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LAMPIRAN

Lampiran 1. Sintaks solusi numerik persamaan gerak sistem pegas terkopel menggunakan aplikasi Scilab

```
clear
clc
close
// Sebelum eksekusi lihat Waspada, x1 dan x2
m1=1;
m2=1;
k1=1;
k2=2;

a=m1*m2;
b=m2*k1+k2*(m1+m2);
c=k1*k2;

dt=0.05;
Ti=0;
Tf=10;

akar0=sqrt(-(k1/2)-k2+(0.5*sqrt(k1^2+4*k2*k2)))
akar1=sqrt(akar0*akar0*-1)

// x1
// RK x1
x11(1,1)=2;
x12(1,1)=2;
x13(1,1)=2;
x14(1,1)=2;
z111(1,1)=0;
z112(1,1)=0;
z113(1,1)=0;
z114(1,1)=0;
z121(1,1)=-2*(akar1^2);
z122(1,1)=-2*(akar1^2);
z123(1,1)=-2*(akar1^2);
z124(1,1)=-2*(akar1^2);
z131(1,1)=0;
z132(1,1)=0;
z133(1,1)=0;
z134(1,1)=0;

nt=1+(Tf-Ti)/dt;

for mt=1:1:nt
    T(mt,1)=Ti+(mt-1)*dt;
end

// Untuk x1 Euler
for mt=2:1:nt
    fx11(mt,1)=z111(mt-1,1);
    fz111(mt,1)=z121(mt-1,1);
    fz121(mt,1)=z131(mt-1,1);
    fz131(mt,1)=-((b*z121(mt-1,1)+c*x11(mt-1,1))/a);
    x11(mt,1)=x11(mt-1,1)+(fx11(mt,1)*dt);
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z111(mt,1)=z111(mt-1,1)+(fz111(mt,1)*dt);
z121(mt,1)=z121(mt-1,1)+(fz121(mt,1)*dt);
z131(mt,1)=z131(mt-1,1)+(fz131(mt,1)*dt);
end

// untuk x1 RK 2
for mt=2:1:nt
    flx12(mt,1)=z112(mt-1,1);
    flz112(mt,1)=z122(mt-1,1);
    flz122(mt,1)=z132(mt-1,1);
    flz132(mt,1)=-(b*z122(mt-1,1)+c*x12(mt-1,1))/a;
    f2x12(mt,1)=z112(mt-1,1)+dt*flz112(mt-1,1);
    f2z112(mt,1)=z122(mt-1,1)+dt*flz122(mt-1,1);
    f2z122(mt,1)=z132(mt-1,1)+dt*flz132(mt-1,1);
    f2z132(mt,1)=-(b*(z122(mt-1,1)+dt*flz122(mt-1,1))+c*(x12(mt-1,1)+dt*flx12(mt-1,1)))/a;
    x12(mt,1)=x12(mt-1,1)+(dt*(flx12(mt,1)+f2x12(mt,1)))/2;
    z112(mt,1)=z112(mt-1,1)+(dt*(flz112(mt,1)+f2z112(mt,1)))/2;
    z122(mt,1)=z122(mt-1,1)+(dt*(flz122(mt,1)+f2z122(mt,1)))/2;
    z132(mt,1)=z132(mt-1,1)+(dt*(flz132(mt,1)+f2z132(mt,1)))/2;
end

// untuk x1 RK 3
for mt=2:1:nt
    flx13(mt,1)=z113(mt-1,1);
    flz113(mt,1)=z123(mt-1,1);
    flz123(mt,1)=z133(mt-1,1);
    flz133(mt,1)=-(b*z123(mt-1,1)+c*x13(mt-1,1))/a;
    f2x13(mt,1)=z113(mt-1,1)+dt*flz113(mt-1,1)/2;
    f2z113(mt,1)=z123(mt-1,1)+dt*flz123(mt-1,1)/2;
    f2z123(mt,1)=z133(mt-1,1)+dt*flz133(mt-1,1)/2;
    f2z133(mt,1)=-(b*(z123(mt-1,1)+dt*flz123(mt-1,1)/2)+c*(x13(mt-1,1)+dt*flx13(mt-1,1)/2))/a;
    f3x13(mt,1)=z113(mt-1,1)-flz113(mt-1,1)*dt+2*f2z113(mt-1,1)*dt;
    f3z113(mt,1)=z123(mt-1,1)-flz123(mt-1,1)*dt+2*f2z123(mt-1,1)*dt;
    f3z123(mt,1)=z133(mt-1,1)-flz133(mt-1,1)*dt+2*f2z133(mt-1,1)*dt;
