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LAMPIRAN

Lampiran 1. Polinomial Berderajat Enam untuk Solusi Kesetimbangan Sel yang Tidak Bernilai Nol (Solusi Non-trivial, $C^{**} \neq 0$)

```
> restart : with(PolynomialTools) : with(RootFinding) : with(plots, implicitplot) :
> P := a1·C^6 + a2·C^5 + a3·C^4 + a4·C^3 + a5·C^2 + a6·C + a7 :
> E := Yce·C :
  G := Gb/ρ − C/Ycg − α·C^2/(ρ·(Ks + C)) :
  F := (μmax·G/(Km + G))·(1 − G/Gcrit)·(1 − E/Ecrit)·(1 − C/Ccrit) − ρ :
> f1 := simplify(F)
```

$$
\begin{aligned}
f1 := {} & \Big(\mu max\, Yce\,(Ycg\,\alpha+\rho)^2\,C^6+(Ycg\,\alpha+\rho)\,\mu max\Big(\big(-Ccrit\,Yce\,\alpha+Gcrit\,Yce\,\rho\\
& -Ecrit\,\alpha-2\,Gb\,Yce\big)\,Ycg+2\,\rho\Big(Ks\,Yce-\frac{1}{2}\,Yce\,Ccrit-\frac{1}{2}\,Ecrit\Big)\Big)\,C^5\\
& +\mu max\Big(\big(Gcrit\,(-Ccrit\,Yce\,\alpha+Ks\,Yce\,\alpha-Ecrit\,\alpha-Gb\,Yce)\,\rho-2\,\alpha\,Gb\,Ks\,Yce\\
& +\alpha\,(Ecrit\,\alpha+2\,Gb\,Yce)\,Ccrit+2\,\alpha\,Gb\,Ecrit+Gb^2\,Yce\big)\,Ycg^2+2\,\rho\Big(Gcrit\Big(Ks\,Yce\\
& -\frac{1}{2}\,Yce\,Ccrit-\frac{1}{2}\,Ecrit\Big)\rho+(-Ccrit\,Yce\,\alpha-Ecrit\,\alpha-2\,Gb\,Yce)\,Ks+(Ecrit\,\alpha\\
& +Gb\,Yce)\,Ccrit+Gb\,Ecrit\Big)\,Ycg+\rho^2\,(Ks^2\,Yce+(-2\,Ccrit\,Yce-2\,Ecrit)\,Ks\\
& +Ccrit\,Ecrit)\Big)\,C^4+\Big(\Big(\Big(-Gcrit\,((Ccrit\,Yce\,\alpha+Ecrit\,\alpha+2\,Gb\,Yce)\,Ks+(-Ecrit\,\alpha\\
& -Gb\,Yce)\,Ccrit-Gb\,Ecrit)\,\rho+2\,Gb\Big((Ccrit\,Yce\,\alpha+Ecrit\,\alpha+Gb\,Yce)\,Ks+\Big(\\
& -\alpha\,Ecrit-\frac{1}{2}\,Gb\,Yce\Big)\,Ccrit-\frac{1}{2}\,Gb\,Ecrit\Big)\Big)\,\mu max-Gcrit\,\alpha\,\rho^2\,Ccrit\,Ecrit\Big)\,Ycg^2\\
& +\rho\,((Gcrit\,(Ks^2\,Yce+(-2\,Ccrit\,Yce-2\,Ecrit)\,Ks+Ccrit\,Ecrit)\,\rho-2\,Gb\,Yce\,Ks^2\\
& +((2\,Ecrit\,\alpha+4\,Gb\,Yce)\,Ccrit+4\,Gb\,Ecrit)\,Ks-2\,Gb\,Ccrit\,Ecrit)\,\mu max\\
& -Gcrit\,\rho^2\,Ccrit\,Ecrit)\,Ycg-\rho^2\,((Ccrit\,Yce+Ecrit)\,Ks-2\,Ccrit\,Ecrit)\,\mu max\,Ks)\,C^3\\
& +(((((-Gb\,Yce\,Ks^2+((Ecrit\,\alpha+2\,Gb\,Yce)\,Ccrit+2\,Gb\,Ecrit)\,Ks\\
& -Gb\,Ccrit\,Ecrit)\,Gcrit\,\rho+Gb^2\,Yce\,Ks^2+((-2\,Ecrit\,Gb\,\alpha-2\,Gb^2\,Yce)\,Ccrit\\
& -2\,Gb^2\,Ecrit)\,Ks+Gb^2\,Ccrit\,Ecrit)\,\mu max-Gcrit\,\rho^2\,Ccrit\,Ecrit\,(-Km\,\rho+Ks\,\alpha\\
& -Gb))\,Ycg^2-\rho\,Ks\,((Ccrit\,Yce+Ecrit)\,Ks-2\,Ccrit\,Ecrit)\,(Gcrit\,\rho-2\,Gb)\,\mu max\\
& +2\,Gcrit\,\rho^2\,Ccrit\,Ecrit)\,Ycg+\mu max\,Ks^2\,\rho^2\,Ccrit\,Ecrit)\,C^2+((Gb\,((Ccrit\,Yce\\
& +Ecrit)\,Ks-2\,Ccrit\,Ecrit)\,(Gcrit\,\rho-Gb)\,\mu max+2\,Gcrit\,\rho^2\,Ccrit\,Ecrit\,(Km\,\rho+Gb))\\
& Ycg+Ecrit\,\rho\,Ccrit\,Ks\,((Gcrit\,\rho-2\,Gb)\,\mu max-Gcrit\,\rho^2))\,Ycg\,Ks\,C\\
& -Ecrit\,(Gb\,(Gcrit\,\rho-Gb)\,\mu max-Gcrit\,\rho^2\,(Km\,\rho+Gb))\,Ccrit\,Ycg^2\,Ks^2)\Big/\\
& (Ecrit\,\rho\,Gcrit\,Ccrit\,((Ycg\,\alpha+\rho)\,C^2+((-Km\,\rho-Gb)\,Ycg+Ks\,\rho)\,C-Ks\,Ycg\,(Km\,\rho\\
& +Gb))\,(Ks+C)\,Ycg)
\end{aligned}
$$

