha" + 2\,\partial_r\left(\frac{1}{r}\right) + \alpha'^2 + \frac{1}{r^2} + \frac{1}{r^2} - \left(\alpha'\beta' + \frac{2\beta'}{r}\right)$$

$$R_{11} = \alpha" - \frac{2}{r^2} + \alpha'^2 + \frac{2}{r^2} - \alpha'\beta' - \frac{2\beta'}{r}$$

$$R_{11} = \alpha" + \alpha'^2 + - \alpha'\beta' - \frac{2\beta'}{r}$$

3.Untuk $\mu = \nu = 2$

$$R_{22} = \partial_2\Gamma^\sigma_{2\sigma} - \partial_\sigma\Gamma^\sigma_{22} + \Gamma^\rho_{2\sigma}\Gamma^\sigma_{\rho 2} - \Gamma^\rho_{22}\Gamma^\sigma_{\rho\sigma}$$

$$R_{22} = \partial_2\Gamma^3_{23} - \partial_1\Gamma^1_{22} + (\Gamma^1_{2\sigma}\Gamma^\sigma_{12} + \Gamma^2_{2\sigma}\Gamma^\sigma_{22} + \Gamma^3_{2\sigma}\Gamma^\sigma_{32} + \Gamma^3_{1\sigma}\Gamma^\sigma_{31}) - \Gamma^1_{22}\Gamma^\sigma_{1\sigma}$$

$$R_{22} = \partial_2\Gamma^3_{23} - \partial_1\Gamma^1_{22} + (\Gamma^1_{22}\Gamma^2_{12} + \Gamma^2_{21}\Gamma^1_{22} + \Gamma^3_{23}\Gamma^3_{32}) - \Gamma^1_{22}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13})$$

$$R_{22} = \partial_2\Gamma^3_{23} - \partial_1\Gamma^1_{22} + (\Gamma^1_{22}\Gamma^2_{12} + \Gamma^2_{21}\Gamma^1_{22} + \Gamma^3_{23}\Gamma^3_{32}) - (\Gamma^1_{22}\Gamma^0_{10} + \Gamma^1_{22}\Gamma^1_{11} + \Gamma^1_{22}\Gamma^2_{12} + \Gamma^1_{22}\Gamma^3_{13})$$

$$R_{22} = \partial_\theta(\cot\theta) - \partial_r\left(-re^{-2\beta}\right) + \left(\frac{1}{r}\left(-re^{-2\beta}\right) + \cot\theta(\cot\theta)\right) - \left(-re^{-2\beta}(\alpha') + \left(-re^{-2\beta}\right)\beta' + \left(-re^{-2\beta}\right)\left(\frac{1}{r}\right)\right)$$

$$R_{22} = \frac{1}{\sin^2\theta} - (-e^{-2\beta} + (-r)(-2\beta')e^{-2\beta}) + \left(-e^{-2\beta} + \cot^2\theta\right)$$
$$- ((-\alpha' r - \beta' r - 1)e^{-2\beta})$$

$$R_{22} = (1 - 2\beta' r)e^{-2\beta} - e^{-2\beta} - \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - (-\alpha' r - \beta' r - 1)e^{-2\beta}$$

$$R_{22} = (1 - 2\beta' r - 1 + \alpha' r + \beta' r + 1)e^{-2\beta} + \frac{\cos^2\theta - 1}{\sin^2\theta}$$

$$R_{22} = (\alpha' r - \beta' r + 1)e^{-2\beta} - \frac{\sin^2\theta}{\sin^2\theta}$$

$$R_{22} = (1 - \beta' r + \alpha' r)e^{-2\beta} - 1$$

4. Untuk $\mu = \nu = 3$

$$R_{33} = \partial_3 \Gamma^{\sigma}_{3\sigma} - \partial_\sigma \Gamma^{\sigma}_{33} + \Gamma^{\rho}_{3\sigma}\Gamma^{\sigma}_{\rho 3} - \Gamma^{\rho}_{33}\Gamma^{\sigma}_{\rho\sigma}$$

$$R_{33} = -(\partial_1 \Gamma^1_{33} + \partial_2 \Gamma^2_{33}) + (\Gamma^1_{3\sigma}\Gamma^{\sigma}_{13} + \Gamma^2_{3\sigma}\Gamma^{\sigma}_{23} + \Gamma^3_{3\sigma}\Gamma^{\sigma}_{33}) - (\Gamma^1_{33}\Gamma^{\sigma}_{1\sigma} + \Gamma^2_{33}\Gamma^{\sigma}_{2\sigma})$$

$$R_{33} = -(\partial_1 \Gamma^1_{33} + \partial_2 \Gamma^2_{33}) + (\Gamma^1_{33}\Gamma^3_{13} + \Gamma^2_{33}\Gamma^3_{23} + (\Gamma^3_{31}\Gamma^1_{33} + \Gamma^3_{32}\Gamma^2_{33})) - (\Gamma^1_{33}(\Gamma^0_{10}$$
$$+ \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}) + \Gamma^2_{33}\Gamma^3_{23})$$

$$R_{33} = -(\partial_1 \Gamma^1_{33} + \partial_2 \Gamma^2_{33}) + (\Gamma^1_{33}\Gamma^3_{13} + \Gamma^2_{33}\Gamma^3_{23} + \Gamma^3_{31}\Gamma^1_{33} + \Gamma^3_{32}\Gamma^2_{33}) - (\Gamma^1_{33}\Gamma^0_{10}$$
$$+ \Gamma^1_{33}\Gamma^1_{11} + \Gamma^1_{33}\Gamma^2_{12} + \Gamma^1_{33}\Gamma^3_{13} + \Gamma^2_{33}\Gamma^3_{23})$$

$$R_{33} = -(\partial_1 \Gamma^1_{33} + \partial_2 \Gamma^2_{33}) + (\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} - \Gamma^3_{31})\Gamma^1_{33} + \Gamma^2_{33}\Gamma^3_{23})$$

$$R_{33} = -\partial_r\left(-r sin^2\theta e^{-2\beta}\right) - \partial_\theta\left(-sin\theta(cos\theta)\right)$$
$$- \left(\alpha' + \beta' + \frac{1}{r} - \frac{1}{r}\right)\left(-r sin^2\theta e^{-2\beta}\right) + (\cot\theta(-sin\theta\ cos\theta))$$

$$R_{33} = -\sin^2\theta\left(-r\left(-2\beta'^{e^{-2\beta}}\right) - e^{-2\beta}\right) + (\cos^2\theta - \sin^2\theta)$$
$$- \left(\alpha' + \beta' + \frac{1}{r} - \frac{1}{r}\right)\left(-r sin^2\theta e^{-2\beta}\right) + (\cot\theta(-sin\theta cos\theta))$$

$$R_{33} = (Sin^2\theta - 2\beta' r Sin^2\theta)e^{-2\beta} + (\cos^2\theta - \sin^2\theta) - (\alpha' + \beta')\left(-r sin^2\theta e^{-2\beta}\right) - \left(\frac{\cos\theta}{\sin\theta}(sin\theta cos\theta)\right)$$

$$R_{33} = (Sin^2\theta - 2\beta' r Sin^2\theta)e^{-2\beta} + \cos^2\theta - \sin^2\theta + (\alpha' r sin^2\theta + \beta' r \sin^2\theta)e^{-2\beta} - \cos^2\theta$$

$$R_{33} = (Sin^2\theta - 2\beta' r Sin^2\theta)e^{-2\beta} - sin^2\theta + (\alpha' r sin^2\theta + \beta' r \sin^2\theta)e^{-2\beta}$$

$$R_{33} = (1 - 2\beta' r)e^{-2\beta} Sin^2\theta - sin^2\theta + (\alpha' r + \beta' r)e^{-2\beta} sin^2\theta$$

$$R_{33} = ((1 - 2\beta' r)e^{-2\beta} - 1 + (\alpha' r + \beta' r)e^{-2\beta}) sin^2\theta$$