    f3z133(mt,1)=-(b*(z123(mt-1,1)-flz123(mt-1,1)*dt+2*f2z123(mt-1,1)*dt)+c*(x13(mt-1,1)-flx13(mt-1,1)*dt+2*f2x13(mt-1,1)*dt))/a;
    x13(mt,1)=x13(mt-1,1)+(flx13(mt,1)+4*f2x13(mt,1)+f3x13(mt,1))*dt/6;
    z113(mt,1)=z113(mt-1,1)+(flz113(mt,1)+4*f2z113(mt,1)+f3z113(mt,1))*dt/6;
    z123(mt,1)=z123(mt-1,1)+(flz123(mt,1)+4*f2z123(mt,1)+f3z123(mt,1))*dt/6;
    z133(mt,1)=z133(mt-1,1)+(flz133(mt,1)+4*f2z133(mt,1)+f3z133(mt,1))*dt/6;
end

// Untuk x1 RK 4
for mt=2:1:nt
    flx14(mt,1)=z114(mt-1,1);
    flz114(mt,1)=z124(mt-1,1);
    flz124(mt,1)=z134(mt-1,1);
    flz134(mt,1)=-(b*z124(mt-1,1)+c*x14(mt-1,1))/a;
    f2x14(mt,1)=z114(mt-1,1)+dt*flz114(mt-1,1)/2;
    f2z114(mt,1)=z124(mt-1,1)+dt*flz124(mt-1,1)/2;
    f2z124(mt,1)=z134(mt-1,1)+dt*flz134(mt-1,1)/2;
    f2z134(mt,1)=-(b*(z124(mt-1,1)+dt*flz124(mt-1,1)/2)+c*(x14(mt-1,1)+dt*flx14(mt-1,1)/2))/a;
    f3x14(mt,1)=z114(mt-1,1)+dt*f2z114(mt-1,1)/2;
    f3z114(mt,1)=z124(mt-1,1)+dt*f2z124(mt-1,1)/2;

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f3z124(mt,1)=z134(mt-1,1)+dt*f2z134(mt-1,1)/2;
f3z134(mt,1)=-(b*(z124(mt-1,1)+dt*f2z124(mt-1,1)/2)+c*(x14(mt-1,1)+dt*f2x14(mt-
1,1)/2))/a;
f4x14(mt,1)=z114(mt-1,1)+dt*f3z114(mt-1,1);
f4z114(mt,1)=z124(mt-1,1)+dt*f3z124(mt-1,1);
f4z124(mt,1)=z134(mt-1,1)+dt*f3z134(mt-1,1);
f4z134(mt,1)=-(b*(z124(mt-1,1)+dt*f3z124(mt-1,1))+c*(x14(mt-1,1)+dt*f3x14(mt-1,1)))/a;
x14(mt,1)=x14(mt-1,1)+(f1x14(mt,1)+2*f2x14(mt,1)+2*f3x14(mt,1)+f4x14(mt,1))*dt/6;
z114(mt,1)=z114(mt-
1,1)+(f1z114(mt,1)+2*f2z114(mt,1)+2*f3z114(mt,1)+f4z114(mt,1))*dt/6;
z124(mt,1)=z124(mt-
1,1)+(f1z124(mt,1)+2*f2z124(mt,1)+2*f3z124(mt,1)+f4z124(mt,1))*dt/6;
z134(mt,1)=z134(mt-
1,1)+(f1z134(mt,1)+2*f2z134(mt,1)+2*f3z134(mt,1)+f4z134(mt,1))*dt/6;
end

// Untuk analitik x1
for mt=1:1:nt
    xa1(mt,1)=2*cos(akar1*T(mt,1)); //Waspada sin atau cos
end

Err11=abs((x11-xa1)./xa1);
Err12=abs((x12-xa1)./xa1);
Err13=abs((x13-xa1)./xa1);
Err14=abs((x14-xa1)./xa1);

for mx=1:1:nt-1
SE11(mx,1)=Err11(mx+1,1)-Err11(mx,1);
SE12(mx,1)=Err12(mx+1,1)-Err12(mx,1);
SE13(mx,1)=Err13(mx+1,1)-Err13(mx,1);
SE14(mx,1)=Err14(mx+1,1)-Err14(mx,1);
end

// x2
// RK x2
x21(1,1)=0;
x22(1,1)=0;
x23(1,1)=0;
x24(1,1)=0;
z211(1,1)=2*(akar1);
z212(1,1)=2*(akar1);
z213(1,1)=2*(akar1);
z214(1,1)=2*(akar1);
z221(1,1)=0;
z222(1,1)=0;
z223(1,1)=0;
z224(1,1)=0;
z231(1,1)=-2*(akar1^3);
z232(1,1)=-2*(akar1^3);
z233(1,1)=-2*(akar1^3);
z234(1,1)=-2*(akar1^3);

// Untuk x2 Euler
for mt=2:1:nt
