Annals of Operations Research
https://doi.org/10.1007/s10479-024-05955-0

ORIGINAL RESEARCH

A double sampling plan for truncated life tests under two-parameter Lindley distribution

Chien-Wei Wu[1] · Armin Darmawan[1,2] · Nien-Yun Wu[1]

Received: 13 May 2023 / Accepted: 14 March 2024

Abstract
The double sampling plan (DSP) is a generalized version of the single sampling plan (SSP) that provides several advantages, such as reduced sample size, increased discriminatory power, and better communication between producers and consumers. This study proposes a DSP for truncated life tests (TLT-DSP) using a two-parameter Lindley distribution. The proposed TLT-DSP's parameters are determined by a mathematical model designed to minimize the average sample number while fulfilling two constraints related to predefined quality levels and tolerated risks. Performance measures of the sampling plans are investigated to evaluate and compare their efficiency and effectiveness. Our results demonstrate that the proposed DSP is more efficient than the traditional SSP, especially in cases where the lot's quality is excellent or poor, and provides necessary protection to both parties involved. Additionally, two examples are presented, discussed and illustrated through a graphical user interface designed to validate the practicability of the proposed approach.

Keywords Acceptance sampling · Failure rate · Product lifetime · Graphical user interface · Truncation time

1 Introduction

Quality plays a critical role in the production process of both products and services. Therefore, quality management utilizes various supporting tools to conduct comprehensive investigations on raw materials, production processes, and finished products. Statistical quality control (SQC) is one such tool used in quality improvement activities. Inspection is a crucial aspect of SQC as it is involved throughout the entire production process. Three modes of inspection exist, including 100% inspection, zero inspection, and acceptance sampling. Acceptance sampling strikes a balance between two opposing views and entails a sampling strategy that defines the sample size and the standards for either approving or declining the product lot.

✉ Chien-Wei Wu
cweiwu@ie.nthu.edu.tw

1 Department of Industrial Engineering and Engineering Management, National Tsing Hua University, Hsinchu, Taiwan

2 Department of Industrial Engineering, Hasanuddin University, Makassar, Indonesia

Published online: 08 April 2024

Implementing a sampling plan can reduce risks, improve cost and time efficiency, optimize human resources, and maintain good relationships between business partners.

Sampling plans are extensively utilized in diverse applications to evaluate the quality of product lots. Researchers have investigated and developed sampling plans from diverse perspectives and contexts, including the works of Pearn and Wu (2007), Wu (2012), Lee et al. (2016), Wu et al. (2018), Liu et al. (2021), Prajapati et al. (2021), Shu et al. (2022), Wang et al., (2022a, 2022b), Wu et al. (2022), Prajapati et al. (2023), and Liu and Wu (2023). In designing a sampling plan, there are several important objectives to consider, such as efficiency, reliable, and ease of use. However, testing until all inspected samples fail can be costly and time-consuming, particularly for products with a lifetime operation. To address this issue, truncated life tests have been developed as an experiment type to estimate the reliability or lifetime of components, products, or systems. In this test, the experiment is stopped at a predetermined time, regardless of whether all the items have failed or not, and the lifetime data is observed and modeled using an appropriate probability distribution to estimate its reliability. Recently, extensive studies have been conducted on sampling plans for truncated life tests using various probability distributions. For instance, investigations have been conducted on the single sampling plan (SSP) for both finite and infinite lot sizes, under probability distributions such as the half-normal distribution (Lu et al., 2013), power Lindley distribution (Shahbaz et al., 2018), Sushila Distribution (Al-Omari, 2018), and the two-parameter Lindley distribution (Wu et al., 2021).

Generally, the SSP serves as a straightforward strategy, yet it does possess certain limitations. Notably, it can occasionally be challenging to maintain a harmonious relationship between producers and customers, and it might necessitate larger sample sizes. Consequently, the double sampling plan (DSP), a more comprehensive model than the SSP, has garnered significant popularity within the industrial sector owing to its advantages. Extensive studies have been conducted on the DSP for truncated life tests. For instance, Gui (2014) explored the Maxwell distribution and considered the zero and one failure schemes by setting $c_1 = 0$ and $c_2 = 1$. Additionally, Al-Omari et al. (2016) investigated the half-normal distribution, calculating minimum sample sizes for both the first and second samples while maintaining a fixed consumer's confidence level to establish the desired mean life. Building upon this, Al-Omari et al. (2017) introduced a new transmuted Weibull-Pareto distribution within the framework of the DSP. Their objective was to mitigate producer risk and derive optimal sample sizes for each stage of the plan.

Although all the previously mentioned distributions are capable of modeling data with lower-bound truncation, each distribution possesses distinct strengths. They are proficient in effectively modeling positive continuous data and addressing various aspects of reliability and survival analysis. The appropriateness of each distribution depends on its unique shape, scale, and tail behavior, which collectively influence its applicability in modeling specific datasets. Particularly noteworthy is the two-parameter Lindley distribution (TPLD), which distinguishes itself through its exceptional flexibility across diverse data types, intuitive parameter interpretations, suitability for small samples, capacity to capture distinct data patterns, and mathematical simplicity.

Consequently, the objective of this paper is to develop a DSP for truncated life tests (TLT-DSP) under the framework of the TPLD, as an extension of the model introduced by Wu et al. (2021). This approach involves performing lifetime tests on samples and recording the number of failures to determine the lot's acceptability. Specifically, when the lot size significantly exceeds the sample size, a random sample of items is tested within a specified timeframe or until a predefined number of failures is reached, whichever comes first. If the outcome remains inconclusive, a secondary evaluation is conducted to determine whether the lot should be accepted or rejected.

The subsequent sections of this paper are arranged as follows. Section 2 focuses on the theoretical foundation of TPLD. Section 3 presents the proposed plan's operational procedure based on TPLD, including the derivation of operating characteristic (OC) function and average sample number (ASN), as well as the formulation of the optimization model for determining plan parameters. Section 4 includes a thorough examination and discussion on plan parameters, and compares the proposed plan's performance with the conventional SSP. Section 5 illustrates how the proposed DSP operates in a truncated life test through an example that also involves a comparison between the Weibull and two-parameter Lindley distributions. Section 6 demonstrates the practical implementation of the proposed plan in conjunction with the designed user-friendly graphical user interface (GUI) application. Finally, Sect. 7 provides a concluding summary of this paper.

2 Two-parameter Lindley distribution (TPLD)

The original one-parameter Lindley distribution (OPLD) was first introduced by Lindley (1958). The probability density function (pdf) of the OPLD with a parameter $\theta > 0$ is given by

$$f(t;\theta) = \frac{\theta^2}{\theta+1}(1+t)e^{-\theta t},\ t > 0. \tag{1}$$

The cumulative distribution function (cdf) and hazard rate function (hrf) are defined as follows:

$$F(t;\theta) = 1 - \frac{\theta+1+\theta t}{\theta+1}e^{-\theta t}, \tag{2}$$

$$h(t;\theta) = \frac{\theta^2(1+t)}{\theta+1+\theta t},\ t > 0. \tag{3}$$

Ghitany et al. (2008) identified several adaptable statistical properties of the OPLD that make it an attractive option for modeling lifetime data, survival data and waiting time compared to the exponential distribution. This discovery generated significant interest in developing the Lindley-like distribution and its practical applications. In the field of lifetime data modeling, Shanker et al. (2016), Shanker and Mishra (2013), and Shanker (2016) proposed a generalization of the Lindley distribution, known as the two-parameter Lindley distribution (TPLD), which includes the OPLD as a special case. The TPLD has two parameters θ and η, and is characterized by its pdf, cdf, and hrf, as follows. Detailed illustrations of its figures with various parameter combinations are available in Wu et al. (2021).

$$f(t;\theta,\eta) = \frac{\theta(\eta+\theta t)}{\eta+1}e^{-\theta t}, \tag{4}$$

$$F(t;\theta,\eta) = 1 - \frac{1+\eta+\theta t}{\eta+1}e^{-\theta t}, \tag{5}$$

$$h(t;\theta,\eta) = \frac{\theta(\eta+\theta t)}{1+\eta+\theta t},\ t > 0,\ \theta > 0,\ \eta > -1. \tag{6}$$

When $\eta = \theta$, it can be observed that all expressions for TPLD in Eqs. (4)–(6) will reduce to Eqs. (1)–(3) for the OPLD. The mean of TPLD, denoted by μ, can be calculated using the following formula:

$$\mu = \frac{\eta + 2}{\theta(\eta + 1)}. \tag{7}$$

3 The proposed TLT-DSP for TPLD

To enhance clarity and improve the readability of the paper, we have summarized the definitions for notations and symbols used in this section in Table 1. Consider a scenario in which a supplier delivers a shipment of N products to a buyer, and the lifetime of the products is assumed to follow a TPLD with a pdf defined by Eq. (4). The mean lifetime of the products is given by Eq. (7).