> $f2 := collect(f1, C)$

$$f2 := \left(\mu max\, Yce\,(Ycg\,\alpha+\rho)^2\,C^6+(Ycg\,\alpha+\rho)\,\mu max\left(\left(-Ccrit\,Yce\,\alpha+Gcrit\,Yce\,\rho-Ecrit\,\alpha-2\,Gb\,Yce\right)Ycg+2\,\rho\left(Ks\,Yce-\frac{1}{2}\,Yce\,Ccrit-\frac{1}{2}\,Ecrit\right)\right)C^5+\mu max\left(\left(Gcrit\left(-Ccrit\,Yce\,\alpha+Ks\,Yce\,\alpha-Ecrit\,\alpha-Gb\,Yce\right)\rho-2\,\alpha\,Gb\,Ks\,Yce+\alpha\left(Ecrit\,\alpha+2\,Gb\,Yce\right)Ccrit+2\,\alpha\,Gb\,Ecrit+Gb^2\,Yce\right)Ycg^2+2\,\rho\left(Gcrit\left(Ks\,Yce-\frac{1}{2}\,Yce\,Ccrit-\frac{1}{2}\,Ecrit\right)\rho+\left(-Ccrit\,Yce\,\alpha-Ecrit\,\alpha-2\,Gb\,Yce\right)Ks+\left(Ecrit\,\alpha+Gb\,Yce\right)Ccrit+Gb\,Ecrit\right)Ycg+\rho^2\left(Ks^2\,Yce+\left(-2\,Ccrit\,Yce-2\,Ecrit\right)Ks+Ccrit\,Ecrit\right)\right)C^4+\left(\left(\left(-Gcrit\left(\left(Ccrit\,Yce\,\alpha+Ecrit\,\alpha+2\,Gb\,Yce\right)Ks+\left(-Ecrit\,\alpha-Gb\,Yce\right)Ccrit-Gb\,Ecrit\right)\rho+2\,Gb\left(\left(Ccrit\,Yce\,\alpha+Ecrit\,\alpha+Gb\,Yce\right)Ks+\left(-\alpha\,Ecrit-\frac{1}{2}\,Gb\,Yce\right)Ccrit-\frac{1}{2}\,Gb\,Ecrit\right)\right)\mu max-Gcrit\,\alpha\,\rho^2\,Ccrit\,Ecrit\right)Ycg^2+\rho\left(\left(Gcrit\left(Ks^2\,Yce+\left(-2\,Ccrit\,Yce-2\,Ecrit\right)Ks+Ccrit\,Ecrit\right)\rho-2\,Gb\,Yce\,Ks^2+\left(\left(2\,Ecrit\,\alpha+4\,Gb\,Yce\right)Ccrit+4\,Gb\,Ecrit\right)Ks-2\,Gb\,Ccrit\,Ecrit\right)\mu max-Gcrit\,\rho^2\,Ccrit\,Ecrit\right)Ycg-\rho^2\left(\left(Ccrit\,Yce+Ecrit\right)Ks-2\,Ccrit\,Ecrit\right)\mu max\,Ks\right)C^3+\left(\left(\left(\left(-Gb\,Yce\,Ks^2+\left(\left(Ecrit\,\alpha+2\,Gb\,Yce\right)Ccrit+2\,Gb\,Ecrit\right)Ks-Gb\,Ccrit\,Ecrit\right)Gcrit\,\rho+Gb^2\,Yce\,Ks^2+\left(\left(-2\,Ecrit\,Gb\,\alpha-2\,Gb^2\,Yce\right)Ccrit-2\,Gb^2\,Ecrit\right)Ks+Gb^2\,Ccrit\,Ecrit\right)\mu max-Gcrit\,\rho^2\,Ccrit\,Ecrit\left(-Km\,\rho+Ks\,\alpha-Gb\right)\right)Ycg^2-\rho\,Ks\left(\left(\left(Ccrit\,Yce+Ecrit\right)Ks-2\,Ccrit\,Ecrit\right)\left(Gcrit\,\rho-2\,Gb\right)\mu max+2\,Gcrit\,\rho^2\,Ccrit\,Ecrit\right)Ycg+\mu max\,Ks^2\,\rho^2\,Ccrit\,Ecrit\right)C^2+\left(\left(Gb\left(\left(Ccrit\,Yce+Ecrit\right)Ks-2\,Ccrit\,Ecrit\right)\left(Gcrit\,\rho-Gb\right)\mu max+2\,Gcrit\,\rho^2\,Ccrit\,Ecrit\left(Km\,\rho+Gb\right)\right)Ycg+Ecrit\,\rho\,Ccrit\,Ks\left(\left(Gcrit\,\rho-2\,Gb\right)\mu max-Gcrit\,\rho^2\right)\right)Ycg\,Ks\,C-Ecrit\left(Gb\left(Gcrit\,\rho-Gb\right)\mu max-Gcrit\,\rho^2\left(Km\,\rho+Gb\right)\right)Ccrit\,Ycg^2\,Ks^2\right)\Big/\left(Ecrit\,\rho\,Gcrit\,Ccrit\left(\left(Ycg\,\alpha+\rho\right)C^2+\left(\left(-Km\,\rho-Gb\right)Ycg+Ks\,\rho\right)C-Ks\,Ycg\left(Km\,\rho+Gb\right)\right)\left(Ks+C\right)Ycg\right)$$

> $g := Ecrit\,Ycg\,Ccrit\,Gcrit\left(\left(Ycg\,\alpha+\rho\right)C^2+\left(\left(-Km\,\rho-Gb\right)Ycg+Ks\,\rho\right)C-Ks\,Ycg\left(Km\,\rho+Gb\right)\right)\left(Ks+C\right)\rho:$

> $f3 := simplify(f2\cdot g)$

$$f3 := \mu max\,Yce\,(Ycg\,\alpha+\rho)^2\,C^6+(Ycg\,\alpha+\rho)\,\mu max\left(\left(-Ccrit\,Yce\,\alpha+Gcrit\,Yce\,\rho-Ecrit\,\alpha-2\,Gb\,Yce\right)Ycg+2\,\rho\left(Ks\,Yce-\frac{1}{2}\,Yce\,Ccrit-\frac{1}{2}\,Ecrit\right)\right)C^5+\mu max\left(\left(Gcrit\left(-Ccrit\,Yce\,\alpha+Ks\,Yce\,\alpha-Ecrit\,\alpha-Gb\,Yce\right)\rho-2\,\alpha\,Gb\,Ks\,Yce+\alpha\left(Ecrit\,\alpha\right.\right.\right.$$

$$+ 2\,Gb\,Yce)\,Ccrit + 2\,\alpha\,Gb\,Ecrit + Gb^2\,Yce)\,Ycg^2 + 2\,\rho\left(Gcrit\left(Ks\,Yce - \frac{1}{2}\,Yce\,Ccrit\right.\right.$$

$$\left.\left. - \frac{1}{2}\,Ecrit\right)\rho + (-Ccrit\,Yce\,\alpha - Ecrit\,\alpha - 2\,Gb\,Yce)\,Ks + (Ecrit\,\alpha + Gb\,Yce)\,Ccrit\right.$$

$$\left. + Gb\,Ecrit\right)Ycg + \rho^2\,(Ks^2\,Yce + (-2\,Ccrit\,Yce - 2\,Ecrit)\,Ks + Ccrit\,Ecrit)\Big)\,C^4 + \Big(\Big(\Big($$

$$-Gcrit\,((Ccrit\,Yce\,\alpha + Ecrit\,\alpha + 2\,Gb\,Yce)\,Ks + (-Ecrit\,\alpha - Gb\,Yce)\,Ccrit$$