$$R_{33} = ((1 - 2\beta' r + \alpha' r + \beta' r)e^{-2\beta} - 1) sin^2\theta$$

$$R_{33} = ((1 - \beta' r + \alpha' r)e^{-2\beta} - 1) sin^2\theta$$

$$R_{33} = R_{22}\, Sin^2\theta$$

Sehingga, diperoleh komponen-komponen tensor Ricci yang tidak nol,

$$R_{00} = \left(-\alpha'' + \alpha'\beta' - \alpha'^2 - \frac{2\alpha'}{r}\right)e^{2\alpha - 2\beta}$$

$$R_{11} = \alpha'' + \alpha'^2 + -\alpha'\beta' - \frac{2\beta'}{r}$$

$$R_{22} = (1 - \beta' r + \alpha' r)e^{-2\beta} - 1$$

$$R_{33} = R_{22}\, Sin^2\theta \qquad (B.2)$$

A.3 Skalar Ricci

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$R = g^{00} R_{00} + g^{11} R_{11} + g^{22} R_{22} + g^{33} R_{33}$$

$$R = -e^{-2\alpha}\left(-\alpha'' + \alpha'\beta' - \alpha'^2 - \frac{2\alpha'}{r}\right)e^{2\alpha-2\beta} + e^{-2\beta}\left(\alpha'' + \alpha'^2 - \alpha'\beta' - \frac{2\beta'}{r}\right) + r^{-2}\left((1 - \beta' r + \alpha' r)e^{-2\beta} - 1\right) + r^{-2}\sin^{-2}\theta\,(1 - \beta' r + \alpha' r)e^{-2\beta} - 1)sin^2\theta$$

$$R = \left(\alpha'' - \alpha'\beta' + \alpha'^2 + \frac{2\alpha'}{r}\right)e^{-2\alpha+2\alpha-2\beta} + \left(\alpha'' + \alpha'^2 - \alpha'\beta' - \frac{2\beta'}{r}\right)e^{-2\beta} + \left(\left(\frac{1}{r^2} - \frac{\beta'}{r} + \frac{\alpha'}{r}\right)e^{-2\beta} - \frac{1}{r^2}\right) + \left(\left(\frac{1}{r^2} - \frac{\beta'}{r} + \frac{\alpha'}{r}\right)e^{-2\beta} - \frac{1}{r^2}\right)$$

$$R = \left(\alpha'' - \alpha'\beta' + \alpha'^2 + \frac{2\alpha'}{r}\right)e^{-2\beta} + \left(\alpha'' + \alpha'^2 - \alpha'\beta' - \frac{2\beta'}{r}\right)e^{-2\beta} + 2\left(\left(\frac{1}{r^2} - \frac{\beta'}{r} + \frac{\alpha'}{r}\right)e^{-2\beta} - \frac{1}{r^2}\right)$$

$$R = \left(\alpha'' - \alpha'\beta' + \alpha'^2 + \frac{2\alpha'}{r}\right)e^{-2\beta} + \left(\alpha'' + \alpha'^2 - \alpha'\beta' - \frac{2\beta'}{r}\right)e^{-2\beta} + \left(\frac{2}{r^2} - \frac{2\beta'}{r} + \frac{2\alpha'}{r}\right)e^{-2\beta} - \frac{2}{r^2}$$

$$R = \left(\alpha'' - \alpha'\beta' + \alpha'^2 + \frac{2\alpha'}{r} + \alpha'' + \alpha'^2 - \alpha'\beta' - \frac{2\beta'}{r} + \frac{2}{r^2} - \frac{2\beta'}{r} + \frac{2\alpha'}{r}\right)e^{-2\beta} - \frac{2}{r^2}$$

$$R = \left(2\alpha'' - 2\alpha'\beta' + 2\alpha'^2 + \frac{4\alpha'}{r} - \frac{4\beta'}{r} + \frac{2}{r^2}\right)e^{-2\beta} - \frac{2}{r^2}$$

$$R = 2\left\{\left(\alpha'' - \alpha'\beta' + \alpha'^2 + \frac{2\alpha'}{r} - \frac{2\beta'}{r} + \frac{1}{r^2}\right)e^{-2\beta} - \frac{1}{r^2}\right\}$$

LAMPIRAN B

PENURUNAN PERSAMAAN MEDAN EINSTEIN STANDAR

Kita mulai dari aksi Einstein-Hilbert

$$S = S_g + S_m \qquad (B.1)$$

dimana

$S_g = \int d^4x\sqrt{-g}\left(\mathcal{L}_g\right)$ dan $S_m = -\int d^4x\sqrt{-g}(\mathcal{L}_m) \qquad (B.2)$

aksi total menjadi

$$S = \int\ d^4x\sqrt{-g}\left(\mathcal{L}_g + \mathcal{L}_m\right) \qquad \text{(A.3)}$$

dengan $\mathcal{L}_g$ merupakan rapat langrangian medan gravitasi sedangkan $\mathcal{L}_m$ merupakan rapat langrangian dari materi.

Bentuk dari $\mathcal{L}_g$

$$\mathcal{L}_g = \frac{R}{2\kappa^2} \qquad (B.4)$$

Sehingga persamaan (B.3), menjadi:

$$S = \int\ d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \mathcal{L}_m\right) \qquad \text{(B.5)}$$

Berdasarkan prinsip aksi terkecil, variasi dari suatu aksi haruslah sama dengan nol $\delta S = 0$, sehingga

$$\delta S = \int \delta\{\ d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \mathcal{L}_m\right)\} = 0 \qquad \text{(B.6)}$$

$$\frac{1}{2\kappa^2}\int\ d^4x\ \delta(\sqrt{-g}\,R)\ - \int\ d^4x\ \delta(\sqrt{-g}\,\mathcal{L}_m) = 0 \qquad \text{(B.7)}$$

Tinjau suku pertama persamaan (A.7)

$$\delta(\sqrt{-g}R) = \delta(\sqrt{-g}\ g^{\mu\nu}R_{\mu\nu})$$

$$\delta(\sqrt{-g}R) = (\delta\ \sqrt{-g})g^{\mu\nu}R_{\mu\nu} + \sqrt{-g}\ (\delta g^{\mu\nu})R_{\mu\nu} + \sqrt{-g}g^{\mu\nu}\left(\delta R_{\mu\nu}\right) \qquad \text{(A.8)}$$

Variasi suku ketiga persamaan (B.8)

$$R_{\mu\nu} = \partial_\alpha\Gamma^\alpha_{\mu\nu} - \partial_\mu\Gamma^\alpha_{\alpha\nu} + \Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\beta\alpha} - \Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\alpha\nu}$$

$$\delta R_{\mu\nu} = \delta(\partial_\alpha\Gamma^\alpha_{\mu\nu}) - \delta(\partial_\mu\Gamma^\alpha_{\alpha\nu}) + \delta(\Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\beta\alpha}) - \delta(\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\alpha\nu})$$

$$\delta R_{\mu\nu} = \delta\left(\partial_\alpha\Gamma^\alpha_{\mu\nu}\right) - \delta\left(\partial_\mu\Gamma^\alpha_{\alpha\nu}\right) + \delta\left(\Gamma^\alpha_{\mu\nu}\right)\Gamma^\beta_{\beta\alpha} + \Gamma^\alpha_{\mu\nu}\delta\left(\Gamma^\beta_{\beta\alpha}\right)$$

$$-\delta\left(\Gamma^\alpha_{\mu\beta}\right)\Gamma^\beta_{\alpha\nu} - \Gamma^\alpha_{\mu\beta}\delta(\Gamma^\beta_{\alpha\nu})$$

$$\delta R_{\mu\nu} = \left[\delta\left(\partial_\alpha \Gamma^\alpha_{\mu\nu}\right) - \delta\left(\Gamma^\alpha_{\mu\beta}\right)\Gamma^\beta_{\alpha\nu} - \Gamma^\alpha_{\mu\beta}\delta\left(\Gamma^\beta_{\alpha\nu}\right)\right]$$
$$- \left[\delta\left(\partial_\mu \Gamma^\alpha_{\alpha\nu}\right) - \delta\left(\Gamma^\alpha_{\mu\nu}\right)\Gamma^\beta_{\beta\alpha} - \Gamma^\alpha_{\mu\nu}\delta\left(\Gamma^\beta_{\beta\alpha}\right)\right]$$
$$\delta R_{\mu\nu} = D_\alpha\left(\delta\Gamma^\alpha_{\mu\nu}\right) - D_\mu\left(\delta\Gamma^\alpha_{\alpha\nu}\right) \quad (B.9)$$