The contractual agreement established between the supplier and the buyer outlines the acceptable quality level (AQL) and rejectable quality level (RQL) of the lot, which are characterized by the product's mean life denoted as μ_{AQL} and μ_{RQL}, respectively. The primary

Table 1 The list of symbols and notations

Notation	Definition
η	The first parameter of two-parameter Lindley distribution
θ	The second parameter of two-parameter Lindley distribution
α	Producer's risk
β	Consumer's risk
μ	Mean life of the products
t_u	Truncation time of the life test
μ_0	A specified mean life of the products
μ_{AQL}	Required mean life of the products at acceptable quality level (AQL)
μ_{RQL}	Required mean life of the products at rejectable quality level (RQL)
q	Ratio of the truncation time to the specified mean life, i.e.,t_u/μ_0
r	Ratio of the actual mean life to the specified mean life, i.e.,μ/μ_0
r_{AQL}	Ratio of the required mean life at AQL to the specified mean life, i.e.,μ_{AQL}/μ_0
r_{RQL}	Ratio of the required mean life at RQL to the specified mean life, i.e.,μ_{RQL}/μ_0
N	Lot size
n_1	Required number of sample items in the first-stage sampling
n_2	Required number of sample items in the second-stage sampling
k	Ratio of the second sample size to the first sample size,n_2/n_1
c_1	Acceptance number for failed items in the first sample
c_2	Acceptance number of failed items in both samples
d	Cumulative number of failed items in both samples
d_1	Number of failed items in the first-stage sampling
d_2	Number of failed items in the second-stage sampling
$\pi_1(r)$	Probability of lot acceptance for the first sample
$\pi_2(r)$	Probability of lot acceptance for the second sample
$\Pi(r)$	Total lot acceptance probability the proposed DSP
$p(r)$	Product's failure probability at r

aim is to ensure that the product's lifespan aligns with the predetermined quality standards, effectively minimizing risks for all parties involved. The choice of risk parameters is based on the critical balance between the risk of accepting a bad lot (referred to producer's risk, α) and the risk of rejecting a satisfactory lot (referred to consumer's risk, β). Through the manipulation of these parameters, it becomes feasible to tailor the sampling strategy to fulfill the desired risk thresholds of both stakeholders.

Hence, the suggested plan must adhere to the two-point condition (quality level and risk suffered) on the OC curve. This condition requires that: (1) The lot must have a probability of being approved at the defined μ_{AQL} of at least $100(1-\alpha)\%$. This criterion ensures that the lot meets the specified μ_{AQL} and reflects the producer's acceptable level of risk. (2) The acceptance probability of the lot must not exceed $100\beta\%$ at the specified μ_{RQL}. This criterion maintains a balance by preventing an excessive risk of accepting a lot that falls below the defined quality threshold.

The operational procedure for implementing the proposed TLT-DSP for the TPLD is depicted in Fig. 1 and also outlined below.

1. Determine mean life and risk requirements $(\mu_{\text{AQL}}, 1-\alpha)$ and $(\mu_{\text{RQL}}, \beta)$ for the products, the specified mean life (μ_0), and the truncation time (t_u) for the life test.
2. Compute the ratio quantities $r_{\text{AQL}} = \mu_{\text{AQL}} \big/ \mu_0$, $r_{\text{RQL}} = \mu_{\text{RQL}} \big/ \mu_0$, and $q = t_u/\mu_0$ using μ_0 as a scaling factor.
3. Determine the plan parameters (n_1, n_2, c_1, c_2) of the TLT-DSP.
4. Collect a sample of size n_1 randomly from the lot and conduct a truncated life test for the first-stage sampling. Observe, identify, and count the number of failed items d_1.
5. Accept the lot if $d_1 \le c_1$, or reject the lot if $d_1 > c_2$ and immediately stop the test.
6. If decision is not reached on the basis of first stage, i.e., $c_1 < d_1 \le c_2$, conduct a truncated life test for the second-stage sampling with size n_2. Observe, identify, count the failed items d_2 from n_2, and calculate the cumulative failed items $d = d_1 + d_2$ from $n_1 + n_2$ samples in two stages.
7. Reject the lot if $d > c_2$, otherwise, accept the lot.

To derive the lot acceptance probability, OC function, of TLT-DSP based on the TPLD, some probability functions should be defined first. Based on Eq. (5), the product's life has the failure probability before the truncation time t_u as

$$P(T < t_u) = F(t_u; \theta, \eta) = 1 - \frac{1 + \eta + \theta t_u}{\eta + 1} e^{-\theta t_u}. \tag{8}$$

As the products' mean life shown in Eq. (7), it implies $\theta = (\eta + 2)/[\mu(\eta + 1)]$. Hence, by substituting θ into Eq. (8), it can be rewritten as follows:

$$F(t_u; \mu, \eta) = 1 - \frac{1 + \eta + \left(\frac{\eta+2}{\eta+1}\right)\frac{t_u}{\mu}}{\eta + 1} e^{-\left(\frac{\eta+2}{\eta+1}\right)\frac{t_u}{\mu}}. \tag{9}$$

The expression for $F(t_u; \mu, \eta)$ as shown in Eq. (9) involves time units, including the truncation time t_u and the mean life μ, which can vary in different real-world scenarios. This makes it difficult to create general tables of plan parameters that can be adapted to various situations.

To address this issue, we introduce a scaling factor, namely the specified mean life (μ_0), and define two quantities ratios as a function of μ_0. The first is the truncation time to the specified mean life ratio $(q = t_u/\mu_0)$, while the second is the mean life to the specified mean

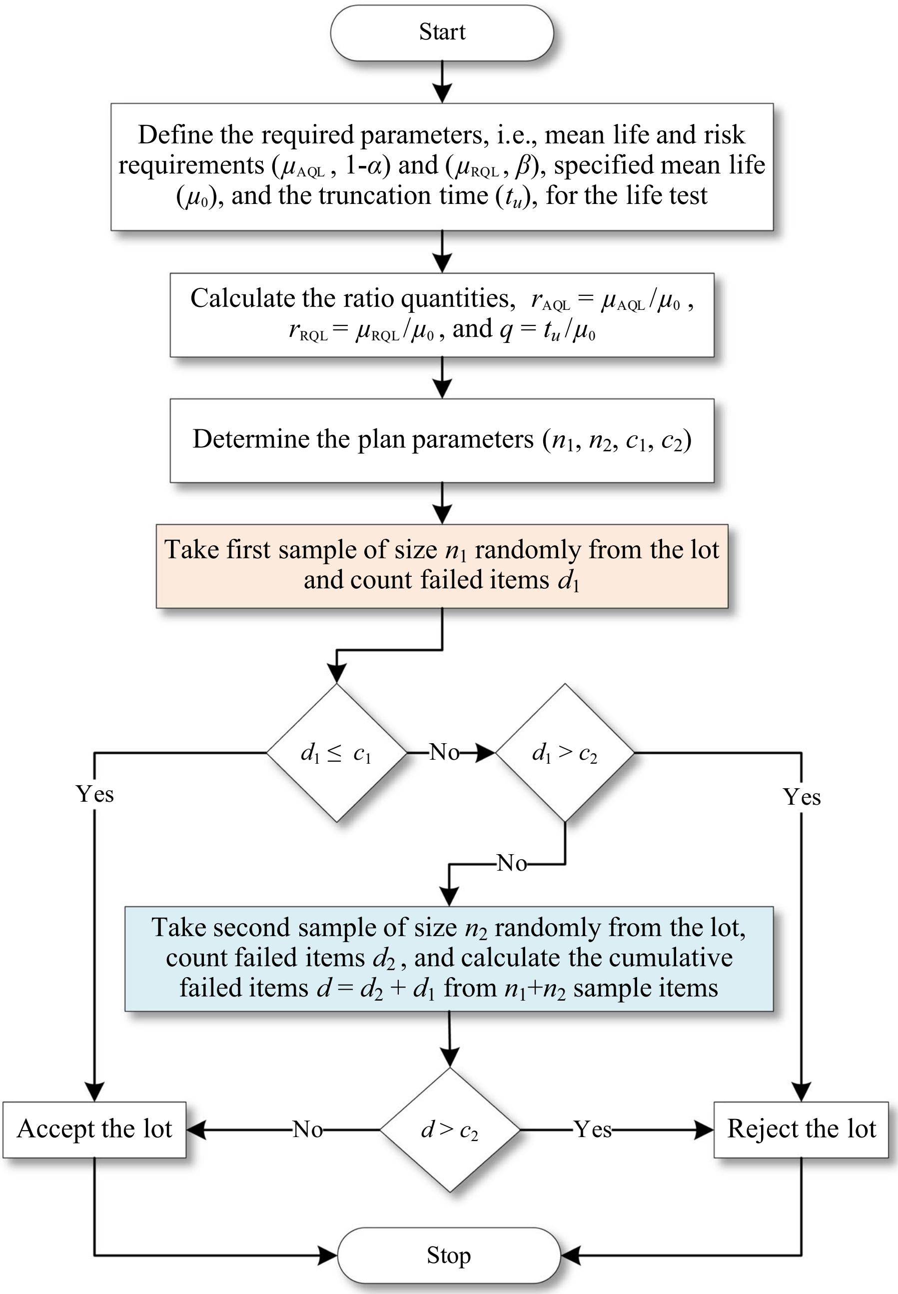


Fig. 1 Flow chart for the operating procedure of TLT-DSP for TPLD

life ratio ($r = \mu/\mu_0$). This allows us to transform Eq. (9) into the following form:

$$F\left(\frac{t_u}{\mu_0};\ \frac{\mu}{\mu_0},\ \eta\right) = F(q; r, \eta) = 1 - \frac{1 + \eta + \left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}{\eta + 1} e^{-\left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}. \tag{10}$$

The OC function plays a crucial role in evaluating the performance of a sampling plan. By utilizing Eq. (10) with the given values of $(t_u,\ \mu_0,\ \eta)$, $q = t_u/\mu_0$ can be obtained, and the product's failure probability can be defined as a function of $r = \mu/\mu_0$, i.e., $p(r) = F(q; r, \eta)$. Given that the lot size N is much greater than n, we can employ the binomial distribution to calculate the lot acceptance probability for the first sample of size n_1, $\pi_1(r)$. It can be represented by the following equation:

$$\begin{aligned}\pi_1(r) &= P(d_1 \le c_1) = \sum_{i=0}^{c_1} \binom{n_1}{i} p(r)^i [1 - p(r)]^{n_1 - i} \\ &= \sum_{i=0}^{c_1} \binom{n_1}{i} \left(1 - \frac{1 + \eta + \left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}{\eta + 1} e^{-\left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}\right)^i \left(\frac{1 + \eta + \left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}{\eta + 1} e^{-\left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}\right)^{n_1 - i}.\end{aligned} \tag{11}$$

Similarly, the lot acceptance probability for the second sample of n_2, $\pi_1(r)$, can be formulated as

$$\begin{aligned}\pi_2(r) &= P(d = d_1 + d_2 < c_2 |\ c_1 < d_1 \le c_2) \\ &= \left(\sum_{x=c_1+1}^{c_2} \binom{n_1}{x} p(r)^x [1 - p(r)]^{n_1 - x}\right) \times \left\{\sum_{j=0}^{c_2 - x} \binom{n_2}{j} p(r)^j [1 - p(r)]^{n_2 - j}\right\} \\ &= \sum_{x=c_1+1}^{c_2} \binom{n_1}{x} \left(1 - \frac{1 + \eta + \left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}{\eta + 1} e^{-\left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}\right)^x \left(\frac{1 + \eta + \left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}{\eta + 1} e^{-\left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}\right)^{n_1 - x} \\ &\quad \times \left\{\sum_{j=0}^{c_2 - x} \binom{n_2}{j} \left(1 - \frac{1 + \eta + \left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}{\eta + 1} e^{-\left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}\right)^j \left(\frac{1 + \eta + \left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}{\eta + 1} e^{-\left(\frac{\eta+2}{\eta+1}\right)\frac{q}{r}}\right)^{n_2 - j}\right\}.\end{aligned} \tag{12}$$

Thus, the OC function, total lot acceptance probability, of the proposed TLT-DSP under the TPLD can be calculated as

$$\begin{aligned}\Pi(r) &= \pi_1(r) + \pi_2(r) \\ &= \sum_{i=0}^{c_1} \binom{n_1}{i} p(r)^i [1 - p(r)]^{n_1 - i} \\ &\quad + \sum_{x=c_1+1}^{c_2} \binom{n_1}{x} p(r)^x [1 - p(r)]^{n_1 - x} \left\{\sum_{j=0}^{c_2 - x} \binom{n_2}{j} p(r)^j [1 - p(r)]^{n_2 - j}\right\}.\end{aligned} \tag{13}$$

As previously mentioned, the supplier–buyer contract specifies the AQL and RQL for the product's mean life at μ_{AQL} and μ_{RQL}, respectively. Hence, using the specified mean life μ_0 as a scaling factor, two ratios, $r_{\mathrm{AQL}} = \mu_{\mathrm{AQL}} / \mu_0$ and $r_{\mathrm{RQL}} = \mu_{\mathrm{RQL}} / \mu_0$, can be further determined. By utilizing Eq. (13), the probability of acceptance from both parties' perspectives can be expressed. In other words, the lot acceptance probability must meet the two-point conditions on the OC curve, represented symbolically as $\Pi(r_{\text{AQL}}) \geq 1-\alpha$ and $\Pi(r_{\text{RQL}}) \leq \beta$.

The proposed TLT-DSP incorporates a second opportunity for each submitted lot before a conclusive judgment is reached. This feature highlights the utilization of the Average Sample Number (ASN) as the preferred metric for evaluating two-stage inspections and assessing submitted lots. The ASN refers the expected number of sample items required for inspection to make a final decision regarding the disposition of the lot. This measure takes into account the entire sampling endeavor, encompassing both the initial examination and the potential subsequent sampling phase. The ASN function within the proposed TLT-DSP can be determined using the following formula.

$$\begin{aligned}
\text{ASN}(r) &= n_1 \times [1 - P(c_1 < d_1 \leq c_2)] + (n_1 + n_2) \times P(c_1 < d_1 \leq c_2) \\
&= n_1 + n_2 \times P(c_1 < d_1 \leq c_2) \\
&= n_1 + n_2 \times \sum_{x=c_1+1}^{c_2} \binom{n_1}{x} p(r)^x [1 - p(r)]^{n_1 - x}.
\end{aligned} \tag{14}$$

Thus, to ascertain the parameters of the proposed plan, n_1, n_2, c_1 and c_2, an optimization model is established. The model aims to minimize the average ASN value evaluated at two specified quality levels and consists of two non-linear constraints associated with the required quality and risk conditions (i.e., r_{AQL}, r_{AQL}, α, and β).

$$\text{Min ASN}^* = \frac{\text{ASN}(r_{\text{AQL}}) + \text{ASN}(r_{\text{RQL}})}{2} \tag{15}$$

Subject to

$$\begin{aligned}
\Pi(r_{\text{AQL}}) = &\sum_{i=0}^{c_1} \binom{n_1}{i} p(r_{\text{AQL}})^i [1 - p(r_{\text{AQL}})]^{n_1 - i} \\
&+ \sum_{x=c_1+1}^{c_2} \binom{n_1}{x} p(r_{\text{AQL}})^x [1 - p(r_{\text{AQL}})]^{n_1 - x} \left\{ \sum_{j=0}^{c_2 - x} \binom{n_2}{j} p(r_{\text{AQL}})^j [1 - p(r_{\text{AQL}})]^{n_2 - j} \right\} \geq 1 - \alpha, \\
\Pi(r_{\text{RQL}}) = &\sum_{i=0}^{c_1} \binom{n_1}{i} p(r_{\text{RQL}})^i [1 - p(r_{\text{RQL}})]^{n_1 - i} \\
&+ \sum_{x=c_1+1}^{c_2} \binom{n_1}{x} p(r_{\text{RQL}})^x [1 - p(r_{\text{RQL}})]^{n_1 - x} \left\{ \sum_{j=0}^{c_2 - x} \binom{n_2}{j} p(r_{\text{RQL}})^j [1 - p(r_{\text{RQL}})]^{n_2 - j} \right\} \leq \beta,
\end{aligned}$$

$$n_1,\ n_2 \in \mathbb{N}^+, \quad c_1,\ c_2 \in \mathbb{N},\ c_2 \geq c_1.$$

4 Analysis and discussion

The developed optimization model for the proposed TLT-DSP expressed in Eq. (15) involves four plan parameters (n_1, n_2, c_1 and c_2). To enhance practical application and implementation, a specific configuration of the DSP assumes that the required sample size for the second-stage sampling (n_2) is a multiple times (k) of the required sample size for the first stage (n_1), i.e., $n_2 = kn_1$. This approach has been suggested by Duncan (1986), Schilling and Neubauer (2009), and Luca et al. (2020), who highlight its pragmatic usability. Moreover, to explore the proposed TLT-DSP's parameters, we consider various combinations of quality levels r_{AQL} =(2, 4, 6), $r_{\text{RQL}} = 1$, $q = (0.5,\ \ 1,\ \ 2)$, risk levels (α,β) = (0.01, 0.05) and (0.05, 0.05) under $k = (0.5,\ \ 1)$ and $\eta = (0, 1, 2)$. The plan parameters are solved using the