    fx21(mt,1)=z211(mt-1,1);
    fz211(mt,1)=z221(mt-1,1);

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fz221(mt,1)=z231(mt-1,1);
fz231(mt,1)=-(b*z221(mt-1,1)+c*x21(mt-1,1))/a;
x21(mt,1)=x21(mt-1,1)+(fx21(mt,1)*dt);
z211(mt,1)=z211(mt-1,1)+(fz211(mt,1)*dt);
z221(mt,1)=z221(mt-1,1)+(fz221(mt,1)*dt);
z231(mt,1)=z231(mt-1,1)+(fz231(mt,1)*dt);
end

// untuk x2 RK 2
for mt=2:1:nt
    flx22(mt,1)=z212(mt-1,1);
    flz212(mt,1)=z222(mt-1,1);
    flz222(mt,1)=z232(mt-1,1);
    flz232(mt,1)=-(b*z222(mt-1,1)+c*x22(mt-1,1))/a;
    f2x22(mt,1)=z212(mt-1,1)+dt*flz212(mt-1,1);
    f2z212(mt,1)=z222(mt-1,1)+dt*flz222(mt-1,1);
    f2z222(mt,1)=z232(mt-1,1)+dt*flz232(mt-1,1);
    f2z232(mt,1)=-(b*(z222(mt-1,1)+dt*flz222(mt-1,1))+c*(x22(mt-1,1)+dt*flx22(mt-1,1)))/a;
    x22(mt,1)=x22(mt-1,1)+(dt*(flx22(mt,1)+f2x22(mt,1)))/2;
    z212(mt,1)=z212(mt-1,1)+(dt*(flz212(mt,1)+f2z212(mt,1)))/2;
    z222(mt,1)=z222(mt-1,1)+(dt*(flz222(mt,1)+f2z222(mt,1)))/2;
    z232(mt,1)=z232(mt-1,1)+(dt*(flz232(mt,1)+f2z232(mt,1)))/2;
end

// untuk x2 RK 3
for mt=2:1:nt
    flx23(mt,1)=z213(mt-1,1);
    flz213(mt,1)=z223(mt-1,1);
    flz223(mt,1)=z233(mt-1,1);
    flz233(mt,1)=-(b*z223(mt-1,1)+c*x23(mt-1,1))/a;
    f2x23(mt,1)=z213(mt-1,1)+dt*flz213(mt-1,1)/2;
    f2z213(mt,1)=z223(mt-1,1)+dt*flz223(mt-1,1)/2;
    f2z223(mt,1)=z233(mt-1,1)+dt*flz233(mt-1,1)/2;
    f2z233(mt,1)=-(b*(z223(mt-1,1)+dt*flz223(mt-1,1)/2)+c*(x23(mt-1,1)+dt*flx23(mt-1,1)/2))/a;
    f3x23(mt,1)=z213(mt-1,1)-flz213(mt-1,1)*dt+2*f2z213(mt-1,1)*dt;
    f3z213(mt,1)=z223(mt-1,1)-flz223(mt-1,1)*dt+2*f2z223(mt-1,1)*dt;
    f3z223(mt,1)=z233(mt-1,1)-flz233(mt-1,1)*dt+2*f2z233(mt-1,1)*dt;
    f3z233(mt,1)=-(b*(z223(mt-1,1)-flz223(mt-1,1)*dt+2*f2z223(mt-1,1)*dt)+c*(x23(mt-1,1)-flx23(mt-1,1)*dt+2*f2x23(mt-1,1)*dt))/a;
    x23(mt,1)=x23(mt-1,1)+(flx23(mt,1)+4*f2x23(mt,1)+f3x23(mt,1))*dt/6;
    z213(mt,1)=z213(mt-1,1)+(flz213(mt,1)+4*f2z213(mt,1)+f3z213(mt,1))*dt/6;
    z223(mt,1)=z223(mt-1,1)+(flz223(mt,1)+4*f2z223(mt,1)+f3z223(mt,1))*dt/6;
    z233(mt,1)=z233(mt-1,1)+(flz233(mt,1)+4*f2z233(mt,1)+f3z233(mt,1))*dt/6;
end

// Untuk x2 RK 4
for mt=2:1:nt
    flx24(mt,1)=z214(mt-1,1);
    flz214(mt,1)=z224(mt-1,1);
    flz224(mt,1)=z234(mt-1,1);
    flz234(mt,1)=-(b*z224(mt-1,1)+c*x24(mt-1,1))/a;
    f2x24(mt,1)=z214(mt-1,1)+dt*flz214(mt-1,1)/2;
    f2z214(mt,1)=z224(mt-1,1)+dt*flz224(mt-1,1)/2;
    f2z224(mt,1)=z234(mt-1,1)+dt*flz234(mt-1,1)/2;