$$- Gb\,Ecrit)\,\rho + 2\,Gb\left((Ccrit\,Yce\,\alpha + Ecrit\,\alpha + Gb\,Yce)\,Ks + \left(-\alpha\,Ecrit\right.\right.$$

$$\left.\left. - \frac{1}{2}\,Gb\,Yce\right)Ccrit - \frac{1}{2}\,Gb\,Ecrit\right)\Big)\,Ycg^2 + \rho\,(Gcrit\,(Ks^2\,Yce + (-2\,Ccrit\,Yce$$

$$- 2\,Ecrit)\,Ks + Ccrit\,Ecrit)\,\rho - 2\,Gb\,Yce\,Ks^2 + ((2\,Ecrit\,\alpha + 4\,Gb\,Yce)\,Ccrit$$

$$+ 4\,Gb\,Ecrit)\,Ks - 2\,Gb\,Ccrit\,Ecrit)\,Ycg - \rho^2\,((Ccrit\,Yce + Ecrit)\,Ks$$

$$- 2\,Ccrit\,Ecrit)\,Ks)\,\mu max - Gcrit\,Ycg\,\rho^2\,Ccrit\,Ecrit\,(Ycg\,\alpha + \rho))\,C^3 + ((((($$

$$-Gb\,Yce\,Ks^2 + ((Ecrit\,\alpha + 2\,Gb\,Yce)\,Ccrit + 2\,Gb\,Ecrit)\,Ks - Gb\,Ccrit\,Ecrit)\,Gcrit\,\rho$$

$$+ Gb^2\,Yce\,Ks^2 + ((-2\,Ecrit\,Gb\,\alpha - 2\,Gb^2\,Yce)\,Ccrit - 2\,Gb^2\,Ecrit)\,Ks$$

$$+ Gb^2\,Ccrit\,Ecrit)\,Ycg^2 - \rho\,((Ccrit\,Yce + Ecrit)\,Ks - 2\,Ccrit\,Ecrit)\,(Gcrit\,\rho$$

$$- 2\,Gb)\,Ks\,Ycg + Ks^2\,\rho^2\,Ccrit\,Ecrit)\,\mu max - Ecrit\,((-Km\,\rho + Ks\,\alpha - Gb)\,Ycg$$

$$+ 2\,Ks\,\rho)\,\rho^2\,Gcrit\,Ccrit\,Ycg)\,C^2 + \Big((Gb\,((Ccrit\,Yce + Ecrit)\,Ks$$

$$- 2\,Ccrit\,Ecrit)\,(Gcrit\,\rho - Gb)\,Ycg + Ks\,\rho\,Ccrit\,Ecrit\,(Gcrit\,\rho - 2\,Gb))\,\mu max$$

$$+ 2\,Ecrit\,\rho^2\,Gcrit\,Ccrit\left((Km\,\rho + Gb)\,Ycg - \frac{1}{2}\,Ks\,\rho\right)\Big)\,Ycg\,Ks\,C - Ecrit\,(Gb\,(Gcrit\,\rho$$

$$- Gb)\,\mu max - Gcrit\,\rho^2\,(Km\,\rho + Gb))\,Ccrit\,Ycg^2\,Ks^2$$

> $a1 := coeff(f3, C, 6)$

$$a1 := \mu max\,Yce\,(Ycg\,\alpha + \rho)^2$$

> $a2 := coeff(f3, C, 5)$

$$a2 := (Ycg\,\alpha + \rho)\,\mu max\left((-Ccrit\,Yce\,\alpha + Gcrit\,Yce\,\rho - Ecrit\,\alpha - 2\,Gb\,Yce)\,Ycg\right.$$

$$\left. + 2\,\rho\left(Ks\,Yce - \frac{1}{2}\,Yce\,Ccrit - \frac{1}{2}\,Ecrit\right)\right)$$

> $factor(a2)$

$$-(Ycg\,\alpha + \rho)\,\mu max\,(Ccrit\,Yce\,Ycg\,\alpha - Gcrit\,Yce\,Ycg\,\rho + Ccrit\,Yce\,\rho + Ecrit\,Ycg\,\alpha$$

$$+ 2\,Gb\,Yce\,Ycg - 2\,Ks\,Yce\,\rho + Ecrit\,\rho)$$

> $a3 := coeff(f3, C, 4)$

$$a3 := \mu max\Big((Gcrit\,(-Ccrit\,Yce\,\alpha + Ks\,Yce\,\alpha - Ecrit\,\alpha - Gb\,Yce)\,\rho - 2\,\alpha\,Gb\,Ks\,Yce$$

$$+ \alpha\,(Ecrit\,\alpha + 2\,Gb\,Yce)\,Ccrit + 2\,\alpha\,Gb\,Ecrit + Gb^2\,Yce)\,Ycg^2 + 2\,\rho\left(Gcrit\left(Ks\,Yce\right.\right.$$

$$-\frac{1}{2}\,Yce\,Ccrit-\frac{1}{2}\,Ecrit\Big)\rho+\left(-Ccrit\,Yce\,\alpha-Ecrit\,\alpha-2\,Gb\,Yce\right)Ks+\left(Ecrit\,\alpha+Gb\,Yce\right)Ccrit+Gb\,Ecrit\Big)\,Ycg+\rho^2\left(Ks^2\,Yce+\left(-2\,Ccrit\,Yce-2\,Ecrit\right)Ks+Ccrit\,Ecrit\right)\Big)$$

> $factor(a3)$

$$\mu max\left(-Ccrit\,Gcrit\,Yce\,Ycg^2\,\alpha\,\rho+Gcrit\,Ks\,Yce\,Ycg^2\,\alpha\,\rho+Ccrit\,Ecrit\,Ycg^2\,\alpha^2+2\,Ccrit\,Gb\,Yce\,Ycg^2\,\alpha-Ccrit\,Gcrit\,Yce\,Ycg\,\rho^2-2\,Ccrit\,Ks\,Yce\,Ycg\,\alpha\,\rho-Ecrit\,Gcrit\,Ycg^2\,\alpha\,\rho-Gb\,Gcrit\,Yce\,Ycg^2\,\rho-2\,Gb\,Ks\,Yce\,Ycg^2\,\alpha+2\,Gcrit\,Ks\,Yce\,Ycg\,\rho^2+2\,Ccrit\,Ecrit\,Ycg\,\alpha\,\rho+2\,Ccrit\,Gb\,Yce\,Ycg\,\rho-2\,Ccrit\,Ks\,Yce\,\rho^2+2\,Ecrit\,Gb\,Ycg^2\,\alpha-Ecrit\,Gcrit\,Ycg\,\rho^2-2\,Ecrit\,Ks\,Ycg\,\alpha\,\rho+Gb^2\,Yce\,Ycg^2-4\,Gb\,Ks\,Yce\,Ycg\,\rho+Ks^2\,Yce\,\rho^2+Ccrit\,Ecrit\,\rho^2+2\,Ecrit\,Gb\,Ycg\,\rho-2\,Ecrit\,Ks\,\rho^2\right)$$

