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LAMPIRAN

Lampiran 1: Tabel Mortalitas Social Security Area Tahun 2019

Usia	Peluang Kematian	Usia	Peluang Kematian	Usia	Peluang Kematian
0	0.006081	34	0.002067	67	0.01828
1	0.000425	35	0.002147	68	0.0195
2	0.00026	36	0.002233	69	0.020829
3	0.000194	37	0.002318	70	0.022364
4	0.000154	38	0.002399	71	0.024169
5	0.000142	39	0.002483	72	0.026249
6	0.000135	40	0.002581	73	0.028642
7	0.000127	41	0.002697	74	0.03138
8	0.000117	42	0.002828	75	0.034593
9	0.000104	43	0.002976	76	0.038235
10	0.000097	44	0.003145	77	0.042159
11	0.000106	45	0.003339	78	0.046336
12	0.000145	46	0.003566	79	0.050917
13	0.00022	47	0.003831	80	0.056205
14	0.000324	48	0.004142	81	0.062327
15	0.000437	49	0.004498	82	0.06919
16	0.000552	50	0.004888	83	0.076844
17	0.000676	51	0.005319	84	0.085407
18	0.000806	52	0.005808	85	0.09501
19	0.000939	53	0.00636	86	0.10577
20	0.001079	54	0.00697	87	0.117771
21	0.001215	55	0.007627	88	0.131063
22	0.001327	56	0.00832	89	0.145666
23	0.001406	57	0.009047	90	0.161582
24	0.001461	58	0.009803	91	0.178797
25	0.001508	59	0.010591	92	0.197287
26	0.001559	60	0.011447	93	0.217013
27	0.001612	61	0.012352	94	0.23793
28	0.001671	62	0.013248	95	0.258655
29	0.001734	63	0.014117	96	0.278786
30	0.001798	64	0.014995	97	0.297897
31	0.00186	65	0.015987	98	0.315556
32	0.001926	66	0.017107	99	0.331333
33	0.001994	67	0.01828	100	0.3479

Sumber: Situs Resmi *Social Security Administration USA* www.ssa.gov, 2019

Lampiran 2: Tabel Estimasi Parameter κ dan θ

t	$r(t)$	$r(t+1)$	$\frac{1}{r(t)}$	$\frac{r(t+1)}{r(t)}$
0	0.0550	0.0550	18.18182	1.00000
1	0.0550	0.0525	18.18182	0.95455
2	0.0525	0.0525	19.04762	1.00000
3	0.0525	0.0525	19.04762	1.00000
4	0.0525	0.0500	19.04762	0.95238
5	0.0500	0.0475	20.00000	0.95000
6	0.0475	0.0475	21.05263	1.00000
7	0.0475	0.0475	21.05263	1.00000
8	0.0475	0.0475	21.05263	1.00000
9	0.0475	0.0475	21.05263	1.00000
10	0.0475	0.0475	21.05263	1.00000
11	0.0475	0.0475	21.05263	1.00000
12	0.0475	0.0475	21.05263	1.00000
13	0.0475	0.0475	21.05263	1.00000
14	0.0475	0.0475	21.05263	1.00000
15	0.0475	0.0450	21.05263	0.94737
16	0.0450	0.0425	22.22222	0.94444
17	0.0425	0.0425	23.52941	1.00000
18	0.0425	0.0425	23.52941	1.00000
19	0.0425	0.0425	23.52941	1.00000
20	0.0425	0.0425	23.52941	1.00000
21	0.0425	0.0425	23.52941	1.00000
22	0.0425	0.0425	23.52941	1.00000
23	0.0425	0.0425	23.52941	1.00000
24	0.0425	0.0450	23.52941	1.05882
25	0.0450	0.0475	22.22222	1.05556
26	0.0475	0.0525	21.05263	1.10526
27	0.0525	0.0525	19.04762	1.00000
28	0.0525	0.0550	19.04762	1.04762
29	0.0550	0.0575	18.18182	1.04545
30	0.0575	0.0575	17.39130	1.00000
31	0.0575	0.0600	17.39130	1.04348
32	0.0600	0.0600	16.66667	1.00000
33	0.0600	0.0600	16.66667	1.00000