Mengalikan persamaan (B.9) dengan $g^{\mu\nu}$

$$g^{\mu\nu}\delta R_{\mu\nu} = g^{\mu\nu}\left\{D_\alpha\left(\delta\Gamma^\alpha_{\mu\nu}\right) - D_\mu\left(\delta\Gamma^\alpha_{\alpha\nu}\right)\right\}$$
$$g^{\mu\nu}\delta R_{\mu\nu} = g^{\mu\nu}D_\alpha\left(\delta\Gamma^\alpha_{\mu\nu}\right) - g^{\mu\nu}D_\mu\left(\delta\Gamma^\alpha_{\alpha\nu}\right)$$
$$g^{\mu\nu}\delta R_{\mu\nu} = D_\alpha\left(g^{\mu\nu}\delta\Gamma^\alpha_{\mu\nu}\right) - \left(D_\alpha g^{\mu\nu}\right)\delta\Gamma^\alpha_{\mu\nu} - D_\mu\left(g^{\mu\nu}\delta\Gamma^\alpha_{\alpha\nu}\right)$$
$$+ \left(D_\mu g^{\mu\nu}\right)\delta\Gamma^\alpha_{\alpha\nu} \quad (B.10)$$

turunan kovarian dari tensor metrik sama dengan nol, sehingga suku kedua dan suku keeempat lenyap. Persamaan (B.10) menjadi

$$g^{\mu\nu}\delta R_{\mu\nu} = D_\alpha\left(g^{\mu\nu}\delta\Gamma^\alpha_{\mu\nu}\right) - D_\mu\left(g^{\mu\nu}\delta\Gamma^\alpha_{\alpha\nu}\right) \quad (B.11)$$

Kontraksikan simbol $\alpha = \lambda$ dan $\mu = \lambda$

$$g^{\mu\nu}\delta R_{\mu\nu} = D_\lambda\left(g^{\mu\nu}\delta\Gamma^\lambda_{\mu\nu}\right) - D_\lambda\left(g^{\lambda\nu}\delta\Gamma^\alpha_{\alpha\nu}\right) \quad (B.12)$$

Jika digunakan teorema Gauss maka integral suku ketiga dalam persamaan (B.8) menjadi lenyap karna berubah menjadi integral permukaan yang variasinya sama dengan nol

$$\int d^4x\sqrt{-g}\, g^{\mu\nu}\left(\delta R_{\mu\nu}\right) = 0$$
$$\int_D d^4x\sqrt{-g}\;\delta R_{\mu\nu}\, g^{\mu\nu} = \int_D \frac{\partial}{\partial x^\lambda}\, d^4x\,\sqrt{-g}\left(g^{\mu\nu}\delta\Gamma^\lambda_{\mu\nu} - g^{\lambda\nu}\delta\Gamma^\alpha_{\alpha\nu}\right) = 0 \quad (B.13)$$

tersisa suku pertama dan kedua dari persamaan (B.8)

$$\delta\int d^4x\sqrt{-g}\,R = \int d^4x\; g^{\mu\nu}R_{\mu\nu}\;\delta\left(\sqrt{-g}\right) + \int d^4x\sqrt{-g}\,R_{\mu\nu}\;\left(\delta g^{\mu\nu}\right) \quad \text{(B.14)}$$

Variasi determinan metrik pada suku kedua persamaan (B.13) digunakan hubungan

$$\delta g = g g^{\mu\nu}\delta g_{\mu\nu} = -g g_{\mu\nu}\delta g^{\mu\nu}$$

Maka,

$$\delta\sqrt{-g} = \frac{\partial\sqrt{-g}}{\partial g}\delta g$$
$$\delta\sqrt{-g} = -\frac{1}{2\sqrt{-g}}\delta g$$

$$\delta\sqrt{-g} = -\frac{1}{2\sqrt{-g}}\left(-g g_{\mu\nu}\delta g^{\mu\nu}\right)$$

dikalikan dengan $\frac{\sqrt{-g}}{\sqrt{-g}}$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu} \qquad (B.15)$$

subtitusi persamaan (B.15) ke dalam suku ke dua persamaan (B.14) dan hubungan $g^{\mu\nu}R_{\mu\nu} = R$ maka diperoleh

$$\delta\int d^4x\sqrt{-g}\,R = \int d^4x\; g^{\mu\nu}R_{\mu\nu}(-\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}) \quad + \int d^4x\sqrt{-g}\;R_{\mu\nu}\;(\delta g^{\mu\nu})$$

$$\delta\int d^4x\sqrt{-g}\,R = \int d^4x\;\sqrt{-g}\;R_{\mu\nu}\,(\delta g^{\mu\nu}) - \int d^4x\sqrt{-g}\;\frac{1}{2}g_{\mu\nu}R\;(\delta g^{\mu\nu})$$

$$\delta\int d^4x\sqrt{-g}\,R = \int d^4x\;\sqrt{-g}\;\left\{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right\}\;\delta g^{\mu\nu} \qquad \text{(B.16)}$$

Variasi aksi pada persamaan (B.7) dimana $\mathcal{L}_m = \mathcal{L}_m(g^{\boldsymbol{\mu\nu}}, g^{\mu\nu}_{,\boldsymbol{\alpha}})$**,** maka persamaan (B.7) dapat dituliskan

$$\delta\left(\sqrt{-g}\;\mathcal{L}_m\right) = \frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}}\delta g^{\mu\nu} + \frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\delta(\partial_\alpha g^{\mu\nu})$$

$$\delta\left(\sqrt{-g}\;\mathcal{L}_m\right) = \frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}}\delta g^{\mu\nu} + \frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\partial_\alpha(\delta g^{\mu\nu}) \qquad \text{(B.17))}$$

Jika diterapkan aturan differensial parsial pada suku kedua persamaan (B.17) maka diperoleh

$$\frac{\partial}{\partial x^\alpha}\left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}(\delta g^{\mu\nu})\right] = \left[\frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right]\delta g^{\mu\nu} + \frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\frac{\partial}{\partial x^\alpha}(\delta g^{\mu\nu})$$

$$\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\frac{\partial}{\partial x^\alpha}(\delta g^{\mu\nu}) = \frac{\partial}{\partial x^\alpha}\left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}(\delta g^{\mu\nu})\right] - \left[\frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right]\delta g^{\mu\nu} \quad \text{(B.18)}$$

subtitusi persamaan (B.18) ke persamaan (B.17)

$$\delta\left(\sqrt{-g}\;\mathcal{L}_m\right) = \frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}}\delta g^{\mu\nu} - \left[\frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right]\delta g^{\mu\nu} + \frac{\partial}{\partial x^\alpha}\left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}(\delta g^{\mu\nu})\right]$$

$$\delta\left(\sqrt{-g}\;\mathcal{L}_m\right) = \left\{\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}} - \left[\frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right]\right\}\delta g^{\mu\nu} + \frac{\partial}{\partial x^\alpha}\left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}(\delta g^{\mu\nu})\right] \text{(B.19))}$$

Mengacu pada prinsip Hamilton bahwa variasi dari suatu aksi stasioner ($\delta I = 0$) dan suku kedua pada persamaan (B.19) akan lenyap sebagai konsekuensi integral luasan Gauss, maka

$$\int d^4x\, \delta\left(\sqrt{-g}\,\mathcal{L}_m\right) = \int d^4x \left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right] \delta g^{\mu\nu} \quad \text{(B.20)}$$