> $a4 := coeff(f3, C, 3)$

$$a4 := \Big(\Big(-Gcrit\left(\left(Ccrit\,Yce\,\alpha+Ecrit\,\alpha+2\,Gb\,Yce\right)Ks+\left(-Ecrit\,\alpha-Gb\,Yce\right)Ccrit-Gb\,Ecrit\right)\rho+2\,Gb\left(\left(Ccrit\,Yce\,\alpha+Ecrit\,\alpha+Gb\,Yce\right)Ks+\left(-\alpha\,Ecrit-\frac{1}{2}\,Gb\,Yce\right)Ccrit-\frac{1}{2}\,Gb\,Ecrit\right)\Big)\,Ycg^2+\rho\left(Gcrit\left(Ks^2\,Yce+\left(-2\,Ccrit\,Yce-2\,Ecrit\right)Ks+Ccrit\,Ecrit\right)\rho-2\,Gb\,Yce\,Ks^2+\left(\left(2\,Ecrit\,\alpha+4\,Gb\,Yce\right)Ccrit+4\,Gb\,Ecrit\right)Ks-2\,Gb\,Ccrit\,Ecrit\right)Ycg-\rho^2\left(\left(Ccrit\,Yce+Ecrit\right)Ks-2\,Ccrit\,Ecrit\right)Ks\Big)\,\mu max-Gcrit\,Ycg\,\rho^2\,Ccrit\,Ecrit\left(Ycg\,\alpha+\rho\right)$$

> $factor(a4)$

$$-\mu max\,Ccrit\,Gcrit\,Ks\,Yce\,Ycg^2\,\alpha\,\rho+\mu max\,Ccrit\,Ecrit\,Gcrit\,Ycg^2\,\alpha\,\rho+\mu max\,Ccrit\,Gb\,Gcrit\,Yce\,Ycg^2\,\rho+2\,\mu max\,Ccrit\,Gb\,Ks\,Yce\,Ycg^2\,\alpha-2\,\mu max\,Ccrit\,Gcrit\,Ks\,Yce\,Ycg\,\rho^2-\mu max\,Ecrit\,Gcrit\,Ks\,Ycg^2\,\alpha\,\rho-2\,\mu max\,Gb\,Gcrit\,Ks\,Yce\,Ycg^2\,\rho+\mu max\,Gcrit\,Ks^2\,Yce\,Ycg\,\rho^2-Ccrit\,Ecrit\,Gcrit\,Ycg^2\,\alpha\,\rho^2-2\,\mu max\,Ccrit\,Ecrit\,Gb\,Ycg^2\,\alpha+\mu max\,Ccrit\,Ecrit\,Gcrit\,Ycg\,\rho^2+2\,\mu max\,Ccrit\,Ecrit\,Ks\,Ycg\,\alpha\,\rho-\mu max\,Ccrit\,Gb^2\,Yce\,Ycg^2+4\,\mu max\,Ccrit\,Gb\,Ks\,Yce\,Ycg\,\rho-\mu max\,Ccrit\,Ks^2\,Yce\,\rho^2+\mu max\,Ecrit\,Gb\,Gcrit\,Ycg^2\,\rho+2\,\mu max\,Ecrit\,Gb\,Ks\,Ycg^2\,\alpha-2\,\mu max\,Ecrit\,Gcrit\,Ks\,Ycg\,\rho^2+2\,\mu max\,Gb^2\,Ks\,Yce\,Ycg^2-2\,\mu max\,Gb\,Ks^2\,Yce\,Ycg\,\rho-Ccrit\,Ecrit\,Gcrit\,Ycg\,\rho^3-2\,\mu max\,Ccrit\,Ecrit\,Gb\,Ycg\,\rho+2\,\mu max\,Ccrit\,Ecrit\,Ks\,\rho^2-\mu max\,Ecrit\,Gb^2\,Ycg^2+4\,\mu max\,Ecrit\,Gb\,Ks\,Ycg\,\rho-\mu max\,Ecrit\,Ks^2\,\rho^2$$

> $a5 := coeff(f3, C, 2)$

$$a5 := \left(\left(\left(-Gb\,Yce\,Ks^2 + \left(\left(Ecrit\,\alpha + 2\,Gb\,Yce\right)Ccrit + 2\,Gb\,Ecrit\right)Ks - Gb\,Ccrit\,Ecrit\right)Gcrit\,\rho + Gb^2\,Yce\,Ks^2 + \left(\left(-2\,Ecrit\,Gb\,\alpha - 2\,Gb^2\,Yce\right)Ccrit - 2\,Gb^2\,Ecrit\right)Ks + Gb^2\,Ccrit\,Ecrit\right)Ycg^2 - \rho\left(\left(Ccrit\,Yce + Ecrit\right)Ks - 2\,Ccrit\,Ecrit\right)\left(Gcrit\,\rho - 2\,Gb\right)Ks\,Ycg + Ks^2\,\rho^2\,Ccrit\,Ecrit\right)\mu max - Ecrit\left(\left(-Km\,\rho + Ks\,\alpha - Gb\right)Ycg + 2\,Ks\,\rho\right)\rho^2\,Gcrit\,Ccrit\,Ycg$$

> $factor(a5)$

$$\mu max\,Ccrit\,Ecrit\,Gcrit\,Ks\,Ycg^2\,\alpha\,\rho + 2\,\mu max\,Ccrit\,Gb\,Gcrit\,Ks\,Yce\,Ycg^2\,\rho - \mu max\,Ccrit\,Gcrit\,Ks^2\,Yce\,Ycg\,\rho^2 - \mu max\,Gb\,Gcrit\,Ks^2\,Yce\,Ycg^2\,\rho + Ccrit\,Ecrit\,Gcrit\,Km\,Ycg^2\,\rho^3 - Ccrit\,Ecrit\,Gcrit\,Ks\,Ycg^2\,\alpha\,\rho^2 - \mu max\,Ccrit\,Ecrit\,Gb\,Gcrit\,Ycg^2\,\rho - 2\,\mu max\,Ccrit\,Ecrit\,Gb\,Ks\,Ycg^2\,\alpha + 2\,\mu max\,Ccrit\,Ecrit\,Gcrit\,Ks\,Ycg\,\rho^2 - 2\,\mu max\,Ccrit\,Gb^2\,Ks\,Yce\,Ycg^2 + 2\,\mu max\,Ccrit\,Gb\,Ks^2\,Yce\,Ycg\,\rho + 2\,\mu max\,Ecrit\,Gb\,Gcrit\,Ks\,Ycg^2\,\rho - \mu max\,Ecrit\,Gcrit\,Ks^2\,Ycg\,\rho^2 + \mu max\,Gb^2\,Ks^2\,Yce\,Ycg^2 + Ccrit\,Ecrit\,Gb\,Gcrit\,Ycg^2\,\rho^2 - 2\,Ccrit\,Ecrit\,Gcrit\,Ks\,Ycg\,\rho^3 + \mu max\,Ccrit\,Ecrit\,Gb^2\,Ycg^2 - 4\,\mu max\,Ccrit\,Ecrit\,Gb\,Ks\,Ycg\,\rho + \mu max\,Ccrit\,Ecrit\,Ks^2\,\rho^2 - 2\,\mu max\,Ecrit\,Gb^2\,Ks\,Ycg^2 + 2\,\mu max\,Ecrit\,Gb\,Ks^2\,Ycg\,\rho$$