t	$r(t)$	$r(t+1)$	$\frac{1}{r(t)}$	$\frac{r(t+1)}{r(t)}$
34	0.0600	0.0600	16.66667	1.00000
35	0.0600	0.0600	16.66667	1.00000
36	0.0600	0.0600	16.66667	1.00000
37	0.0600	0.0600	16.66667	1.00000
38	0.0600	0.0600	16.66667	1.00000
39	0.0600	0.0575	16.66667	0.95833
40	0.0575	0.0550	17.39130	0.95652
41	0.0550	0.0525	18.18182	0.95455
42	0.0525	0.0500	19.04762	0.95238
43	0.0500	0.0500	20.00000	1.00000
44	0.0500	0.0500	20.00000	1.00000
45	0.0500	0.0500	20.00000	1.00000
46	0.0500	0.0475	20.00000	0.95000
47	0.0475	0.0450	21.05263	0.94737
48	0.0450	0.0450	22.22222	1.00000
49	0.0450	0.0450	22.22222	1.00000
50	0.0450	0.0425	22.22222	0.94444
51	0.0425	0.0400	23.52941	0.94118
52	0.0400	0.0400	25.00000	1.00000
53	0.0400	0.0400	25.00000	1.00000
54	0.0400	0.0400	25.00000	1.00000
55	0.0400	0.0375	25.00000	0.93750
56	0.0375	0.0375	26.66667	1.00000
57	0.0375	0.0375	26.66667	1.00000
58	0.0375	0.0350	26.66667	0.93333
59	0.0350	0.0350	28.57143	1.00000
60	0.0350	0.0350	28.57143	1.00000
61	0.0350	0.0350	28.57143	1.00000
62	0.0350	0.0350	28.57143	1.00000
63	0.0350	0.0350	28.57143	1.00000
64	0.0350	0.0350	28.57143	1.00000

Lampiran 3: Kekonvergenan *Actuarial Present Value* Asuransi Jiwa Seumur Hidup Waktu Kontinu

Actuarial present value asuransi jiwa seumur hidup waktu kontinu pada Persamaan (2.37),

$$\bar{A}_x = \int_0^\infty v^t \ {}_tp_x \, \mu(x+t) \ dt, \qquad x, t \geq 0,$$

adalah konvergen.

Bukti.

Menggunakan Uji Banding Langsung dengan memisalkan $f(t) = v^t \ {}_tp_x \, \mu(x+t)$. Karena $v = \frac{1}{1+r}$, $r \geq 0$, maka batas atas terkecil dari v adalah 1. Dengan memilih $g(t) = \ {}_tp_x \, \mu(x+t)$, dimana $f(t) \leq g(t)$, dan karena $g(t)$ merupakan fungsi kepadatan peluang variabel acak T, $f_T(t)$ (Persamaan 2.19) sehingga

$$\int_0^\infty g(t) \, dt = \int_0^\infty \ {}_tp_x \, \mu(x+t) \ dt = \int_0^\infty f_T(t) \, dt = 1.$$

Karena $\int_0^\infty g(t) \, dt$ konvergen, maka $\int_0^\infty f(t) \, dt = \int_0^\infty v^t \ {}_tp_x \, \mu(x+t) \ dt$ konvergen.

Lampiran 4: Kekonvergenan *Actuarial Present Value* Asuransi Jiwa Seumur Hidup Waktu Diskrit

Actuarial present value asuransi jiwa seumur hidup waktu diskrit pada Persamaan (2.38)

$$A_x = \sum_{k=0}^{\infty} v^{k+1} \, {}_kp_x \, q_{x+k},$$

adalah konvergen.

Bukti.

Menggunakan Uji Banding Langsung dengan memisalkan $f_k = v^{k+1} \, {}_kp_x \, q_{x+k}$. Karena $v = \frac{1}{1+r}$, $r \geq 0$, maka batas atas terkecil dari v adalah 1. Dengan memilih $g_k = {}_kp_x \, q_{x+k}$, dimana $f_k \leq g_k$, dan karena g_k merupakan fungsi massa peluang variabel acak diskrit K, $\Pr(K = k)$ (Persamaan 2.16) sehingga

$$\sum_{k=0}^{\infty} g_k = \sum_{k=0}^{\infty} {}_kp_x \, q_{x+k} = \sum_{k=0}^{\infty} \Pr(K = k) = 1.$$

Karena $\sum_{k=0}^{\infty} g_k$ konvergen, maka $\sum_{k=0}^{\infty} f_k = \sum_{k=0}^{\infty} v^{k+1} \, {}_kp_x \, q_{x+k}$ konvergen.