Subtitusi persamaan (B.20) dan (B.16) ke persamaan (B.7) diperoleh

$$\int \frac{1}{2\kappa^2}\, d^4x\sqrt{-g}\, \left\{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R\right\} \delta g^{\mu\nu} - d^4x \left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right]\delta g^{\mu\nu} = 0$$

$$\int d^4x\, \sqrt{-g}\, \frac{G_{\mu\nu}}{2\kappa^2}\, \delta g^{\mu\nu} - d^4x \left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right] \delta g^{\mu\nu} = 0$$

$$\int d^4x\sqrt{-g}\left\{\frac{G_{\mu\nu}}{2\kappa^2} - \frac{1}{\sqrt{-g}}\left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right]\right\} \delta g^{\mu\nu} = 0 \quad \text{(B.21)}$$

Dengan mengambil suku dalam kurung, maka dari persamaan (B.21) diperoleh:

$$\frac{G_{\mu\nu}}{2\kappa^2} - \frac{1}{\sqrt{-g}}\left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right] = 0 \quad \text{(B.22)}$$

$$G_{\mu\nu} = \frac{2\kappa^2}{\sqrt{-g}}\left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right] \quad \text{(B.23)}$$

Persamaan (B.23) merupakan persamaan medan Einstein, mengingat $G_{\mu\nu} = \kappa^2 T_{\mu\nu}$ maka tensor energi momentumnya adalah:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\left[\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}}\right)\right] \quad \text{(B.24)}$$

Persamaan (B.24) di atas merupakan sajian eksplisit bagi tensor energi-momentum untuk setiap medan materi selain gravitasi.

Secara lengkap persamaan medan gravitasi Einstein dapat ditulis

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = \kappa^2 T_{\mu\nu} \quad (B.25)$$

dengan $\kappa^2 = \frac{8\pi G}{c^4}$.

LAMPIRAN C
SOLUSI SCHWARZSCHILD DALAM TEORI GRAVITASI EINSTEIN STANDAR

Persamaan medan Einstein dalam ruang vakum ($T_{\mu\nu} = 0$) dirumuskan sebagai $G_{\mu\nu} = 0$ dengan $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, sehingga

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

$$R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R \qquad (C.1)$$

Kontraksikan dengan metrik $g^{\mu\nu}$, maka akan di dapatkan

$$g^{\mu\nu} R_{\mu\nu} = \frac{1}{2} g^{\mu\nu} g_{\mu\nu} R$$

$$R = \frac{1}{2} \delta^{\mu}_{\mu} R \qquad (C.2)$$

Dimana $\delta^{\mu}_{\mu} = g^{\mu\nu} g_{\mu\nu} = 4$, maka persamaan diatas menjadi

$$R = \frac{1}{2}(4)R$$

$$R = 2R \rightarrow R = 0 \qquad (C.3)$$

Subtitusi ke persamaan (C.1), diperoleh

$$R_{\mu\nu} = 0 \qquad (C.4)$$

Karena ($R_{\mu\nu} = 0$), maka persamaan medan hanya memberi 3 persamaan

$$-\alpha" + \alpha'\beta' - \alpha'^2 - \frac{2\alpha'}{r} = 0 \qquad (C.5)$$

$$\alpha" + \alpha'^2 - \alpha'\beta' - \frac{2\beta'}{r} = 0 \qquad (C.6)$$

$$(1 - \beta' r + \alpha' r) e^{-2\beta} = 1 \qquad (C.7)$$

dengan menjumlahkan persamaan (C.5) dan persamaan (C.6),

$$\frac{2\alpha'}{r} + \frac{2\beta'}{r} = 0$$

$$\frac{2}{r}(\alpha' + \beta') = 0$$

$$\alpha' + \beta' = 0$$

$$\alpha' = -\beta' \qquad (C.8)$$

Integrasinya memberikan

$$\alpha + \beta = A \qquad (C.9)$$

dimana A adalah konstanta integrasi.

Selanjutnya integrasikan persamaan $(C.7)$ dengan menggunakan persamaan $(C.8)$ sehingga diperoleh

$$(1 - \beta' r + \alpha' r)e^{-2\beta} = 1$$

$$(1 - (-\alpha')r + \alpha' r)e^{-2(-\alpha')} = 1$$

$$(1 + \alpha' r + \alpha' r)e^{2\alpha'} = 1$$

$$(1 + 2\alpha' r)e^{2\alpha} = 1$$

$$e^{2\alpha} + 2\alpha' r e^{2\alpha} = 1$$

$$\frac{d}{dr}[re^{2\alpha}] = 1$$

$$d\,(re^{2\alpha}) = dr \qquad (C.9)$$

dengan mengintegralkan persamaan $(C.9)$, maka

$$\int d[re^{2\alpha}] = \int dr$$

$$re^{2\alpha} = r + B \qquad (C.10)$$

Integralkan

$$\int d(re^{2\alpha}) = \int dr$$

$$re^{2\alpha} = r + B \qquad (C.11)$$

Dimana B adalah adalah konstanta integrasi. Dengan demikian didapatkan hubungan

$$e^{2\alpha} = 1 + \frac{B}{r} \qquad (C.12)$$

Untuk menentukan kontanta integrasi B, diberikan syarat batas untuk jarak yang sangat jauh dari massa sumber ($r \to \infty$), pengaruh gravitasi menjadi sangat lemah sehingga geometri ruang-waktu haruslah tereduksi menjadi ruang Minkowski. Dengan kata lain,

$$g_{00}(r \to \infty) = -e^{2\alpha} = -1 \qquad (C.13)$$

$$g_{11}(r \to \infty) = e^{2\beta} = 1 \qquad (C.14)$$

Yang hanya dipenuhi oleh

$$\alpha(r \to \infty) \to 0 \qquad (C.15)$$

$$\beta(r \to \infty) \to 0 \qquad (C.16)$$

Sehingga $A = 0$, yang memberikan

$$\alpha + \beta = 0 \to \alpha = -\beta \qquad (C.17)$$

Kemudian untuk menentukan nilai konstanta B, ditinjau komponen g_{00} dalam limit medan lemah. Lagrangian non-relativistik berbentuk

$$L = -mc^2 - m\phi + \frac{1}{2}mv^2$$

$$L = -mc\left(c + \frac{\phi}{c} - \frac{\frac{1}{2}v^2}{c}\right)$$

$$L = -mc\left(c + \frac{\phi}{c} - \frac{1}{2} g_{ij} \frac{v^i}{c} v^j\right) \qquad (C.18)$$

Pada koordinat kartesian $g_{ij} = -\delta_{ij}$, sehingga menjadi

$$L = -mc\left(c + \frac{\phi}{c} + \frac{1}{2} g_{ij} \frac{v^i}{c} v^j\right) \qquad (C.19)$$

Integral aksi adalah

$$I = \int L\, dt$$

$$I = \int \left(-mc\left(c + \frac{\phi}{c} + \frac{1}{2} g_{ij} \frac{v^i}{c} v^j\right)\right) dt$$

$$I = -mc \int \left(c + \frac{\phi}{c} + \frac{1}{2} g_{ij} \frac{v^i}{c} v^j\right) dt$$

$$I = -mc \int \left(\left(c + \frac{\phi}{c}\right) + \frac{1}{2} g_{ij} \frac{v^i}{c} \frac{dx^j}{dt}\right) dt$$

$$I = -mc \int \left(\left(1 + \frac{\phi}{c^2}\right) c + \frac{1}{2} g_{ij} \frac{v^i}{c} \frac{dx^j}{dt}\right) dt$$

$$I = -mc \int \left(\left(1 + \frac{\phi}{c^2}\right) cdt + \frac{1}{2} g_{ij} \frac{v^i}{c} dx^j\right)$$

$$I = -mc \int ds \qquad (C.20)$$

Maka

$$ds = \left(1 + \frac{\phi}{c^2}\right) cdt + \frac{1}{2} g_{ij} \frac{v^i}{c} dx^j \qquad (C.21)$$