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f2z234(mt,1)=-(b*(z224(mt-1,1)+dt*f1z224(mt-1,1)/2)+c*(x24(mt-1,1)+dt*f1x24(mt-
1,1)/2))/a;
f3x24(mt,1)=z214(mt-1,1)+dt*f2z214(mt-1,1)/2;
f3z214(mt,1)=z224(mt-1,1)+dt*f2z224(mt-1,1)/2;
f3z224(mt,1)=z234(mt-1,1)+dt*f2z234(mt-1,1)/2;
f3z234(mt,1)=-(b*(z224(mt-1,1)+dt*f2z224(mt-1,1)/2)+c*(x24(mt-1,1)+dt*f2x24(mt-
1,1)/2))/a;
f4x24(mt,1)=z214(mt-1,1)+dt*f3z214(mt-1,1);
f4z214(mt,1)=z224(mt-1,1)+dt*f3z224(mt-1,1);
f4z224(mt,1)=z234(mt-1,1)+dt*f3z234(mt-1,1);
f4z234(mt,1)=-(b*(z224(mt-1,1)+dt*f3z224(mt-1,1))+c*(x24(mt-1,1)+dt*f3x24(mt-1,1)))/a;
x24(mt,1)=x24(mt-1,1)+(f1x24(mt,1)+2*f2x24(mt,1)+2*f3x24(mt,1)+f4x24(mt,1))*dt/6;
z214(mt,1)=z214(mt-
1,1)+(f1z214(mt,1)+2*f2z214(mt,1)+2*f3z214(mt,1)+f4z214(mt,1))*dt/6;
z224(mt,1)=z224(mt-
1,1)+(f1z224(mt,1)+2*f2z224(mt,1)+2*f3z224(mt,1)+f4z224(mt,1))*dt/6;
z234(mt,1)=z234(mt-
1,1)+(f1z234(mt,1)+2*f2z234(mt,1)+2*f3z234(mt,1)+f4z234(mt,1))*dt/6;
end

// untuk analitik
for mt=1:1:nt
    xa2(mt,1)=2*sin(akar1*T(mt,1)); // Waspada sin atau cos
end

Err21=abs((x21-xa2)./xa2);
Err22=abs((x22-xa2)./xa2);
Err23=abs((x23-xa2)./xa2);
Err24=abs((x24-xa2)./xa2);

for mx=1:1:nt-1
SE21(mx,1)=Err21(mx+1,1)-Err21(mx,1);
SE22(mx,1)=Err22(mx+1,1)-Err22(mx,1);
SE23(mx,1)=Err23(mx+1,1)-Err23(mx,1);
SE24(mx,1)=Err24(mx+1,1)-Err24(mx,1);
end

// GAMBAR
plot(T,xa1,T,xa2)
title("Grafik x1 analitik dan x2 analitik")
xlabel("T")
ylabel("x1 dan x2")
h1=legend(['x1 Analitik','x2 Analitik'],pos=4)
figure

subplot(2,2,1)
plot (T,x11,'r',T,x12,'c',T,x13,'m',T,x14,'g',T,xa1,'k');
title("Grafik Perbandingan Solusi Numerik untuk x1")
xlabel("T")
ylabel("x1")
h1=legend(['Euler','RK 2','RK 3','RK 4','Analitik'],pos=4)

subplot(2,2,2)
plot (T,x21,'r',T,x22,'c',T,x23,'m',T,x24,'g',T,xa2,'k');
title("Grafik Perbandingan Solusi Numerik untuk x2")
xlabel("T")

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ylabel("x2")
h1=legend(['Euler','RK 2','RK 3','RK 4'],'Analitik',pos=4)

subplot(2,2,3)
plot (T,Err11,'r',T,Err12,'c',T,Err13,'m',T,Err14,'g');
title("Grafik Perbandingan Kesalahan Numerik untuk x1")
xlabel("T")
ylabel("Kesalahan x1")
h1=legend(['Euler','RK 2','RK 3','RK 4'],pos=2)

subplot(2,2,4)
plot (T,Err21,'r',T,Err22,'c',T,Err23,'m',T,Err24,'g');
title("Grafik Perbandingan Kesalahan Numerik untuk x2")
xlabel("T")
ylabel("Kesalahan x2")
h1=legend(['Euler','RK 2','RK 3','RK 4'],pos=2)

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