Table 2 Plan parameters of TLT-DSP under $(\alpha, \beta) = (0.01, 0.05)$

k	q	$r_{RQL} = 1$	$\eta = 0$				$\eta = 1$				$\eta = 2$			
		r_{AQL}	n_1	c_1	c_2	ASN*	n_1	c_1	c_2	ASN*	n_1	c_1	c_2	ASN*
0.5	0.5	2	52	7	13	59.19	74	15	30	95.09	78	18	34	99.86
		4	18	0	3	20.87	26	3	8	30.04	28	5	9	30.88
		6	16	0	2	17.35	20	2	5	21.78	20	2	6	22.50
	1	2	26	9	17	31.89	42	17	31	54.07	46	20	35	58.99
		4	10	2	4	10.54	12	1	7	15.63	16	5	10	18.75
		6	6	0	2	6.63	10	2	5	11.12	12	3	6	13.05
	2	2	28	22	22	28.00	44	34	34	44.00	48	37	37	48.00
		4	6	2	6	7.79	10	6	9	11.84	10	5	10	12.80
		6	4	1	3	4.42	6	3	5	6.83	8	4	7	9.39
1	0.5	2	35	2	12	61.46	56	11	30	92.89	60	13	35	99.75
		4	14	0	3	19.40	20	2	8	30.34	21	3	9	30.83
		6	12	0	2	14.77	15	1	5	20.91	13	0	5	21.28
	1	2	20	7	17	31.05	32	13	31	50.90	35	15	35	55.59
		4	7	1	4	9.44	11	3	8	16.34	11	2	9	18.43
		6	5	0	2	6.30	8	1	5	11.67	7	0	5	12.13
	2	2	28	22	22	28.00	43	33	33	43.00	48	37	37	48.00
		4	8	5	5	8.00	11	7	11	16.36	14	9	9	14.00
		6	3	1	3	4.55	7	4	5	7.62	10	6	6	10.00

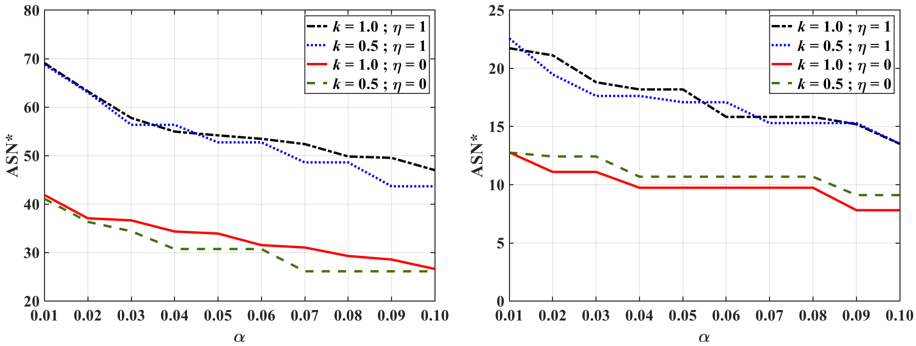
Table 3 Plan parameters of TLT-DSP under $(\alpha, \beta) = (0.05, 0.05)$

k	q	$r_{RQL} = 1$	$\eta = 0$				$\eta = 1$				$\eta = 2$			
		r_{AQL}	n_1	c_1	c_2	ASN*	n_1	c_1	c_2	ASN*	n_1	c_1	c_2	ASN*
0.5	0.5	2	38	3	9	46.35	50	7	19	68.08	56	12	23	71.53
		4	16	0	2	17.99	22	2	6	25.46	20	2	6	23.43
		6	12	0	1	12.72	14	0	3	16.81	16	1	4	18.16
	1	2	18	5	11	22.37	30	11	21	39.21	32	13	23	41.43
		4	6	0	2	6.91	10	2	5	11.51	14	5	5	14.00
		6	6	1	1	6.00	8	1	3	8.80	8	2	3	8.36
	2	2	20	15	15	20.00	32	24	24	32.00	36	27	27	36.00
		4	4	1	3	4.60	6	3	5	6.96	8	4	7	9.59
		6	4	2	2	4.00	4	1	3	4.65	4	1	3	4.70
1	0.5	2	27	2	8	42.68	40	7	20	68.58	40	7	22	71.47
		4	12	0	2	15.57	14	0	5	23.67	15	1	6	23.76
		6	11	0	1	12.41	11	0	3	16.06	10	0	3	14.83
	1	2	14	4	11	22.72	23	8	21	39.17	24	9	23	40.64
		4	6	1	2	6.68	9	2	6	13.27	7	0	5	12.64
		6	4	0	1	4.63	6	1	3	7.68	5	0	3	7.87
	2	2	20	15	15	20.00	31	23	23	31.00	35	26	26	35.00
		4	4	2	3	4.66	7	4	6	9.15	10	6	6	10.00
		6	2	0	2	3.26	3	0	3	5.34	3	0	3	5.40

model expressed in Eq. (15) and tabulated in Tables 2 and 3, their corresponding values of ASN* are also included. Furthermore, we have examined the relationships between ASN* and other parameters, as depicted in Figs. 2, 3 and 4.

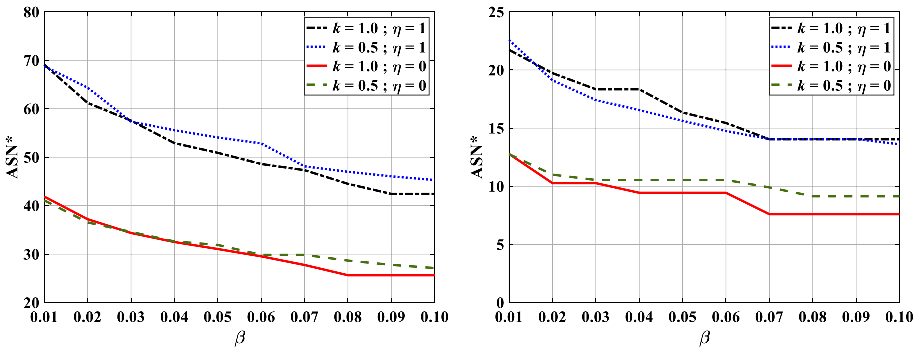
After examining these tables and figures, the following phenomena can be observed:

1. An increase in either α or β (or both) results in a smaller sample size and a lower acceptance number of failed items for the first and second samples being required, i.e., n_1, n_2, ASN*, c_1 and c_2. This could be because the producer and/or consumer accommodating a greater risk of arriving at an incorrect conclusion. Conversely, a decrease in either α or β (or both) leads to an increase in the required n_1, n_2, and ASN.
2. With a fixed $r_{\mathrm{RQL}} = 1$, a decrease in r_{AQL} leads to an increase in the required sample sizes and acceptance numbers of failed items for the sampling in the first and second stages, i.e., n_1, n_2, ASN*, c_1 and c_2. This implies that a closer of two quality levels (r_{AQL} and r_{RQL}) requires a larger sample size and ASN* for making a judgement. Conversely, as the value of r_{AQL} increases, a smaller sample size and ASN is sufficient.
3. Either a lower and higher value of q can result in an increase in the sample size needed and the acceptance number of failed items. This is because a larger or smaller value of



(a) $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (2,1)$ (b) $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (4,1)$

Fig. 2 Plots of ASN* versus various α values when $\beta = 0.01$ under $q = 1$



(a) $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (2,1)$ (b) $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (4,1)$

Fig. 3 Plots of ASN* versus various β values when $\alpha = 0.01$ under $q = 1$

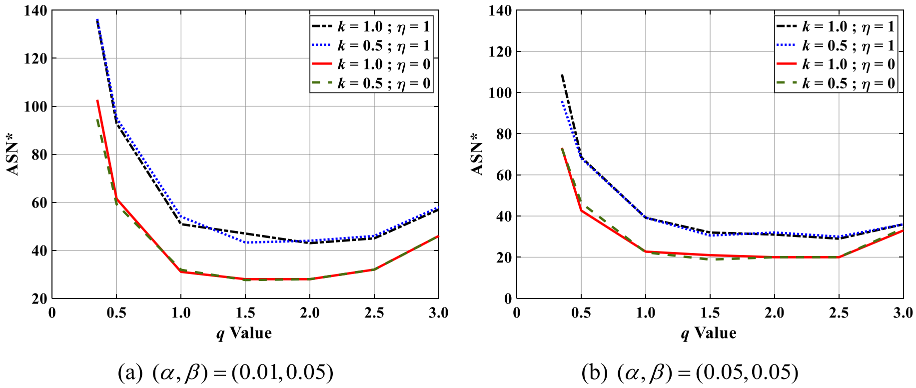


Fig. 4 ASN* plot versus various q under $(r_{\mathrm{AQL}},\ r_{\mathrm{RQL}}) = (2,\ 1)$

q indicates that the termination time is significantly greater or smaller than the specified mean life, and the product failure rate is relatively high or low before the life test is terminated. As a result, a larger sample size and ASN are necessary in both scenarios to differentiate between good and poor product lots.