Sehingga

$$ds^2 = \left[\left(1+\frac{\phi}{c^2}\right)cdt + \frac{1}{2}g_{ij}\frac{v^i}{c}dx^j\right]^2$$

$$ds^2 = \left(1+\frac{\phi}{c^2}\right)^2 c^2dt^2 + \frac{1}{2}\left(1+\frac{\phi}{c^2}\right)g_{ij}\frac{v^i}{c}dx^jcdt + \frac{1}{2}\left(1+\frac{\phi}{c^2}\right)g_{ij}\frac{v^i}{c}dx^jcdt + \frac{1}{4}\left(g_{ij}\frac{v^i}{c}dx^j\right)^2$$

$$ds^2 = \left(1+\frac{\phi}{c^2}\right)^2 c^2dt^2 + \left(1+\frac{\phi}{c^2}\right)g_{ij}\frac{v^i}{c}dx^jcdt + \frac{1}{4}\left(g_{ij}\frac{v^i}{c}dx^j\right)^2 \qquad (C.22)$$

dimana $\frac{1}{4}\left(g_{ij}\frac{v^i}{c}dx^j\right)^2 \approx 0$

$$ds^2 \approx \left(1+\frac{\phi}{c^2}\right)^2 c^2dt^2 + \left(1+\frac{\phi}{c^2}\right)g_{ij}\frac{v^i}{c}dx^jcdt$$

$$ds^2 \approx \left(1+\frac{\phi}{c^2}\right)^2 c^2dt^2 + \left(1+\frac{\phi}{c^2}\right)g_{ij}v^idt\,dx^j$$

$$ds^2 \approx \left(1+\frac{\phi}{c^2}\right)^2 c^2dt^2 + \left(1+\frac{\phi}{c^2}\right)g_{ij}dx^idx^j \qquad (C.23)$$

dengan

$$\left(1+\frac{\phi}{c^2}\right)^2 = 1 + \frac{\phi}{c^2} + \frac{\phi}{c^2} + \frac{\phi^2}{c^4}$$

$$\left(1+\frac{\phi}{c^2}\right)^2 = 1 + \frac{2\phi}{c^2} + \frac{\phi^2}{c^4} \approx 1 + \frac{2\phi}{c^2} \qquad (C.24)$$

Serta

$$\left(1+\frac{\phi}{c^2}\right)g_{ij}dx^idx^j \approx g_{ij}dx^idx^j \qquad (C.25)$$

Sehingga

$$ds^2 \approx \left(1+\frac{2\phi}{c^2}\right)c^2dt^2 + g_{ij}dx^idx^j \qquad C.26)$$

didapatkan komponen g_{00}

$$g_{00} = \left(1 + \frac{2\phi}{c^2}\right) \qquad (C.27)$$

Dibandingkan solusi Schwarzschild pada persamaan $(C.12)$ dan $(C.27)$

$$1 + \frac{B}{r} = 1 + \frac{2\phi}{c^2}$$

$$\frac{B}{r} = \frac{2\phi}{c^2}$$

$$B = \phi \frac{2r}{c^2} \qquad (C.21)$$

dengan $\phi = -\frac{GM}{r}$ adalah potensial gravitasi Newton.

Sehingga

$$B = -\frac{GM}{r}\frac{2r}{c^2}$$

$$B = -\frac{2GM}{c^2} \qquad (C.22)$$

Jika dimisalkan $m = \frac{GM}{c^2}$, maka $B = -2m$

Elemen garisnya akan menjadi

$$ds^2 = -\left(1 - \frac{2m}{r}\right)c^2dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (C.23)$$

Persamaan $(C.23)$ dapat ditulis kembali dalam bentuk

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)c^2dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (C.25)$$

Dengan $r_s = 2m$

Persamaan diatas merupakan metrik Schwarzschild dalam teori gravitasi Einstein.

LAMPIRAN D
PENURUNAN PERSAMAAN MEDAN EINSTEIN TERMODIFIKASI DALAM TEORI GRAVITASI $f(R)$

Aksi dari teori gravitasi $f(R)$ diberikan oleh persamaan (2.21):

$$S = S_g(g_{\mu\nu}) + S_m(g_{\mu\nu}, \psi) \qquad (D.1)$$

dimana S_g adalah aksi gravitasi dan S_m adalah aksi materi.

aksi total gravitasi $f(R)$ dapat ditulis,

$$S = \int d^4x\sqrt{-g}\left(\frac{f(R)}{2\kappa^2}\right) + S_m(g_{\mu\nu}, \psi)$$

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + S_m \qquad (D.2)$$

Untuk mendapatkan persamaan medan gravitasi $f(R)$, kita terlebih dahulu meninjau suku pertama dengan memvariasikan terhadap metrik $g_{\mu\nu}$:

$$\delta S = \frac{1}{2\kappa^2}\int d^4x\delta(\sqrt{-g}f(R)) \qquad (D.3)$$

$$\delta S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\delta(f(R)) + \frac{1}{2\kappa^2}\int d^4x\delta(\sqrt{-g})f(R) \qquad (D.4)$$

Kita tinjau suku pertama pada persamaan di atas,

$$\delta S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\delta(f(R)) \qquad (D.5)$$

Variasi $f(R)$:

$$\delta f(R) = \frac{\partial f(R)}{\partial R}\delta R = \frac{\partial f(R)}{\partial R}\delta(g^{\mu\nu}R_{\mu\nu}) = \frac{\partial f(R)}{\partial R}(\delta g^{\mu\nu})R_{\mu\nu} + \frac{\partial f(R)}{\partial R}g^{\mu\nu}\delta R_{\mu\nu} \quad \text{(D.6)}$$

Subtitusi ke persamaan (D.5),

$$\delta S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(\frac{\partial f(R)}{\partial R}(\delta g^{\mu\nu})R_{\mu\nu} + \frac{\partial f(R)}{\partial R}g^{\mu\nu}\delta R_{\mu\nu}\right) + \frac{1}{2\kappa^2}\int d^4x\delta\left(\sqrt{-g}\right)f(R)$$

$$\delta S = \frac{1}{2\kappa^2}\int d^4x\left[\sqrt{-g}\left(\frac{\partial f(R)}{\partial R}(\delta g^{\mu\nu})R_{\mu\nu} + \frac{\partial f(R)}{\partial R}g^{\mu\nu}\delta R_{\mu\nu}\right) + \delta\left(\sqrt{-g}\right)f(R)\right]$$

$$\delta S = \frac{1}{2\kappa^2}\int d^4x\left[\left(\sqrt{-g}\frac{\partial f(R)}{\partial R}(\delta g^{\mu\nu})R_{\mu\nu} + \sqrt{-g}\frac{\partial f(R)}{\partial R}g^{\mu\nu}\delta R_{\mu\nu}\right) + \delta(\sqrt{-g})f(R)\right] \qquad \text{(D.7)}$$

Sehingga variasi aksi terbagi 3 bagian,

$$\delta S = (\delta S1) + (\delta S2) + (\delta S3) \qquad (D.8)$$

dimana

$$\delta(S1) = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\frac{\partial f(R)}{\partial R}(\delta g^{\mu\nu})R_{\mu\nu}\right] \qquad (D.9)$$

$$\delta(S2) = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\frac{\partial f(R)}{\partial R}g^{\mu\nu}\delta R_{\mu\nu}\right] \qquad (D.10)$$

$$\delta(S3) = \frac{1}{2\kappa^2}\int d^4x\left[\delta\left(\sqrt{-g}\right)f(R)\right] \qquad (D.11)$$

atau bisa ditulis kembali, dengan $\delta(\sqrt{-g}) = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\delta g^{\mu\nu}$

$$\delta(S1) = \int d^4x\sqrt{-g}\left[\frac{\partial f(R)}{\partial R}R_{\mu\nu}\right]\delta g^{\mu\nu} \qquad (D.12)$$

$$\delta(S2) = \int d^4x\sqrt{-g}\left[\frac{\partial f(R)}{\partial R}g^{\mu\nu}\right]\delta R_{\mu\nu} \qquad (D.13)$$

$$\delta(S3) = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g_{\mu\nu}f(R)\right]\delta g^{\mu\nu} \qquad (D.14)$$