Lampiran 5: Kode Lengkap Program Python

```
import math
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt

print('PERHITUNGAN PREMI ASURANSI JIWA DWIGUNA\nMODEL STOKAST
       IK COX-INGERSOLL-ROSS\n')

# 4.2 ESTIMASI PARAMETER MODEL COX-INGERSOLL-ROSS

# 4.2.1 ESTIMASI PARAMETER kappa DAN theta

tsb = [0.055,0.055,0.0525,0.0525,0.0525,0.05,0.0475,0.0475,0.
      0475,0.0475,0.0475,0.0475,0.0475,0.0475,0.0475,0.0475,
       0.045,0.0425,0.0425,0.0425,0.0425,0.0425,0.0425,0.0425
      ,0.0425,0.045,0.0475,0.0525,0.0525,0.055,0.0575,0.0575,
       0.06,0.06,0.06,0.06,0.06,0.06,0.06,0.06,0.0575,0.055,0
      .0525,0.05,0.05,0.05,0.05,0.0475,0.045,0.045,0.045,
       0.0425,0.04,0.04,0.04,0.04,0.0375,0.0375,0.0375,0.035,
      0.035,0.035,0.035,0.035,0.035,0.035]
N = len(tsb)

ip = []
rtp = []
rt1p = []
rt2p = []
rt3p = []
srt = 0
srt1 = 0
srt2 = 0
srt3 = 0
for i in range(N-1):
  rt = tsb[i]
  rt1 = tsb[i+1]
  rt2 = 1/rt
  rt3 = rt1/rt
  SRT = srt+rt
  SRT1 = srt1 + rt1
  SRT2 = srt2 + rt2
  SRT3 = srt3 + rt3
  srt = SRT
  srt1 = SRT1
  srt2 = SRT2
  srt3 = SRT3
```

```
  ip.append(i)
  rtp.append(rt)
  rt1p.append(rt1)
  rt2p.append(rt2)
  rt3p.append(rt3)
cir_est_data = {'t':ip,'r(t)':rtp,'r(t+1)':rt1p,'1/r(t)':rt2p
                ,'r(t+1)/r(t)':rt3p}
df = pd.DataFrame(cir_est_data)
print(df)
print('Diperoleh: ','\nR1 = ',"%.9f"%SRT,'\nR2 = ',"%.9f"%SRT
      1,'\nR3 = ',"%.9f"%SRT2,'\nR4 = ',"%.9f"%SRT3)

k = (N**2-2*N+1+SRT1*SRT2-SRT*SRT2-(N-1)*SRT3)/(N**2-2*N+1-
     SRT*SRT2)
theta = ((N-1)*SRT1-SRT3*SRT)/(N**2-2*N+1+SRT1*SRT2-SRT*SRT2-
         (N-1)*SRT3)

# 4.2.2 ESTIMASI PARAMETER sigma

srt4 = 0
for i in range(N-1):
  rtt = tsb[i]
  rtt1 = tsb[i+1]
  rt4 = ((rtt1-rtt)/math.sqrt(rtt)-
         theta/math.sqrt(rtt)+k*math.sqrt(rtt))**2
  SRT4 = srt4 + rt4
  srt4 = SRT4
print('R5 = ',"%.9f"%SRT4)
sigma = math.sqrt((1/(N-2))*SRT4)

print('\nDiperoleh Estimasi Parameter:')
print('Estimasi k     =',"%.9f"%k)
print('Estimasi theta =',"%.9f"%theta)
print('Estimasi sigma =',"%.9f"%sigma,'\n')

# 4.3 SIMULASI MODEL TINGKAT SUKU BUNGA STOKASTIK COX-
INGERSOLL-ROSS

def modelCIR(t):
  f = r0+k*(theta-
      r0)+sigma*math.sqrt(r0)*np.random.normal(0,1)
  return(f)

# 4.3.1 SIMULASI PERBANDINGAN DATA AKTUAL DAN MODEL CIR
```

```
print('Simulasi Perbandingan Data Aktual dan Model Cox-
       Ingersoll-Ross')
plt.figure(figsize=(12,6))
time = range(N)
for i in range(4):
  tpoints = []
  rpoints = []
  r0 = tsb[0]
  rsum0 = 0
  for t in time:
    r = modelCIR(t)
    rsum = rsum0+r
    rsum0 = rsum
    r0=r
    tpoints.append(t)
    rpoints.append(r)
  plt.subplot(2,  2,i+1)
  plt.plot(time,tsb)
  plt.plot(tpoints,rpoints)
  plt.xlabel('Waktu (Bulan)')
  plt.ylabel('Tingkat Suku Bunga')
  plt.legend(['Data Aktual','Model CIR'],bbox_to_anchor=(1.0,