Furthermore, we conducted a comparison between SSP and DSP for truncated life tests under the TPLD. Our study examined various conditions, such as $k = (0.5, 1)$, $q = (0.5, 1, 2)$, $\eta = (0, 1)$, and the ASN curves of SSP and DSP with $k = (0.5, 1)$ under $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (4, 1)$ are shown in Figs. 5 and 6 for $(\alpha, \beta) = (0.01, 0.05)$ and $(0.05, 0.05)$, respectively. For instance, consider the given conditions of $(\alpha, \beta) = (0.01, 0.05)$, $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (4, 1)$, $k = 1$, $q = 0.5$, and $\eta = 0$. Under these circumstances, SSP requires a fixed sample size of 27, regardless of the actual quality of the lot. In contrast, our proposed DSP only mandates 14 sample items, achieving an ASN ranging from 14.00 to 24.64 for various values of r between 0.01 and 4.00, respectively. Furthermore, in comparison to SSP, DSP requires a sample size (n) of 14, yielding an ASN that varies from 14.00 to 24.64 for different values of r between 0.01 and 4.00. Specifically, at $r = 3.0$, the corresponding ASN is 23.83, indicating a potential reduction of approximately 23.85% in required inspection samples. However, for cases involving marginal or ambiguous lot quality (e.g., $r = 2.0$), a slight increase in ASN to 23.83 for inspection samples may be needed.

It can be identified from Figs. 5 and 6 that DSP is superior in terms of requiring a smaller ASN especially when the ratio r is not in the middle range, in comparison to SSP under the same conditions. That is, DSP is more suitable when the quality of the product batch is either very good or very poor, as it enables the lot to be either accepted or declined immediately during the first stage of sampling. On the other hand, SSP is still more appropriate when the lot's quality falls within the average range, as a secondary sample might be necessary to reach a conclusive decision.

Additionally, it's important to emphasize that the selection of appropriate sampling plans should take into account various factors, such as the organization's objectives, operational constraints, quality profile, and historical data. Therefore, the analysis presented in this study aims to offer insights into the potential benefits of both SSP and DSP, rather than establish rigid criteria. For instance, in scenarios where there is limited knowledge about a supplier, the initial inclination might be towards using SSP. However, as the supplier's quality and reputation progressively improve, transitioning to DSP could become more advantageous. This pragmatic approach ensures alignment with evolving quality assurance strategies.

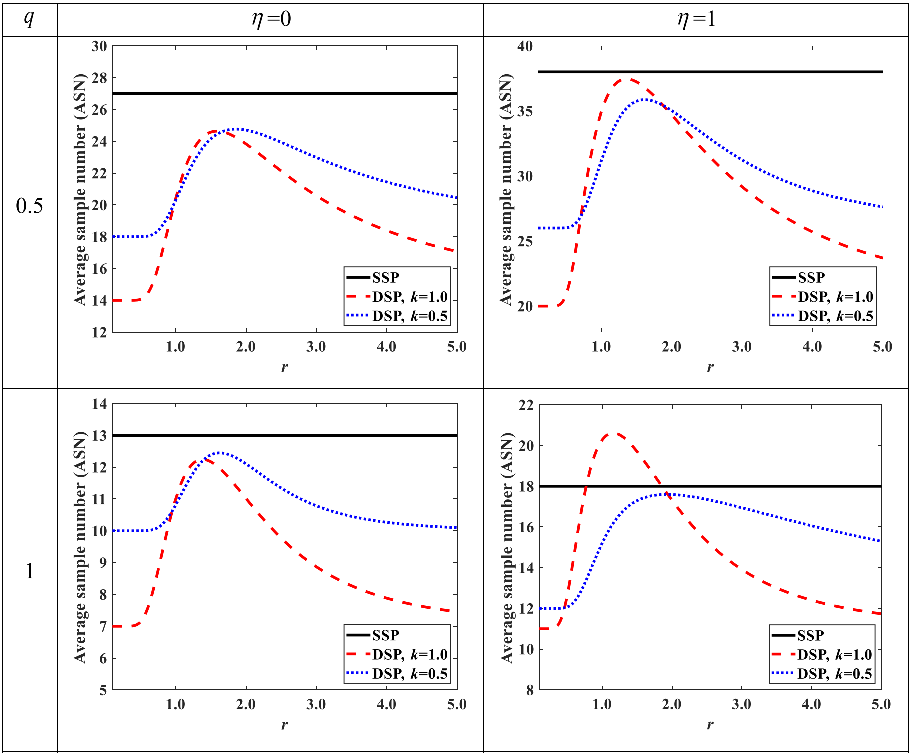


Fig. 5 ASN curves of SSP and DSP with $k = (0.5,\ 1)$ under $(r_{\mathrm{AQL}},\ r_{\mathrm{RQL}}) = (4,\ 1)$ and $(\alpha,\ \beta) = (0.01, 0.05)$

Furthermore, the OC function is also an essential performance measure that can be utilized to assess the discriminatory power of a sampling plan. A steeper slope in the OC curve indicates a higher discriminatory power. Typically, two methods illustrate the superiority of an OC curve: (a) achieving an equivalent discriminatory power (i.e., a similar OC curve shape) with fewer samples, and (b) maintaining a steeper slope in the OC curve while utilizing the same sample size.

Figures 7 and 8 depict the OC curves of both the SSP and DSP with $k = (0.5, 1)$, $q = (0.5, 1)$, $\eta = (0, 1)$ under $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (4, 1)$, along with different combinations of risks $(\alpha, \beta) = (0.01, 0.05)$ and $(0.05, 0.05)$, respectively. From Figs. 7 and 8, it is evident that these OC curves are quite similar, demonstrating relative equivalence and satisfying the required quality-and-risk conditions. This indicates that the proposed plan would offer comparable protection for both the producer and the consumer, even with a smaller sample size.

In contrast, the OC curves of the SSP and DSP with $k = (0.5, 1)$, $q = 0.5$, $\eta = 0$ were also evaluated under an equal sample size and presented in Fig. 9a and b for $n = 20$ and $n = 40$, respectively. It can be observed from Fig. 9a and b that the proposed DSP's OC curve exhibits better discriminatory power than the SSP under a fixed sample size, thus aiding users in making more reliable lot determinations.