Untuk mendapatkan suku kedua $\delta(S2)$ bentuknya sama dengan $\delta(S1)$ dan $\delta(S3)$, maka dilakukan variasi tensor Ricci:

$$\delta R_{\mu\nu} = \delta\left(\partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\alpha} + \Gamma^\beta_{\mu\nu}\Gamma^\alpha_{\beta\alpha} - \Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\alpha\nu}\right)$$

$$\delta R_{\mu\nu} = \delta\partial_\alpha \Gamma^\alpha_{\mu\nu} - \delta\partial_\nu \Gamma^\alpha_{\mu\alpha} + \left(\delta\Gamma^\beta_{\mu\nu}\right)\Gamma^\alpha_{\beta\alpha} + \Gamma^\beta_{\mu\nu}\left(\delta\Gamma^\alpha_{\beta\alpha}\right) - \left(\delta\Gamma^\alpha_{\mu\beta}\right)\Gamma^\beta_{\alpha\nu} - \Gamma^\alpha_{\mu\beta}\left(\delta\Gamma^\beta_{\alpha\nu}\right)$$

$$\delta R_{\mu\nu} = \delta\partial_\alpha \Gamma^\alpha_{\mu\nu} - \left(\delta\Gamma^\alpha_{\mu\beta}\right)\Gamma^\beta_{\alpha\nu} - \Gamma^\alpha_{\mu\beta}\left(\delta\Gamma^\beta_{\alpha\nu}\right) - \delta\partial_\nu \Gamma^\alpha_{\mu\alpha} + \left(\delta\Gamma^\beta_{\mu\nu}\right)\Gamma^\alpha_{\beta\alpha} + \Gamma^\beta_{\mu\nu}\left(\delta\Gamma^\alpha_{\beta\alpha}\right)$$

$$\delta R_{\mu\nu} = \left[\delta\partial_\alpha \Gamma^\alpha_{\mu\nu} - \left(\delta\Gamma^\alpha_{\mu\beta}\right)\Gamma^\beta_{\alpha\nu} - \Gamma^\alpha_{\mu\beta}\left(\delta\Gamma^\beta_{\alpha\nu}\right)\right] - \left[\delta\partial_\nu \Gamma^\alpha_{\mu\alpha} - \left(\delta\Gamma^\beta_{\mu\nu}\right)\Gamma^\alpha_{\beta\alpha} - \Gamma^\beta_{\mu\nu}\left(\delta\Gamma^\alpha_{\beta\alpha}\right)\right] \qquad (D.15)$$

Dengan menggunakan hubungan turunan kovarian,

$$D_\alpha\delta\Gamma^\alpha_{\mu\nu} = \delta\partial_\alpha \Gamma^\alpha_{\mu\nu} - \left(\delta\Gamma^\alpha_{\mu\beta}\right)\Gamma^\beta_{\alpha\nu} - \Gamma^\alpha_{\mu\beta}\left(\delta\Gamma^\beta_{\alpha\nu}\right) \qquad (D.16)$$

$$D_\nu\delta\Gamma^\alpha_{\mu\alpha} = \delta\partial_\nu \Gamma^\alpha_{\mu\alpha} - \left(\delta\Gamma^\beta_{\mu\nu}\right)\Gamma^\alpha_{\beta\alpha} - \Gamma^\beta_{\mu\nu}\left(\delta\Gamma^\alpha_{\beta\alpha}\right) \qquad (D.17)$$

Sehingga variasi tensor Ricci menjadi,

$$\delta R_{\mu\nu} = D_\alpha\delta\Gamma^\alpha_{\mu\nu} - D_\nu\delta\Gamma^\alpha_{\mu\alpha}$$

$$\delta R_{\mu\nu} = \nabla_\alpha\delta\Gamma^\alpha_{\mu\nu} - \nabla_\nu\delta\Gamma^\alpha_{\mu\alpha} \qquad (D.18)$$

Kontraksikan tensor Ricci dengan $g^{\mu\nu}$,

$$g^{\mu\nu}\delta R_{\mu\nu} = g^{\mu\nu}\left(\nabla_\alpha\delta\Gamma^\alpha_{\mu\nu} - \nabla_\nu\delta\Gamma^\alpha_{\mu\alpha}\right)$$

$$g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\alpha g^{\mu\nu}\delta\Gamma^\alpha_{\mu\nu} - \nabla_\nu g^{\mu\nu}\delta\Gamma^\alpha_{\mu\alpha} \qquad (D.19)$$

Dengan mengkontraksikan $\alpha = \beta$ pada suku pertama dan $\nu = \beta$ pada suku kedua:

$$g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\beta g^{\mu\nu}\delta\Gamma^\beta_{\mu\nu} - \nabla_\beta g^{\mu\beta}\delta\Gamma^\alpha_{\mu\alpha}$$

$$g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\beta\left(g^{\mu\nu}\delta\Gamma^\beta_{\mu\nu} - g^{\mu\beta}\delta\Gamma^\alpha_{\mu\alpha}\right) \qquad (D.20)$$

Subtitusi variasi dari $\delta\Gamma^{\beta}_{\mu v}$ dan $\delta\Gamma^{\alpha}_{\mu\alpha}$ terkait dengan $\delta g^{\mu v}$:

$$\delta\Gamma^{\beta}_{\mu v} = -\tfrac{1}{2}\left(g_{\alpha\mu}\nabla_{v}\left(\delta g^{\alpha\beta}\right) + g_{\alpha\mu}\nabla_{\mu}\left(\delta g^{\alpha\beta}\right) - g_{\mu\lambda}g_{v\sigma}\nabla^{\beta}\left(\delta g^{\lambda\sigma}\right)\right) \tag{D.21}$$

$$\delta\Gamma^{\alpha}_{\mu\alpha} = -\tfrac{1}{2}\left(g_{\rho\mu}\nabla_{\alpha}(\delta g^{\alpha\rho}) + g_{\rho\mu}\nabla_{\mu}(\delta g^{\alpha\rho}) - g_{\mu\sigma}g_{\alpha\lambda}\nabla^{\alpha}\left(\delta g^{\lambda\sigma}\right)\right) \tag{D.22}$$

Subtitusi persamaan (D.21) dan (D.22) ke persamaan (D.20):