> $a6 := coeff(f3, C, 1)$

$$a6 := \left(\left(Gb\left(\left(Ccrit\,Yce + Ecrit\right)Ks - 2\,Ccrit\,Ecrit\right)\left(Gcrit\,\rho - Gb\right)Ycg + Ks\,\rho\,Ccrit\,Ecrit\left(Gcrit\,\rho - 2\,Gb\right)\right)\mu max + 2\,Ecrit\,\rho^2\,Gcrit\,Ccrit\left(\left(Km\,\rho + Gb\right)Ycg - \frac{1}{2}\,Ks\,\rho\right)\right)Ycg\,Ks$$

> $factor(a6)$

$$\left(\mu max\,Ccrit\,Gb\,Gcrit\,Ks\,Yce\,Ycg\,\rho + 2\,Ccrit\,Ecrit\,Gcrit\,Km\,Ycg\,\rho^3 - 2\,\mu max\,Ccrit\,Ecrit\,Gb\,Gcrit\,Ycg\,\rho + \mu max\,Ccrit\,Ecrit\,Gcrit\,Ks\,\rho^2 - \mu max\,Ccrit\,Gb^2\,Ks\,Yce\,Ycg + \mu max\,Ecrit\,Gb\,Gcrit\,Ks\,Ycg\,\rho + 2\,Ccrit\,Ecrit\,Gb\,Gcrit\,Ycg\,\rho^2 - Ccrit\,Ecrit\,Gcrit\,Ks\,\rho^3 + 2\,\mu max\,Ccrit\,Ecrit\,Gb^2\,Ycg - 2\,\mu max\,Ccrit\,Ecrit\,Gb\,Ks\,\rho - \mu max\,Ecrit\,Gb^2\,Ks\,Ycg\right)Ycg\,Ks$$

> $a7 := coeff(f3, C, 0)$

$$a7 := -Ecrit\left(Gb\left(Gcrit\,\rho - Gb\right)\mu max - Gcrit\,\rho^2\left(Km\,\rho + Gb\right)\right)Ccrit\,Ycg^2\,Ks^2$$

Lampiran 2. *Plotting* Polinomial (4.33) terhadap ρ dan G_b

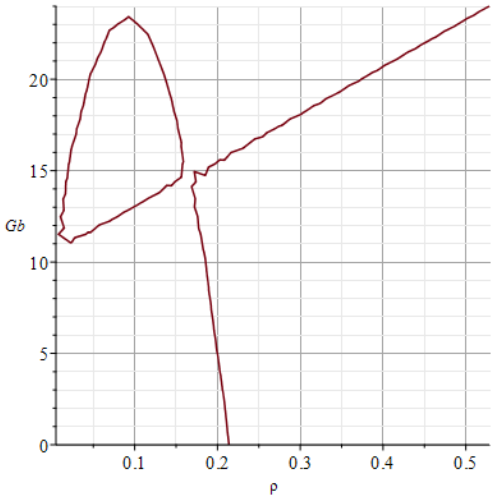
Setelah memperoleh polinomial (4.33) pada Lampiran 1, lanjutkan dengan menginput semua nilai-nilai parameter dari Tabel 4.3 ke dalam Maple, kecuali nilai ρ dan nilai-nilai parameter pada G_b.

```
> μmax := 0.408 :
  Km := 0.22 :
  Gcrit := 260 :
  Ecrit := 76 :
  Ccrit := 23 :
  Ycg := 0.5 :
  Yce := 0.35 :
  α := 0.84 :
  Ks := 0.6894 :
```

```
> P
```

$$0.14280\,C^6\,(\rho+0.420)^2+0.408\,C^5\,(\rho+0.420)\,(-0.350\,Gb-38.06742000\,\rho-35.30100)+C^4\,(26.52000\,(-0.35\,Gb-70.39931640)\,\rho+14.36146055\,Gb+125.8056576+0.0357000\,Gb^2+0.4080\,(-10863.76460\,\rho+83.567420\,Gb+1419.646981)\,\rho+665.9695076\,\rho^2)+C^3\,(-26.52000\,(-83.567420\,Gb-1419.646981)\,\rho+0.20400\,Gb\,(-41.78371000\,Gb-1419.646981)+0.2040\,(4.243923333\,10^5\,\rho-3264.556411\,Gb+2024.519616)\,\rho+967.0398693\,\rho^2-2.272400\,10^5\,\rho^2\,(\rho+0.420))+C^2\,(26.52000\,(-1632.278205\,Gb+1012.259808)\,\rho+166.4923769\,Gb^2-206.5010008\,Gb+483.5199346\,(-2\,Gb+260\,\rho)\,\rho+338.9566428\,\rho^2-2.272400\,10^5\,\rho^2\,(-0.5\,Gb+1.2688\,\rho+0.2895480))+C\,(-241.7599673\,Gb\,(-Gb+260\,\rho)+169.4783214\,(-2\,Gb+260\,\rho)\,\rho+3.133185120\,10^5\,\rho^2\,(0.5\,Gb-0.2347000000\,\rho))-84.73916069\,Gb\,(-Gb+260\,\rho)+54000.44554\,\rho^2\,(Gb+0.22\,\rho) \quad \textbf{(16)}$$

```
> Explore( plots[:-implicitplot](16, ρ = 0 .. 1, Gb = 0 .. 24, labels = [ρ, Gb]), ':-parameters' = [C = 0. .. 220.], ':-initialvalues' = [C = 13.027] )
```