             1.0),loc='upper left')
plt.subplots_adjust(wspace=0.7,hspace=0.3)
plt.show()
print()

# 4.3.2 SIMULASI DAN PENENTUAN RATA-
RATA PERGERAKAN TINGKAT SUKU BUNGA MODEL CIR

print('Simulasi dan Penentuan Rata-
       rata Pergerakan Tingkat Suku Bunga Model CIR')
rav0 = 0
for i in range(1):
  tpoints = []
  rpoints = []
  r0 = tsb[0]
  rsum0 = 0
  for t in range(684):
    r = modelCIR(t)
    rsum = rsum0+r
    rsum0 = rsum
    r0=r
    tpoints.append(t)
    rpoints.append(r)
  rav = rav0 + rsum/684
```

```
  rav0 = rav
  plt.plot(tpoints,rpoints)
  plt.xlabel('Waktu (Bulan)')
  plt.ylabel('Tingkat Suku Bunga')
plt.legend(['Simulasi 1','Simulasi 2','Simulasi 3','Simulasi
           4'],bbox_to_anchor=(1.0, 1.0),loc='upper left')
plt.savefig('Simulasi CIR',dpi=300,bbox_inches='tight') #SIMP
            AN GAMBAR HASIL SIMULASI
plt.show()

ravv = rav
d = ravv/(1+ravv)
print('\nRata-
      rata Pergerakan Tingkat Suku Bunga CIR =',"%.4f"%ravv)
print('Tingkat Diskon =',"%.9f"%d)

# 4.5 PERHITUNGAN PREMI ASURANSI JIWA DWIGUNA

print('\n\nPERHITUNGAN PREMI ASURANSI JIWA DWIGUNA\n')
lamda = 0.00022     # parameter model Makeham
alpha = 0.0000027
beta = 0.11689

n = 57          # jangka waktu asuransi (tahun)
b = 180000000   # manfaat asuransi (rupiah)

# 4.5.1 PERHITUNGAN PELUANG KEMATIAN

def PelKematian(x):
  f = 1-math.exp(-lamda*n+(alpha/beta)*math.exp(beta*x)*(1-
      math.exp(beta*n)))
  return(f)
usia = []
peluang_kematianp = []
for x in range(61):
  peluang_kematian = PelKematian(x)
  usia.append(x)
  peluang_kematianp.append(peluang_kematian)
peluang_kematian_data = {'Usia':usia,'Peluang Kematian':pelua
                          ng_kematianp}
df1 = pd.DataFrame(peluang_kematian_data)
print(df1)

print('\nGrafik Peluang Kematian')
plt.plot(usia,peluang_kematianp)
plt.xlabel('Usia')
```

```
plt.ylabel('Peluang Kematian')
#plt.savefig('Grafik Peluang Kematian.png',dpi=300,bbox_inche
              s = 'tight')        #SIMPAN GAMBAR GRAFIK
plt.show()
print()

# 4.5.2 PERHITUNGAN PERCEPATAN KEMATIAN

def PercKematian(x):
  f = lamda+alpha*math.exp(beta*x)
  return(f)
usia = []
perc_kematianp = []
for x in range(61):
  perc_kematian = PercKematian(x)
  usia.append(x)
  perc_kematianp.append(perc_kematian)
perc_kematian_data = {'Usia':usia,'Percepatan Kematian':perc_
                      kematianp}
df2 = pd.DataFrame(perc_kematian_data)
print(df2)

print('\nGrafik Percepatan Kematian')
plt.plot(usia,perc_kematianp)
plt.xlabel('Usia')
plt.ylabel('Percepatan Kematian')
#plt.savefig('Grafik Percepatan Kematian.png',dpi=300,bbox_in
              ches = 'tight')     #SIMPAN GAMBAR GRAFIK
plt.show()
print()

# 4.5.3 PERHITUNGAN ACTUARIAL PRESENT VALUE

def APVBerjangka(x,k):
  f = apv+((1-d)**(k+1))*math.exp(-
      lamda*k+(alpha/beta)*math.exp(beta*x)*(1-