$$g^{\mu v}\delta R_{\mu v} = \nabla_{\beta}\left[g^{\mu v}\left(-\tfrac{1}{2}\left\{g_{\alpha\mu}\nabla_{v}\left(\delta g^{\alpha\beta}\right) + g_{\alpha\mu}\nabla_{\mu}\left(\delta g^{\alpha\beta}\right) - g_{\mu\lambda}g_{v\sigma}\nabla^{\beta}\left(\delta g^{\lambda\sigma}\right)\right\}\right) - g^{\mu\beta}\left(-\tfrac{1}{2}\left\{g_{\rho\mu}\nabla_{\alpha}(\delta g^{\alpha\rho}) + g_{\rho\mu}\nabla_{\mu}(\delta g^{\alpha\rho}) - g_{\mu\sigma}g_{\alpha\lambda}\nabla^{\alpha}\left(\delta g^{\lambda\sigma}\right)\right\}\right)\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\tfrac{1}{2}\nabla_{\beta}\left[g^{\mu v}\left(g_{\alpha\mu}\nabla_{v}\left(\delta g^{\alpha\beta}\right) + g_{\alpha\mu}\nabla_{\mu}\left(\delta g^{\alpha\beta}\right) - g_{\mu\lambda}g_{v\sigma}\nabla^{\beta}\left(\delta g^{\lambda\sigma}\right)\right) - g^{\mu\beta}\left(g_{\rho\mu}\nabla_{\alpha}(\delta g^{\alpha\rho}) + g_{\rho\mu}\nabla_{\mu}(\delta g^{\alpha\rho}) - g_{\mu\sigma}g_{\alpha\lambda}\nabla^{\alpha}\left(\delta g^{\lambda\sigma}\right)\right)\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\tfrac{1}{2}\nabla_{\beta}\left[g^{\mu v}g_{\alpha\mu}\nabla_{v}\left(\delta g^{\alpha\beta}\right) + g^{\mu v}g_{\alpha\mu}\nabla_{\mu}\left(\delta g^{\alpha\beta}\right) - g^{\mu v}g_{\mu\lambda}g_{v\sigma}\nabla^{\beta}\left(\delta g^{\lambda\sigma}\right) - g^{\mu\beta}g_{\rho\mu}\nabla_{\alpha}(\delta g^{\alpha\rho}) - g^{\mu\beta}g_{\rho\mu}\nabla_{\mu}(\delta g^{\alpha\rho}) + g^{\mu\beta}g_{\mu\sigma}g_{\alpha\lambda}\nabla^{\alpha}\left(\delta g^{\lambda\sigma}\right)\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\tfrac{1}{2}\nabla_{\beta}\left[\delta^{v}_{\alpha}\nabla_{v}\left(\delta g^{\alpha\beta}\right) + \delta^{v}_{\alpha}\nabla_{\mu}\left(\delta g^{\alpha\beta}\right) - g_{\lambda\sigma}\nabla^{\beta}\left(\delta g^{\lambda\sigma}\right) - \delta^{\beta}_{\rho}\nabla_{\alpha}(\delta g^{\alpha\rho}) - g^{\mu\beta}g_{\rho\mu}\nabla_{\mu}(\delta g^{\alpha\rho}) + \delta^{\beta}_{\sigma}g_{\alpha\lambda}\nabla^{\alpha}\left(\delta g^{\lambda\sigma}\right)\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\tfrac{1}{2}\nabla_{\beta}\left[\delta^{v}_{\alpha}\nabla_{v}\left(\delta g^{\alpha\beta}\right) - \delta^{\beta}_{\rho}\nabla_{\alpha}(\delta g^{\alpha\rho}) + \delta^{v}_{\alpha}\nabla_{\mu}\left(\delta g^{\alpha\beta}\right) - g_{\lambda\sigma}\nabla^{\beta}\left(\delta g^{\lambda\sigma}\right) - g^{\mu\beta}g_{\rho\mu}\nabla_{\mu}(\delta g^{\alpha\rho}) + \delta^{\beta}_{\sigma}g_{\alpha\lambda}\nabla^{\alpha}\left(\delta g^{\lambda\sigma}\right)\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\frac{1}{2}\nabla_{\beta}\left[\nabla_{\alpha}\left(\delta g^{\alpha\beta}\right) - \nabla_{\alpha}\left(\delta g^{\alpha\beta}\right) + \nabla_{\mu}\left(\delta g^{\mu\beta}\right) - g_{\lambda\sigma}\nabla^{\beta}\left(\delta g^{\lambda\sigma}\right) - g^{\mu\beta}g_{\rho\mu}\nabla_{\mu}(\delta g^{\alpha\rho}) + \delta^{\beta}_{\sigma}g_{\alpha\lambda}\nabla^{\alpha}\left(\delta g^{\lambda\sigma}\right)\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\frac{1}{2}\nabla_{\beta}\left[\nabla_{\mu}\left(\delta g^{\mu\beta}\right) - g_{\lambda\sigma}\nabla^{\beta}\left(\delta g^{\lambda\sigma}\right) - g_{\rho\mu}\nabla^{\beta}(\delta g^{\alpha\rho}) + g_{\alpha\lambda}\nabla^{\alpha}\left(\delta g^{\lambda\beta}\right)\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\frac{1}{2}\nabla_\beta\left[\nabla_\mu\left(\delta g^{\mu\beta}\right) + g_{\alpha\lambda}\nabla^\alpha\left(\delta g^{\lambda\beta}\right) - g_{\lambda\sigma}\nabla^\beta\left(\delta g^{\lambda\sigma}\right) - g_{\rho\mu}\nabla^\beta(\delta g^{\alpha\rho})\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\frac{1}{2}\nabla_\beta\left[\nabla_\mu\left(\delta g^{\mu\beta}\right) + \nabla_\lambda\left(\delta g^{\lambda\beta}\right) - g_{\lambda\sigma}\nabla^\beta\left(\delta g^{\lambda\sigma}\right) - g_{\rho\mu}\nabla^\beta(\delta g^{\alpha\rho})\right] \quad (4.23)$$

Dengan mengkontraksikan $\alpha, \lambda = \mu$ dan $\rho, \sigma = v$ diperoleh:

$$g^{\mu v}\delta R_{\mu v} = -\frac{1}{2}\nabla_\beta\left[\nabla_\mu\left(\delta g^{\mu\beta}\right) + \nabla_\mu\left(\delta g^{\mu\beta}\right) - g_{\mu v}\nabla^\beta(\delta g^{\mu v}) - g_{\mu v}\nabla^\beta(\delta g^{\mu v})\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\frac{1}{2}\nabla_\beta\left[\, 2\,\nabla_\mu\left(\delta g^{\mu\beta}\right) - 2\, g_{\mu v}\nabla^\beta(\delta g^{\mu v})\right]$$

$$g^{\mu v}\delta R_{\mu v} = -\,\nabla_\beta\left[\,\nabla_\mu\left(\delta g^{\mu\beta}\right) - \, g_{\mu v}\nabla^\beta(\delta g^{\mu v})\right]$$

$$g^{\mu v}\delta R_{\mu v} = \; g_{\mu v}\,\nabla_\beta\,\nabla^\beta(\delta g^{\mu v}) - \nabla_\mu\,\nabla_\beta\,\left(\delta g^{\mu\beta}\right)$$

$$g^{\mu v}\delta R_{\mu v} = \; g_{\mu v}\,\Box\,(\delta g^{\mu v}) - \nabla_\mu\,\nabla_v\,(\delta g^{\mu v}) \quad (D.24)$$

didapatkan variasi aksi total dengan $F(R) = \frac{\partial f(R)}{\partial R}$,

$$\delta S = \frac{1}{2\kappa^2}\Big[\int\sqrt{-g}\,F(R)R_{\mu v}\delta g^{uv}\;d^4x + \int\sqrt{-g}\,F(R)\,\Big(g_{\mu v}\,\Box\,(\delta g^{\mu v}) - \nabla_\mu\,\nabla_v\,(\delta g^{\mu v})\Big)\,d^4x - \frac{1}{2}\int\sqrt{-g}\,f(R)\,g_{\mu v}\,\delta g^{\mu v}\,d^4x\Big] + \delta S_m = 0$$

$$\delta S = \frac{1}{2\kappa^2}\Big[\int\sqrt{-g}\,F(R)R_{\mu v}\delta g^{uv}\;d^4x + \int\sqrt{-g}\;\,g_{\mu v}\,\Box\,F(R)\,\delta g^{\mu v}\,d^4x - \int\sqrt{-g}\;\,\nabla_\mu\,\nabla_v\,F(R)\,\delta g^{\mu v}\,d^4x - \frac{1}{2}\int\sqrt{-g}\,f(R)\,g_{\mu v}\,\delta g^{\mu v}\,d^4x\Big] + \delta S_m = 0$$

$$\delta S = \frac{1}{2\kappa^2}\Big[\int\sqrt{-g}\;\Big\{F(R)R_{\mu v} + \; g_{\mu v}\,\Box\,F(R) - \;\nabla_\mu\,\nabla_v F(R) - \frac{1}{2}f(R)\,g_{\mu v}\Big\}\,\delta g^{uv}\;d^4x\Big] + \delta S_m = 0 \quad (D.25)$$

Tinjau suku terakhir, dimana variasi aksi medan-materi, didefinisikan sebagai:

$$\delta S_m = -\int\delta(\sqrt{-g}\,\mathcal{L}_M)\,d^4x \quad (D.26)$$

dengan memasukkan persamaan (2.15), diperoleh:

$$\delta S_m = -\int \left[\frac{\partial(\sqrt{-g}\mathcal{L}_M)}{\partial g^{\mu v}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_M)}{\partial g^{\mu v}_{,\alpha}}\right)\right] \delta g^{\mu v}\, d^4x \qquad \text{(D.27)}$$

dari persamaan tensor energi-momentum (2.20) diperoleh hubungan:

$$\left[\frac{\partial(\sqrt{-g}\mathcal{L}_M)}{\partial g^{\mu v}} - \frac{\partial}{\partial x^\alpha}\left(\frac{\partial(\sqrt{-g}\mathcal{L}_M)}{\partial g^{\mu v}_{,\alpha}}\right)\right] = \frac{T_{\mu v}\sqrt{-g}}{2} \qquad \text{(D.28)}$$