As a result, through the analysis of two performance measures, ASN and OC curves, meaningful insights can be gleaned from the evaluation of the proposed plan's behavior. It is evident

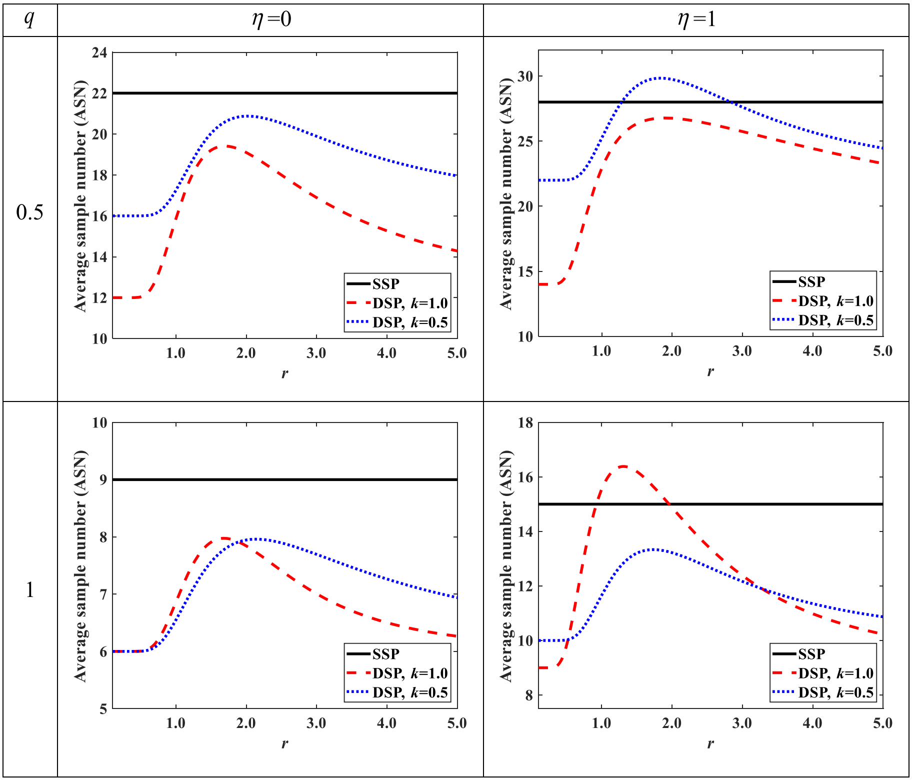


Fig. 6 ASN curves of SSP and DSP with $k = (0.5,\ \ 1)$ under $(r_{\text{AQL}},\ \ r_{\text{RQL}}) = (4,\ \ 1)$ and $(\alpha,\ \ \beta) = (0.05,\ 0.05)$

that the proposed plan demonstrates efficient performance with a reduced sample size, ensuring comparable protection for both the producer and customer. Additionally, the proposed DSP's two-stage inspection provides a nuanced approach to quality assessment. A successful initial sample indicates acceptable quality, facilitating efficient lot disposition decisions. In contrast, a failed initial sample initiates more thorough scrutiny, indicating potential quality issues and triggering corrective actions, along with a larger second sample for further evaluation. This transparent process encourages open discussions among stakeholders, enhancing their understanding of quality standards and facilitating collaborative quality control efforts, ultimately improving communication between producers and consumers.

5 A comparative study

In this section, we present an example taken from Lawless (2003) to illustrate how the proposed DSP plan operates in a truncated life test. The example also involves a comparison between TPLD and Weibull distribution, which are commonly used probability distributions for analyzing life data. Initially, it was presumed that the data conforms to a Weibull distribution having a shape parameter of 1.97 (as shown in Table 4 for 23 ball bearing life data). However, Shanker et al. (2016) presented an argument suggesting that the TPLD is a better

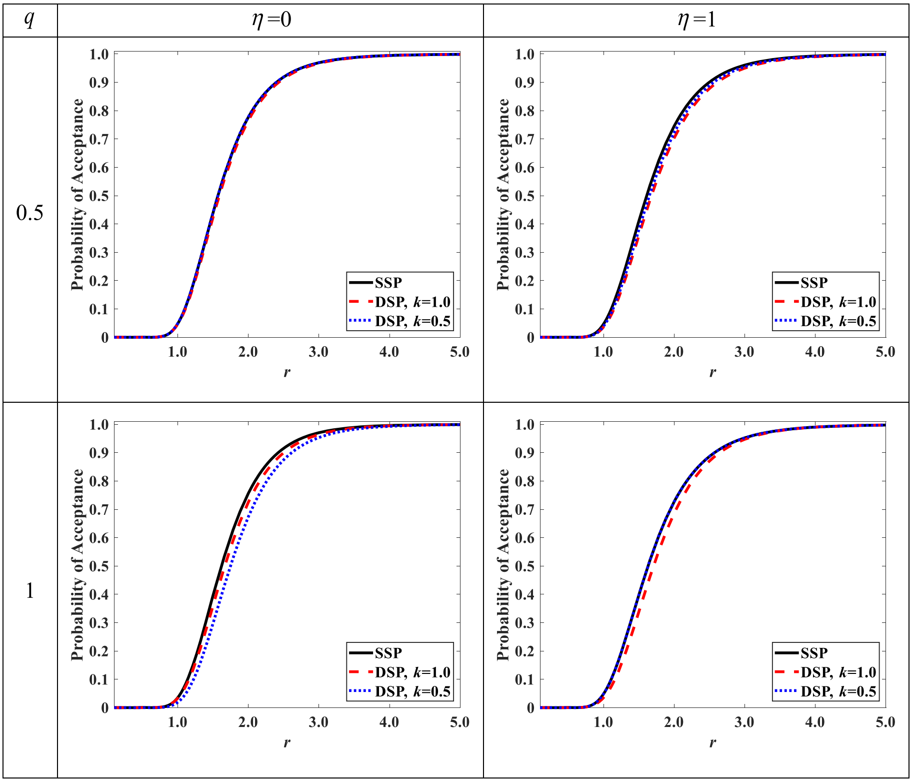


Fig. 7 OC curves of SSP and DSP with $k = (0.5, 1)$, $q = (0.5, 1)$, $\eta = (0, 1)$ under $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (4, 1)$ and $(\alpha, \beta) = (0.01, 0.05)$

fit for this data under $\hat{\eta} = -0.358716$ and $\hat{\theta} = 0.035434$, as it has a higher p-value. This claim is further supported by the distribution fit test performed on the data, which showed that the data can be adapted to both the TPLD and Weibull distribution (refer to Fig. 10 and Table 5).

Suppose that the product's mean lifetime (μ_0) is specified at 20, and the truncation time (t_u) of the experiment is also 20, implying that $q = t_u/\mu_0 = 1$. Based on these parameters, two ratio quantities, $r_{\mathrm{AQL}} = 2$ and $r_{\mathrm{RQL}} = 1$, can be further determined. Table 6 shows the plan parameters of the DSP when the two-parameter Lindley and Weibull distributions are applied to the ball bearing life data under the above given parameters and risk conditions ($\alpha = 0.05$ and $\beta = 0.05$).

If the data are adapted to the TPLD, this batch will be accepted as there are 0 records below 20 in the first 6 records, which do not exceed the first allowable quantity of 0. Consequently, this batch will be accepted, and there is no need to draw the next stage sample. Similarly, if the data is adapted to the Weibull distribution, this lot will also be accepted as there are 0 transactions below 20 in the first 14 data, which does not exceed the first allowable number of 4. Hence, there is no need to draw the next stage sample for either of the cases.

In addition, the ASN curves of DSP for TPLD and Weibull distribution under the truncated life test are displayed in Fig. 11. It can be inferred that the ASN required by the two-parameter Lindley distribution is smaller than that required by the Weibull distribution. Therefore, if

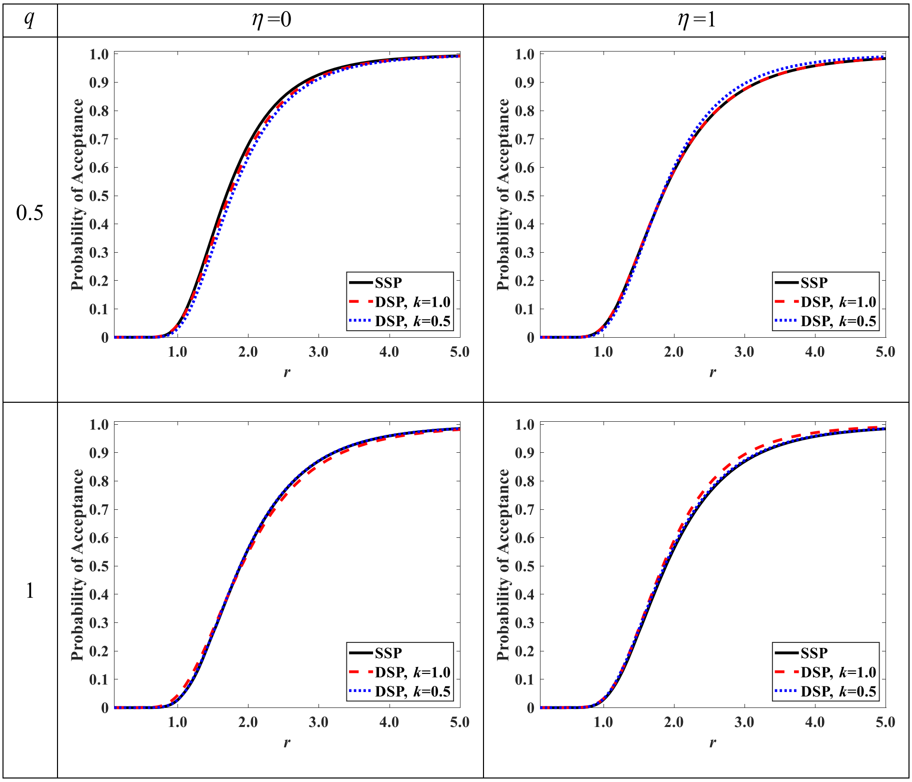
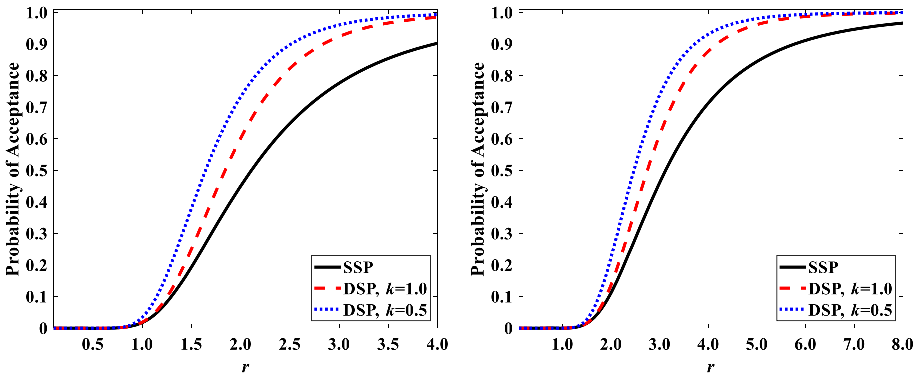