Lampiran 3. Matriks Jacobian Sistem Persamaan (4.20) untuk E_1 dan E_2

```
> restart : with(VectorCalculus) : with(LinearAlgebra) : with(plots, implicitplot) :
> f1 := Cu·ρ·C : f2 := Gb − Gk − ρ·G : f3 := Eb − ρ·E :
>
```

$$Cu := C\cdot\left(\frac{\mu max\cdot G}{Km+G}\right)\cdot\left(1-\frac{G}{Gcrit}\right)\cdot\left(1-\frac{E}{Ecrit}\right)\cdot\left(1-\frac{C}{Ccrit}\right):$$

$$Gk := \frac{Cu}{Ycg}+\alpha\cdot\left(\frac{C^2}{Ks+C}\right):$$

$$Eb := Yce\cdot Cu:$$

```
> J := Jacobian([f1, f2, f3], [C, G, E])
```

$$J := \left[\left[\frac{2\,C\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)\rho}{Km+G} - \frac{C^2\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\rho}{(Km+G)\,Ccrit},\right.\right.$$

$$\frac{C^2\,\mu max\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)\rho}{Km+G} - \frac{C^2\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)\rho}{(Km+G)^2}$$

$$- \frac{C^2\,\mu max\,G\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)\rho}{(Km+G)\,Gcrit},$$

$$\left.\left. - \frac{C^2\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{C}{Ccrit}\right)\rho}{(Km+G)\,Ecrit}\right],\right.$$

$$\left[-\frac{\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)}{(Km+G)\,Ycg} + \frac{C\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)}{(Km+G)\,Ccrit\,Ycg} - \frac{2\,\alpha\,C}{Ks+C} + \frac{\alpha\,C^2}{(Ks+C)^2}, -\rho\right.$$

$$- \frac{C\,\mu max\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)}{(Km+G)\,Ycg} + \frac{C\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)}{(Km+G)^2\,Ycg}$$

$$\left. + \frac{C\,\mu max\,G\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)}{(Km+G)\,Gcrit\,Ycg}, \frac{C\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{C}{Ccrit}\right)}{(Km+G)\,Ecrit\,Ycg}\right],$$

$$\left[\frac{Yce\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)}{Km+G}\right.$$
$$-\frac{Yce\,C\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)}{(Km+G)\,Ccrit},$$
$$\frac{Yce\,C\,\mu max\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)}{Km+G}$$
$$-\frac{Yce\,C\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)}{(Km+G)^2}$$
$$-\frac{Yce\,C\,\mu max\,G\left(1-\frac{E}{Ecrit}\right)\left(1-\frac{C}{Ccrit}\right)}{(Km+G)\,Gcrit},\ -\rho$$
$$\left.-\frac{Yce\,C\,\mu max\,G\left(1-\frac{G}{Gcrit}\right)\left(1-\frac{C}{Ccrit}\right)}{(Km+G)\,Ecrit}\right]\Bigg]$$

> $J1 := subs\left(C=0, G=\frac{Gb}{\rho}, E=0, J\right)$

$$J1 := \begin{bmatrix} 0 & 0 & 0 \\ -\frac{\mu max\,Gb\left(1-\frac{Gb}{\rho\,Gcrit}\right)}{\rho\left(Km+\frac{Gb}{\rho}\right)Ycg} & -\rho & 0 \\ \frac{Yce\,\mu max\,Gb\left(1-\frac{Gb}{\rho\,Gcrit}\right)}{\rho\left(Km+\frac{Gb}{\rho}\right)} & 0 & -\rho \end{bmatrix}$$

> $Eigenvalues(J1)$

$$\begin{bmatrix} 0 \\ -\rho \\ -\rho \end{bmatrix}$$

> $J2 := subs\left(C=C[i], G=\frac{Gb}{\rho}-\frac{C[i]}{Ycg}-\frac{\alpha\cdot(C[i])^2}{\rho\cdot(Ks+C[i])}, E=Yce\cdot C[i], J\right)$

$$J2 := \left[\left[\frac{2\,C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce\,C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)\rho}{Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}\right.\right.$$
$$\left.\left.-\frac{C\_i^2\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce\,C\_i}{Ecrit}\right)\rho}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Ccrit},\right.$$

$$\frac{C\_i^2\,\mu max\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)\rho}{Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}$$

$$-\frac{C\_i^2\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)\rho}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)^2}$$

$$-\frac{C\_i^2\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)\rho}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Gcrit},$$

$$-\frac{C\_i^2\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)\rho}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Ecrit}\Bigg],$$

$$\Bigg[-\frac{\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Ycg}$$

$$+\frac{C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Ccrit\,Ycg}-\frac{2\,\alpha C\_i}{Ks+C\_i}+\frac{\alpha C\_i^2}{(Ks+C\_i)^2},-\rho$$

$$-\frac{C\_i\,\mu max\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Ycg}$$

$$+\frac{C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)^2Ycg}$$

$$+\frac{C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Gcrit\,Ycg},$$

$$\frac{C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Ecrit\,Ycg}\Bigg],$$

$$\Bigg[\frac{Yce\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}$$

$$-\frac{Yce C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce C\_i}{Ecrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha C\_i^2}{\rho\,(Ks+C\_i)}\right)Ccrit},$$

$$\frac{Yce\,C\_i\,\mu max\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce\,C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}}$$

$$-\frac{Yce\,C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{Yce\,C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}\right)^{2}}$$

$$-\frac{Yce\,C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{Yce\,C\_i}{Ecrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}\right)Gcrit},-\rho$$

$$-\frac{Yce\,C\_i\,\mu max\left(\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}\right)\left(1-\frac{\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}}{Gcrit}\right)\left(1-\frac{C\_i}{Ccrit}\right)}{\left(Km+\frac{Gb}{\rho}-\frac{C\_i}{Ycg}-\frac{\alpha\,C\_i^{2}}{\rho\,(Ks+C\_i)}\right)Ecrit}\Bigg]\Bigg]$$

Lampiran 4. Tampilan Lengkap Matriks Jacobian untuk Sistem Persamaan (4.20) yang Dievaluasi pada E_{2_i}

$$J_2 = \begin{bmatrix} \frac{\partial f_1}{\partial C(t)}(C_i^{**}, G_i^{**}, E_i^{**}) & \frac{\partial f_1}{\partial G(t)}(C_i^{**}, G_i^{**}, E_i^{**}) & \frac{\partial f_1}{\partial E(t)}(C_i^{**}, G_i^{**}, E_i^{**}) \\ \frac{\partial f_2}{\partial C(t)}(C_i^{**}, G_i^{**}, E_i^{**}) & \frac{\partial f_2}{\partial G(t)}(C_i^{**}, G_i^{**}, E_i^{**}) & \frac{\partial f_2}{\partial E(t)}(C_i^{**}, G_i^{**}, E_i^{**}) \\ \frac{\partial f_3}{\partial C(t)}(C_i^{**}, G_i^{**}, E_i^{**}) & \frac{\partial f_3}{\partial G(t)}(C_i^{**}, G_i^{**}, E_i^{**}) & \frac{\partial f_3}{\partial E(t)}(C_i^{**}, G_i^{**}, E_i^{**}) \end{bmatrix},$$

dengan

$$\frac{\partial f_1}{\partial C_i^{**}} = -\rho + \frac{\mu_{max}\left(\frac{Gb}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)\left(1 - \frac{\frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(Ks + C_i^{**})}}{G_{crit}}\right)\left(1 - \frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1 - \frac{C_i^{**}}{C_{crit}}\right)}{K_m + \frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}}$$