Masukkan persamaan (D.28) ke persamaan (D.27) diperoleh:

$$\delta S_m = -\int \left(\frac{1}{2}\sqrt{-g}\, T_{\mu v}\right) \delta g^{\mu v}\, d^4x \qquad \text{(D.29)}$$

Sehingga dengan memasukkan persamaan (D.29) pada persamaan (D.25) menjadi:

$$\delta S = \frac{1}{2\kappa^2}\left[\int \sqrt{-g}\, \left\{F(R)R_{\mu v} + g_{\mu v} \Box F(R) - \nabla_\mu \nabla_v F(R) - \frac{1}{2}f(R)\, g_{\mu v}\right\}\delta g^{uv}\, d^4x\right] - \int \left(\frac{1}{2}\sqrt{-g}\, T_{\mu v}\right) \delta g^{\mu v}\, d^4x = 0$$

$$\delta S = \frac{1}{2}\int \sqrt{-g}\left[\frac{F(R)}{\kappa^2}\{R_{\mu v} + g_{\mu v} \Box - \nabla_\mu \nabla_v\} - \frac{1}{2\kappa^2}f(R)\, g_{\mu v} - T_{\mu v}\right]\delta g^{\mu v} d^4x = 0$$

$$\frac{F(R)}{\kappa^2}\{R_{\mu v} + g_{\mu v} \Box - \nabla_\mu \nabla_v\} - \frac{1}{2\kappa^2}f(R)\, g_{\mu v} - T_{\mu v} = 0$$

$$\frac{1}{\kappa^2}\left[F(R)\{R_{\mu v} + g_{\mu v} \Box - \nabla_\mu \nabla_v\} - \frac{1}{2}f(R)\, g_{\mu v}\right] = T_{\mu v} \qquad \text{(D.30)}$$

Sehingga diperoleh

$$F(R)R_{\mu v} - \frac{1}{2}f(R)\, g_{\mu v} + g_{\mu v} \Box F(R) - \nabla_\mu \nabla_v F(R) = \kappa^2 T_{\mu v} \qquad \text{(D.31)}$$

Persamaan diatas adalah persamaan medan dalam teori gravitasi $f(R)$.

LAMPIRAN E
SOLUSI SCHWARZSCHILD TERMODIFIKASI DALAM TEORI GRAVITASI $\boldsymbol{f}(\boldsymbol{R})$

Elemen garis simetri bola

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (E.1)$$

Tensor metrik kovarian

$$g_{\mu\nu} = \begin{pmatrix} -e^{2\alpha} & 0 & 0 & 0 \\ 0 & e^{2\beta} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} \qquad (E.2)$$

Tensor metrik kontravarian

$$g^{\mu\nu} = \begin{pmatrix} -e^{-2\alpha} & 0 & 0 & 0 \\ 0 & e^{-2\beta} & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2}\sin^{-2}\theta \end{pmatrix} \qquad (E.3)$$

Simbol Christoffel jenis kedua

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(\partial_\nu g_{\mu\sigma} + \partial_\nu g_{\nu\sigma} - \partial_\sigma g_{\mu\nu}\right) \qquad (E.4)$$

Beberapa komponen simbol Christoffel yang tidak nol,

$\Gamma^0_{01} = \Gamma^0_{10} = \alpha'$

$\Gamma^1_{00} = \alpha' e^{2\alpha-2\beta}$

$\Gamma^1_{11} = \beta'$

$\Gamma^1_{22} = -re^{-2\beta}$

$\Gamma^1_{33} = -r\, sin^2\theta\, e^{-2\beta}$

$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r}$

$$\Gamma^2_{33} = -\sin\theta\cos\theta$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta \qquad (E.5)$$

Komponen-komponen tensor Ricci yang tidak nol,

$$R_{00} = \left(-\alpha'' + \alpha'\beta' - {\alpha'}^2 - \frac{2\alpha'}{r}\right)e^{2\alpha-2\beta}$$

$$R_{11} = \alpha'' + {\alpha'}^2 - \alpha'\beta' - \frac{2\beta'}{r}$$

$$R_{22} = (1 - \beta' r + \alpha' r)e^{-2\beta} - 1$$

$$R_{33} = \sin^2\theta\, R_{22} \qquad (E.6)$$

Solusi yang dicari adalah solusi dengan konstanta kurvatur $R = R_0$. Sehingga persamaan Einstein menjadi

$$R_{\mu\nu}F(R) - \frac{1}{2}g_{\mu\nu}f(R) = 0$$

$$R_{\mu\nu}F(R) = \frac{1}{2}g_{\mu\nu}f(R) \qquad (E.7)$$

$$R_{\mu\nu}F(R_0) = \frac{1}{2}g_{\mu\nu}f(R_0) \qquad (E.8)$$

diperoleh,

$$R_{\mu\nu} = \frac{1}{2}\frac{f(R_0)}{F(R_0)}g_{\mu\nu} \qquad (E.9)$$

Trace persamaan medan (E.7),

$$g^{\mu\nu}(R_{\mu\nu}F(R)) = g^{\mu\nu}\left(\frac{1}{2}g_{\mu\nu}f(R)\right)$$

$$g^{\mu\nu}R_{\mu\nu}F(R) = g^{\mu\nu}g_{\mu\nu}\frac{1}{2}f(R)$$

$$RF(R) = \frac{4}{2}f(R)$$

$$RF(R) = 2f(R) \qquad (E.10)$$

maka persamaan diatas menjadi

$$R_0F(R_0) = 2f(R_0) \qquad (E.11)$$

Kemudian R_0 didefinisikan,

$$R_0 = \frac{2f(R_0)}{F(R_0)} \qquad (E.12)$$

$$\frac{R_0}{2} = \frac{f(R_0)}{F(R_0)} \qquad (E.13)$$

Sehingga persamaan (E.9), menjadi

$$R_{\mu\nu} = \frac{R_0}{4}g_{\mu\nu} \qquad (E.14)$$

Menggunakan R_{22} dan $\alpha' = -\beta'$

$$(1 + \alpha'r + \alpha'r)e^{2\alpha} - 1 = \frac{R_0}{4}(r^2)$$

$$(1 + 2\alpha'r)e^{2\alpha} - 1 = \frac{R_0}{4}r^2$$

$$e^{2\alpha} + e^{2\alpha}2\alpha'r = 1 + \frac{R_0}{4}r^2$$

$$\frac{d}{dr}(re^{2\alpha}) = 1 + \frac{R_0}{4}r^2$$

$$\int d(re^{2\alpha}) = \int \left(dr + \frac{R_0}{4}r^2dr\right)$$

$$\int d(re^{2\alpha}) = \int dr + \int \frac{R_0}{4} r^2 dr$$

$$re^{2\alpha} = r + \frac{R_0}{12} r^3 + C$$

$$e^{2\alpha} = 1 + \frac{R_0}{12} r^2 + \frac{C}{r}$$

$$e^{2\alpha} = 1 + \frac{R_0}{12} r^2 - \frac{2GM}{rc^2} \qquad (E.15)$$

Dengan $C = -2GM/c\hat{}2$ adalah konstanta integrasi.

Maka elemen garisnya menjadi

$$ds^2 = -\left(1 + \frac{R_0}{12} r^2 - \frac{2GM}{rc^2}\right) dt^2 + \left(1 + \frac{R_0}{12} r^2 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \; (E.16)$$

jika diambil $m = \frac{GM}{c^2}$, maka persamaan diatas dapat ditulis kembali dalam bentuk

$$ds^2 = -\left(1 + \frac{R_0}{12} r^2 - \frac{2m}{r}\right) dt^2 + \left(1 + \frac{R_0}{12} r^2 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (E.17)$$

dengan $d\Omega^2 = (d\theta^2 + \sin^2\theta d\phi^2)$.

Persamaan tersebut merupakan metrik Schwarzschild termodifikasi dalam teori gravitasi $f(R)$.