Fig. 8 OC curves of SSP and DSP with $k = (0.5, 1)$, $q = (0.5, 1)$, $\eta = (0, 1)$ under $(r_{\mathrm{AQL}}, r_{\mathrm{RQL}}) = (4, 1)$ and $(\alpha, \beta) = (0.05, 0.05)$



(a) $n = 20,\ c_1 = 1,\ c_2 = 3$ (b) $n = 40,\ c_1 = 1,\ c_2 = 3$

Fig. 9 OC curves of SSP and DSP with $k = (0.5, 1)$, $q = 0.5$, $\eta = 0$ under a fixed sample size

Table 4 The 23 ball bearing life data

68.64	105.12	67.80	54.12	93.12	42.12	105.84	173.40
33.00	84.12	68.44	48.80	68.88	45.60	51.84	17.88
28.92	128.04	127.92	98.64	41.52	55.56	51.96	

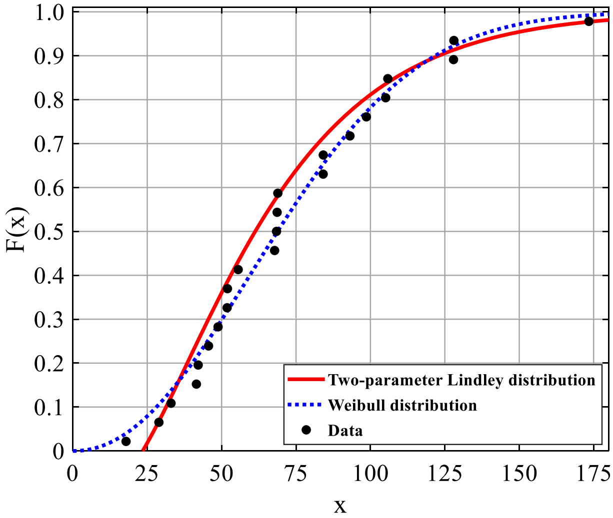


Fig. 10 Probability plots of ball bearing life data for two-parameter Lindley and Weibull distributions

Table 5 Distribution fit test result of ball bearing life data

Distribution model	K-S test statistic	p-value
Two-parameter Lindley distribution (TPLD)	0.0985	0.9629
Weibull distribution	0.1512	0.6159

the goal is to require fewer samples, it is better to use the two-parameter Lindley distribution for analyzing the data in this case.

6 An example demonstration by the developed GUI

This section will demonstrate the use of the proposed TLS-DSP using a practical case as an example. The data used in this case was obtained from Fuller et al. (1994), which consists of 31 data points about the pressure strength of the internal layer of an airplane window measured in MPa (see Table 7). The airplane's window is constructed using a dual-layered acrylic glass that can be separated into an internal and external layer. The space between these two layers is used for controlling the pressure and temperature within the airplane. However, the

Table 6 DSP's parameters under the two-parameter Lindley and Weibull distributions

Distribution Model	n_1	n_2	c_1	c_2
Two-parameter Lindley distribution (TPLD)	6	6	0	4
Weibull distribution	14	14	4	7

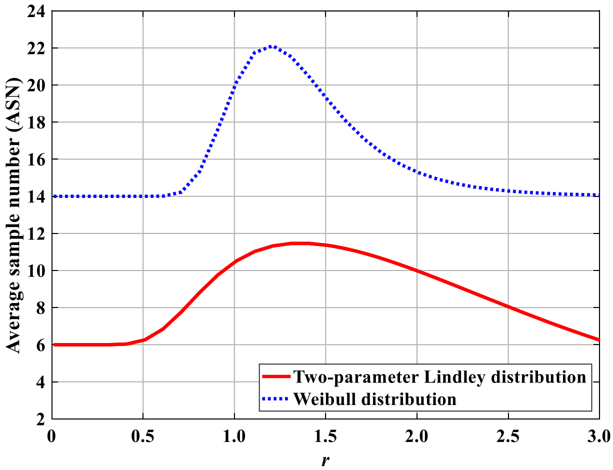


Fig. 11 ASN curves of DSP for Two-parameter Lindley and Weibull distributions

Table 7 The 31 data of pressure strength (unit: MPa) from Fuller et al. (1994)

26.690	**39.580**	**33.760**	**31.110**	**33.730**	24.050	36.980	45.381	34.760
23.230	18.830	27.670	29.900	33.200	25.800	24.321	33.890	21.657
37.080	35.750	37.090	35.910	44.045	25.520	26.780	23.030	26.770
27.050	45.290	25.500	20.800					

intensity of pressure can cause damage to the surface and edge of the window. This damage can be caused by various factors such as poor manufacturing and dust/sand during flight. The lifespan of an airplane window is primarily influenced by pressure intensity and relative humidity. High relative humidity limits the window's ability to handle pressure, resulting in a shorter lifespan, whereas lower relative humidity enables the window to endure more pressure and last longer. In Fuller et al. (1994)'s study, they used 31 data points of borosilicate glass (BK-7 glass) to examine its compressive strength distribution under an inert-gas-atmosphere. The experiment was conducted at room temperature, and the glass was exposed to varying pressures of water to determine the anticipated lifespan of the window.

Shanker et al. (2016) also examined this dataset and presented the results of distribution fit tests, including AIC, AICC and BIC, in Table 8. These information criteria are used to compare the fit of different distribution models to a given set of data. In general, smaller values of AIC, AICC, and BIC indicate a better model fit. Thus, it can be inferred that TPLD is a better

Table 8 Parameter estimate and criteria for distribution fit tests (Shanker et al., 2016)

Data	Distribution Model	Distribution parameters estimated by ML method	Test information criteria		
			AIC	AICC	BIC
The 31 pressure strength data from Fuller et al (1994)	One-parameter Lindley distribution (OPLD)	$\hat{\theta} = 0.062988$	255.99	256.13	257.42
	Exponential distribution	$\hat{\theta} = 0.032455$	276.53	276.67	277.96
	Two-parameter Lindley distribution (TDLP)	$\hat{\theta} = 0.035434 \hat{\eta} = -0.546267$	235.82	236.25	238.69

fit than the OPLD and exponential distribution. Furthermore, the parameters of the TPLD can be estimated through the maximum likelihood (ML) method, yielding $\hat{\eta} = -0.546267$ and $\hat{\theta} = 0.035434$.

Suppose the agreement specifies a mean product strength (μ_0) of 30 MPa, and the maximum strength test (t_u) of the experiment is 30 MPa, That is, the ratio of $q = t_u/\mu_0 = 1$. Under the conditions specified in the contract ($r_{\mathrm{AQL}} = 2$, $r_{\mathrm{RQL}} = 1$, $\alpha = 0.05$ and $\beta = 0.05$), the plan's parameters can be obtained using the model expressed in Eq. (15), i.e., the first sample number n_1 and the second sample number n_2 are both 4, the first acceptable number c_1 is 0, and the second acceptable number c_2 is 2 (see also from Fig. 12).