$$- \frac{C_i^{**}\mu_{max}\left(\frac{Gb}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)\left(1 - \frac{\frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(Ks + C_i^{**})}}{G_{crit}}\right)\left(1 - \frac{Y_{ce}C_i^{**}}{E_{crit}}\right)}{C_{crit}\left(K_m + \frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)},$$

$$\frac{\partial f_1}{\partial G_i^{**}} = \frac{C_i^{**}\mu_{max}\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}}$$

$$-\frac{C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)^2}$$

$$-\frac{C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{G_{crit}\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)},$$

$$\frac{\partial f_1}{\partial E_i^{**}} = -\frac{C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{E_{crit}\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)},$$

$$\frac{\partial f_2}{\partial C_i^{**}} = -\frac{\mu_{max}\left(\frac{Gb}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)\left(1 - \frac{\frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(Ks + C_i^{**})}}{G_{crit}}\right)\left(1 - \frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1 - \frac{C_i^{**}}{C_{crit}}\right)}{Y_{cg}\left(K_m + \frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)}$$

$$+\frac{C_i^{**}\mu_{max}\left(\frac{Gb}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)\left(1 - \frac{\frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(Ks + C_i^{**})}}{G_{crit}}\right)\left(1 - \frac{Y_{ce}C_i^{**}}{E_{crit}}\right)}{C_{crit}Y_{cg}\left(K_m + \frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)} - \frac{2\alpha C_i^{**}}{K_s + C_i^{**}} + \frac{\alpha(C_i^{**})^2}{(K_s + C_i^{**})^2},$$

$$\frac{\partial f_2}{\partial G_i^{**}} = -\rho - \frac{C_i^{**}\mu_{max}\left(1 - \frac{\frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(Ks + C_i^{**})}}{G_{crit}}\right)\left(1 - \frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1 - \frac{C_i^{**}}{C_{crit}}\right)}{Y_{cg}\left(K_m + \frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)}$$

$$+\frac{C_i^{**}\mu_{max}\left(\frac{Gb}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)\left(1 - \frac{\frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(Ks + C_i^{**})}}{G_{crit}}\right)\left(1 - \frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1 - \frac{C_i^{**}}{C_{crit}}\right)}{Y_{cg}\left(K_m + \frac{G_b}{\rho} - \frac{C_i^{**}}{Y_{cg}} - \frac{\alpha(C_i^{**})^2}{\rho(K_s + C_i^{**})}\right)^2}$$

$$+\frac{C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{G_{crit}Y_{cg}\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)},$$

$$\frac{\partial f_2}{\partial E_i^{**}}=\frac{C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{E_{crit}Y_{cg}\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)},$$

$$\frac{\partial f_3}{\partial C_i^{**}}=\frac{Y_{ce}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}}$$

$$-\frac{Y_{ce}C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)}{C_{crit}\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)},$$

$$\frac{\partial f_3}{\partial G_i^{**}} = \frac{Y_{ce}C_i^{**}\mu_{max}\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}}$$

$$-\frac{Y_{ce}C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)^2}$$

$$-\frac{Y_{ce}C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{Y_{ce}C_i^{**}}{E_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{G_{crit}\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)},$$

$$\frac{\partial f_3}{\partial E_i^{**}} = -\rho-\frac{Y_{ce}C_i^{**}\mu_{max}\left(\frac{Gb}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)\left(1-\frac{\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(Ks+C_i^{**})}}{G_{crit}}\right)\left(1-\frac{C_i^{**}}{C_{crit}}\right)}{E_{crit}\left(K_m+\frac{G_b}{\rho}-\frac{C_i^{**}}{Y_{cg}}-\frac{\alpha(C_i^{**})^2}{\rho(K_s+C_i^{**})}\right)}.$$

Lampiran 5. *Script* (m-file) Matlab R2015a untuk Simulasi Numerik Model Matematika (4.16)

- modelmatematika.m

```
function y = modelmatematika(t,x)
global umax Km Gcrit Ecrit Ccrit rho Gs A Ycg alpha Ks Yce F
C=x(1);
G=x(2);
E=x(3);
omega=2*pi*F; %omega: frekuensi gangguan (jumlah gelombang sinus
yang terbentuk dalam 1 jam)
y = [C*(umax*G/(Km+G))*(1-(G/Gcrit))*(1-(E/Ecrit))*(1-(C/Ccrit))-
rho*C; rho*Gs+A*sin(omega*t)-(C*(umax*G/(Km+G))*(1-(G/Gcrit))*(1-
(E/Ecrit))*(1-(C/Ccrit)))/Ycg-alpha*(C^2/(Ks+C))-rho*G;
Yce*(C*(umax*G/(Km+G))*(1-(G/Gcrit))*(1-(E/Ecrit))*(1-(C/Ccrit)))-
rho*E];
end
```