      math.exp(beta*k)))*(1-math.exp(-
      lamda+(alpha/beta)*math.exp(beta*(x+k))*(1-
      math.exp(beta))))
  return(f)
def APVDwiMurni(x):
  f = ((1-d)**n)*math.exp(-
       lamda*n+(alpha/beta)*math.exp(beta*x)*(1-
       math.exp(beta*n)))
  return(f)
usia = []
```

```
apv_dwigunap = []
for x in range(61):
  apv = 0
  for k in range(n):
    apv_berjangka = APVBerjangka(x,k)
    apv = apv_berjangka
  apv_dwimurni = APVDwiMurni(x)
  apv_dwiguna = apv_berjangka+apv_dwimurni
  usia.append(x)
  apv_dwigunap.append(apv_dwiguna)
apv_dwiguna_data = {'Usia':usia,'Actuarial Present Value':apv
                    _dwigunap}
df3 = pd.DataFrame(apv_dwiguna_data)
print(df3)

print('\nGrafik APV Dwiguna')
plt.plot(usia,apv_dwigunap)
plt.xlabel('Usia')
plt.ylabel('Actuarial Present Value')
#plt.savefig('Grafik Actuarial Present Value Benefit.png',dpi
            =300,bbox_inches = 'tight') #SIMPAN GAMBAR GRAFIK
plt.show()
print()

# 4.5.4 PERHITUNGAN ANUITAS HIDUP

def Anuitas(x,k):
  f = anuitas+((1-d)**k)*math.exp(-
      lamda*k+(alpha/beta)*math.exp(beta*x)*(1-
      math.exp(beta*k)))
  return(f)
usia = []
ANUITASp = []
for x in range(61):
  anuitas = 0
  for k in range(n):
    ANUITAS = Anuitas(x,k)
    anuitas = ANUITAS
  usia.append(x)
  ANUITASp.append(ANUITAS)
ANUITAS_data = {'Usia':usia,'Anuitas Hidup':ANUITASp}
df4 = pd.DataFrame(ANUITAS_data)
print(df4)
print('\nGrafik Anuitas Hidup')
plt.plot(usia,ANUITASp)
plt.xlabel('Usia')
```

```
plt.ylabel('Anuitas')
#plt.savefig('Grafik Anuitas Hidup.png',dpi=300,bbox_inches =
             'tight')           #SIMPAN GAMBAR GRAFIK
plt.show()
print()

# 4.5.5 PERHITUNGAN PREMI

usia = []
apv_dwigunap = []
ANUITASp = []
premip = []
for x in range(61):
  apv = 0
  anuitas = 0
  for k in range(n):
    apv_berjangka = APVBerjangka(x,k)
    apv = apv_berjangka
    ANUITAS = Anuitas(x,k)
    anuitas = ANUITAS
  apv_dwimurni = APVDwiMurni(x)
  apv_dwiguna = apv_berjangka+apv_dwimurni
  premi = (b*apv_berjangka)/(ANUITAS - n*apv_dwimurni)
  usia.append(x)
  apv_dwigunap.append(apv_dwiguna)
  ANUITASp.append(ANUITAS)
  premip.append(premi)
premi_data = {'Usia':usia,'Premi Tahunan':premip}
df5 = pd.DataFrame(premi_data)
print(df5)
print('\nGrafik Premi Tahunan')
plt.plot(usia,premip)
plt.xlabel('Usia')
plt.ylabel('Premi')
#plt.savefig('Grafik Premi Tahunan.png',dpi=300,bbox_inches =
             'tight')           #SIMPAN GAMBAR GRAFIK
plt.show()
print()

# SIMPAN TABEL
#df1.to_excel (r'Peluang Kematian.xlsx', index = False, heade
              r=True)
#df2.to_excel (r'Percepatan Kematian.xlsx', index = False, he
              ader=True)
#df3.to_excel (r'APV.xlsx', index = False, header=True)
```

```
#df4.to_excel (r'Anuitas Hidup.xlsx', index = False, header=T
               rue)
#df5.to_excel (r'Premi.xlsx', index = False, header=True)
```