In addition, to assist practitioners in implementing the proposed TLT-DSP in real-world scenarios, a user-friendly graphical user interface (GUI) application has been created, which comprises two segments. The first segment (depicted in Fig. 12) includes (1) Referenced Data, (2) Selected Parameters, and (3) Criteria. The user is required to input necessary conditions, such as the required quality levels ($r_{\mathrm{AQL}},\ r_{\mathrm{RQL}}$), tolerated risk levels ($\alpha,\ \beta$), truncation time (t_u), and specified mean (μ_0), which are agreed upon and approved by both the supplier and the customer. Besides, the user must provide the shape parameter for the TPLD and select the parameters k (or use the default setting of $k = 1$) before clicking the "Calculate" button to obtain the plan parameters, including the required sample sizes and the acceptance numbers for the first and second-stage sampling.

After clicking the "Calculate" button, the GUI will automatically exhibit the second segment (shown on the right in Fig. 13), which includes (1) Input or Import the Data, (2) Result, and (3) Decisions for the first-stage sampling and, if needed, the second-stage sampling.

Thus, the pressure strength data of aircraft windows can be imported into the GUI application to determine the lot's acceptability. In this session, the user can simply input the data information or import the data file by clicking the "Import Data" button, and the data will be automatically displayed. If the "Result" button is clicked, the sample statistics for the data (data size, required sample size, defective items, and sample mean) will be displayed and a decision on the lot disposition will be provided. For example, consider the four data points ($n_1 = 4$): 39.580, 33.760, 31.110, and 33.730. Since no data point is less than 30 ($i.e.,\ d_1 = 0$), which is no more than first acceptable number ($c_1 = 0$), the lot can be accepted. The results are shown in Fig. 13.

If the number of failures (d_1) during the first-stage sampling inspection exceeds the first acceptance number (c_1) but does not exceed the second acceptance number (c_2), a second-stage sampling will be required. In such a scenario, the GUI will display an additional layer

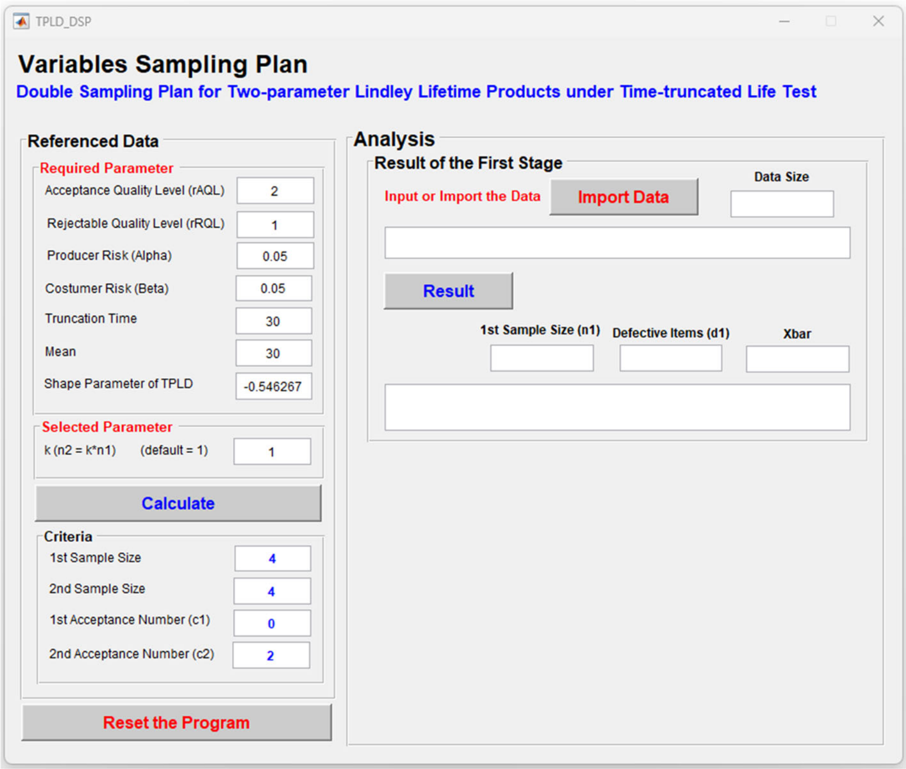


Fig. 12 GUI for plan parameters determination (first segment)

in the window session automatically, allowing the user to input or import data for the second-stage sampling. The sample results and the final decision on the lot will be also given. During the second-stage sampling, if the cumulative number of failed items ($d = d_1 + d_2$) is no more than c_2 items failed from the cumulative samples ($n_1 + n_2$), the lot will be accepted. However, if it exceeds the limit, the lot will be rejected, and the test will be stopped. Therefore, the developed GUI application simplifies the process of determining plan parameters, inputting sample data, computing sample results, and recommending a course of action.

7 Conclusions

An acceptance sampling plan consists of a set of rules and procedures used to determine whether to accept or reject a lot of products or materials based on inspecting samples from the lot. However, testing until all examined samples fail can be both costly and time-consuming, particularly for items with extended lifespans. To address this challenge, truncated life tests have emerged as an experimental method to estimate the reliability or longevity of components, products, or systems.

This study introduces the development of a DSP for truncated life tests, referred to as TLT-DSP, based on the TPLD. A mathematical model is formulated to determine plan parameters by minimizing the ASN while adhering to quality level and sampling risk criteria. Compared

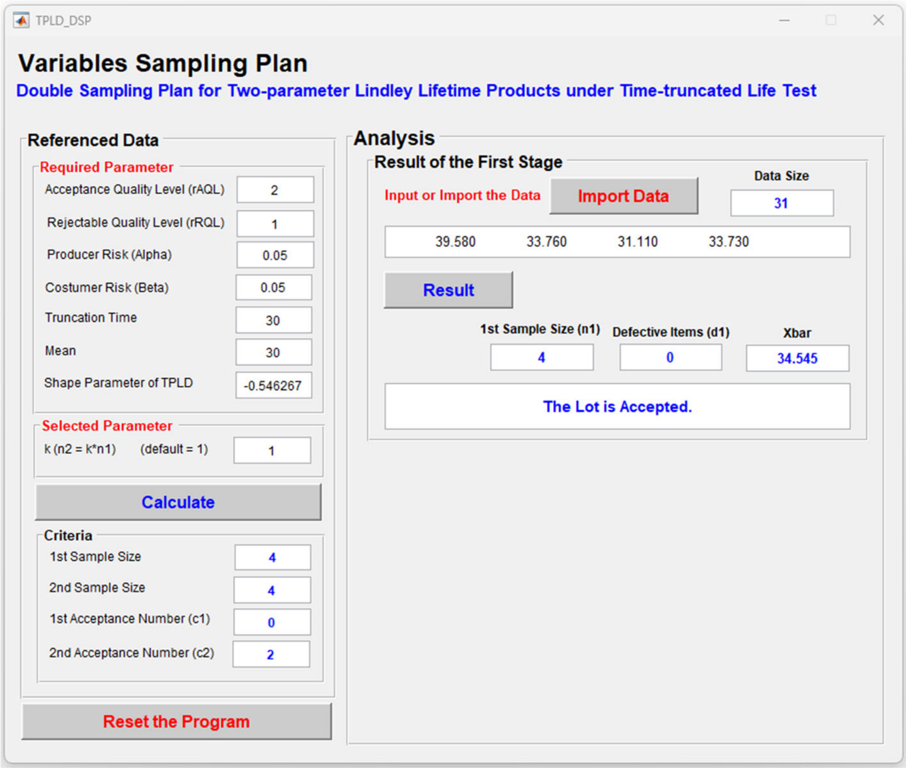


Fig. 13 GUI for sample results calculation and lot disposition (second segment)

to the existing SSP, the TLT-DSP requires fewer samples for lot disposition, particularly for lots with excellent or unsatisfactory quality, thereby leading to reduced inspection costs. The study presents two cases: the first highlights operational procedures, comparing the TPLD and Weibull distribution and concluding the suitability of TPLD for product lifetime data. Additionally, a user-friendly GUI application has been created to simplify the implementation of the TLT-DSP in practical scenarios. The second case illustrates the practical application of TLT-DSP in airplane window pressure strength testing using the designed GUI application.

Lastly, it's essential to note that this study focused on the TPLD and the proposed DSP for truncated life tests. Although the paper introduces a general DSP framework that could extend to other probability distributions, each distribution holds unique characteristics and assumptions that influence their applicability when analyzing truncated life data. A distribution's suitability for modeling specific datasets depends on its distinct shape, scale, and tail behavior. This suggests that future research directions could explore these aspects independently.

Funding This work was partially supported by the Ministry of Science and Technology of Taiwan under grant number MOST107-2221-E-992-064-MY3.

Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose.

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