- SimNum.m

```
clc; clear; close all;
%DATA PARAMETER
global umax Km Gcrit Ecrit Ccrit rho Gs A Ycg alpha Ks Yce F
umax=0.408; %Laju pertumbuhan spesifik maksimum sel
Km=0.22; %Konstanta Monod
Gcrit=260; %"Konsentrasi jenuh yang terukur" dari glukosa
Ecrit=76; %"Konsentrasi jenuh yang terukur" dari etanol
Ccrit=23; %Daya muat sel
rho=0.084; %Laju dilusi glukosa
Gs=150; %Konsentrasi pemberian glukosa
Ycg=0.5; %Faktor yield sel terhadap glukosa yang diberikan
alpha=0.84; %Laju penyerapan spesifik glukosa
Ks=0.6894; %Konstanta Michaelis
Yce=0.35; %Faktor yield sel terhadap etanol yang terbentuk
A=1; %Amplitudo gangguan (PARAMETER YANG DIATUR)
%NILAI AWAL
y0 = [10; 8; 0]; %C(t); G(t); E(t)
%INTERVAL WAKTU
tspan = [0 24]; %Pengamatan dilakukan dalam 24 jam
%UNTUK MENAMPILKAN C(t), G(t), E(t) DALAM 1 PLOT
figure(1)
F=1; %Frekuensi untuk omega (PARAMETER YANG DIATUR)
[t,y] = ode45(@modelmatematika,tspan,y0); %RUNGE-KUTTA ORDE 4-5
Titik_Kesetimbangan = y(end,:)
grid on
hold on; plot(t,y(:,1),'-b','LineWidth',1); hold off;
hold on; plot(t,y(:,2),':r','LineWidth',1.5); hold off;
hold on; plot(t,y(:,3),'-.m','LineWidth',1);
xlabel('t (jam)');
ylabel('Konsentrasi Sel, Glukosa, dan Etanol (g/L)');
legend('Konsentrasi Sel (C(t))','Konsentrasi Glukosa
(G(t))','Konsentrasi Etanol (E(t))');
hold off
figure(2)
F=2;
[t,y] = ode45(@modelmatematika,tspan,y0);
Titik_Kesetimbangan = y(end,:)
grid on
```

```
hold on; plot(t,y(:,1),'-b','LineWidth',1); hold off;
hold on; plot(t,y(:,2),':r','LineWidth',1.5); hold off;
hold on; plot(t,y(:,3),'-.m','LineWidth',1);
xlabel('t (jam)');
ylabel('Konsentrasi Sel, Glukosa, dan Etanol (g/L)');
legend('Konsentrasi Sel (C(t))','Konsentrasi Glukosa
(G(t))','Konsentrasi Etanol (E(t))');
hold off
figure(3)
F=3;
[t,y] = ode45(@modelmatematika,tspan,y0);
Titik_Kesetimbangan = y(end,:)
grid on
hold on; plot(t,y(:,1),'-b','LineWidth',1); hold off;
hold on; plot(t,y(:,2),':r','LineWidth',1.5); hold off;
hold on; plot(t,y(:,3),'-.m','LineWidth',1);
xlabel('t (jam)');
ylabel('Konsentrasi Sel, Glukosa, dan Etanol (g/L)');
legend('Konsentrasi Sel (C(t))','Konsentrasi Glukosa
(G(t))','Konsentrasi Etanol (E(t))');
hold off
%PLOTTING FUNGSI Gb (LAJU PEMBERIAN GLUKOSA SECARA KONSTAN &
SECARA PERIODIK)
figure(4)
tx=0:0.01:2;
grid on
hold on; A=0; F=0; omega=2*pi*F; Gb=rho*Gs + A*sin(omega*tx);
plot(tx,Gb,':b','LineWidth',1.5); hold off;
hold on; A=0.5; F=1; omega=2*pi*F; Gb=rho*Gs + A*sin(omega*tx);
plot(tx,Gb,'-m','LineWidth',1);
xlabel('t (jam)');
ylabel('Laju Pemberian Glukosa (g/L/h)');
ylim([9 15]);
legend('Secara Konstan (A=0)','Secara Periodik (A=0.5,
\omega=2\pi)');
hold off
figure(5)
tx=0:0.01:2;
grid on
hold on; A=0; F=0; omega=2*pi*F; Gb=rho*Gs + A*sin(omega*tx);
plot(tx,Gb,':b','LineWidth',1.5); hold off;
hold on; A=0.5; F=2; omega=2*pi*F; Gb=rho*Gs + A*sin(omega*tx);
plot(tx,Gb,'-m','LineWidth',1);
xlabel('t (jam)');
ylabel('Laju Pemberian Glukosa (g/L/h)');
ylim([9 15]);
legend('Secara Konstan (A=0)','Secara Periodik (A=0.5,
\omega=4\pi)');
hold off
figure(6)
tx=0:0.01:2;
grid on
hold on; A=0; F=0; omega=2*pi*F; Gb=rho*Gs + A*sin(omega*tx);
plot(tx,Gb,':b','LineWidth',1.5); hold off;
hold on; A=0.5; F=3; omega=2*pi*F; Gb=rho*Gs + A*sin(omega*tx);
plot(tx,Gb,'-m','LineWidth',1);
xlabel('t (jam)');
ylabel('Laju Pemberian Glukosa (g/L/h)');
ylim([9 15]);
```

```
legend('Secara Konstan (A=0)','Secara Periodik (A=0.5,
\omega=6\pi)');
hold off
%UNTUK MENAMPILKAN C(t) DAN E(t), MASING-MASING TERHADAP t
F=1;
A=0;
[t,y] = ode45(@modelmatematika,tspan,y0);
figure(7); grid on; hold on; plot(t,y(:,1),':r','LineWidth',1.5);
xlabel('t (jam)'); ylabel('Konsentrasi Sel (g/L)');
figure(8); grid on; hold on; plot(t,y(:,3),':r','LineWidth',1.5);
xlabel('t (jam)'); ylabel('Konsentrasi Etanol (g/L)');
figure(9); grid on; hold on; plot(t,y(:,1),':r','LineWidth',1.5);
xlabel('t (jam)'); ylabel('Konsentrasi Sel (g/L)');
figure(10); grid on; hold on; plot(t,y(:,3),':r','LineWidth',1.5);
xlabel('t (jam)'); ylabel('Konsentrasi Etanol (g/L)');
figure(11); grid on; hold on; plot(t,y(:,1),':r','LineWidth',1.5);
xlabel('t (jam)'); ylabel('Konsentrasi Sel (g/L)');
figure(12); grid on; hold on; plot(t,y(:,3),':r','LineWidth',1.5);
xlabel('t (jam)'); ylabel('Konsentrasi Etanol (g/L)');
fprintf('Jumlah konsentrasi sel saat A=%.1f
adalah %.4f\n',A,trapz(t,y(:,1)));
fprintf('Jumlah konsentrasi etanol saat A=%.1f
adalah %.4f\n\n',A,trapz(t,y(:,3)));
A=0.5;
[t,y] = ode45(@modelmatematika,tspan,y0);
figure(7); plot(t,y(:,1),'-b','LineWidth',0.8);
legend('A=0','A=0.5, \omega=2\pi'); hold off;
figure(8); hold on; plot(t,y(:,3),'-b','LineWidth',0.8);
legend('A=0','A=0.5, \omega=2\pi'); hold off;
fprintf('Jumlah konsentrasi sel saat A=%.1f dan omega=2pi
adalah %.4f\n',A,trapz(t,y(:,1)));
fprintf('Jumlah konsentrasi etanol saat A=%.1f dan omega=2pi
adalah %.4f\n',A,trapz(t,y(:,3)));
A=1;
[t,y] = ode45(@modelmatematika,tspan,y0);
figure(9); plot(t,y(:,1),'-b','LineWidth',0.8); legend('A=0','A=1,
\omega=2\pi'); hold off;
figure(10); hold on; plot(t,y(:,3),'-b','LineWidth',0.8);
legend('A=0','A=1, \omega=2\pi'); hold off;
fprintf('Jumlah konsentrasi sel saat A=%.1f dan omega=2pi
adalah %.4f\n',A,trapz(t,y(:,1)));
fprintf('Jumlah konsentrasi etanol saat A=%.1f dan omega=2pi
adalah %.4f\n',A,trapz(t,y(:,3)));
A=1.5;
[t,y] = ode45(@modelmatematika,tspan,y0);
figure(11); plot(t,y(:,1),'-b','LineWidth',0.8);
legend('A=0','A=1.5, \omega=2\pi'); hold off;
figure(12); hold on; plot(t,y(:,3),'-b','LineWidth',0.8);
legend('A=0','A=1.5, \omega=2\pi'); hold off;
fprintf('Jumlah konsentrasi sel saat A=%.1f dan omega=2pi
adalah %.4f\n',A,trapz(t,y(:,1)));
fprintf('Jumlah konsentrasi etanol saat A=%.1f dan omega=2pi
adalah %.4f\n',A,trapz(t,y(:,3)));
```