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LAMPIRAN

Lampiran 1. Titik ekuilibrium non endemik dan endemik model penyebaran penyakit meningitis.

> restart :

> with(DEtools) :

> with(linalg) : with(VectorCalculus) :

>

$\beta := 0.2 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda$
 $:= 0.00005 : \epsilon := 0.3 : N := 1 : \tau_2 := 0.05 : \tau_1 := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N} \right) - \mu \cdot S :$

> $P2 := \left(\frac{\beta \cdot \tau_1 \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$

> $P3 := \left(\frac{\beta \cdot (1 - \tau_1) \cdot Is}{N} \right) \cdot S - (\mu + \omega) \cdot Ia :$

> $P4 := Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$

> $P5 := \epsilon \cdot C + \chi \cdot \tau_2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$

> $P6 := \chi \cdot (1 - \tau_2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$

> $T := \text{solve}(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, \{S, C, Ia, Is, R1, R2\}) :$

Lampiran 2. Analisis Sensitivitas menggunakan software Maple

> restart :

> with(linalg) :

>

$$R0 := (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon));$$

$$R0 := (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon))$$

> #Untuk Parameter β

> P1 := diff(R0, β)

$$P1 := (\Lambda (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon))$$

$$> P11 := P1 \cdot \frac{\beta}{R0}$$

$$P11 := 1$$

> #Untuk Parameter τl

> P2 := diff(R0, τl)

$$P2 := (\Lambda \beta (\delta \mu - \mu \omega - \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon))$$

$$> P22 := P2 \cdot \frac{\tau l}{R0}$$

$$P22 := \frac{(\delta \mu - \mu \omega - \omega \epsilon) \tau l}{\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon}$$

> #Untuk Parameter Λ

> P3 := diff(R0, Λ)

$$P3 := (\beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon))$$

$$> P33 := P3 \cdot \frac{\Lambda}{R0}$$

$$P33 := 1$$

> #Untuk Parameter χ

> $P4 := \text{diff}(R0, \chi)$

$$P4 := -(\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon) (\delta \mu + \delta \omega + \mu^2 + \mu \omega + \mu \epsilon + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2)$$

> $P44 := P4 \cdot \frac{\chi}{R0}$

$$P44 := -((\delta \mu + \delta \omega + \mu^2 + \mu \omega + \mu \epsilon + \omega \epsilon) \chi) / (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)$$

>

> #Untuk Parameter ω

> $P5 := \text{diff}(R0, \omega)$

$$P5 := (\Lambda \beta (-\mu \tau l - \tau l \epsilon + \delta + \mu + \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) - (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon) (\alpha \delta + \alpha \mu + \alpha \epsilon + \chi \delta + \chi \mu + \chi \epsilon + \delta \mu + \mu^2 + \mu \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2)$$

> $P55 := P5 \cdot \frac{\omega}{R0}$

$$\begin{aligned}
P55 := & \left(\left((\Lambda\beta(-\mu\tau l - \tau l\epsilon + \delta + \mu + \epsilon)) / (\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)) \right) - (\Lambda\beta(\delta\mu\tau l - \mu\omega\tau l - \omega\tau l\epsilon + \delta\omega + \mu\omega + \omega\epsilon)(\alpha\delta + \alpha\mu + \alpha\epsilon + \chi\delta + \chi\mu + \chi\epsilon + \delta\mu + \mu^2 + \mu\epsilon)) / (\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)^2) \right) \omega\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon) \Big/ \\
& (\Lambda\beta(\delta\mu\tau l - \mu\omega\tau l - \omega\tau l\epsilon + \delta\omega + \mu\omega + \omega\epsilon))
\end{aligned}$$

> #Untuk Parameter δ

> $P6 := \text{diff}(R0, \delta)$

$$\begin{aligned}
P6 := & (\Lambda\beta(\mu\tau l + \omega)) / (\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)) \\
& + (\chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)^2)
\end{aligned}$$

> $P66 := P6 \cdot \frac{\delta}{R0}$

$$\begin{aligned}
P66 := & \left(\left((\Lambda\beta(\mu\tau l + \omega)) / (\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)) \right) \right. \\
& \left. + (\chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)^2) \right) \delta\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon) \Big/ \\
& (\Lambda\beta(\delta\mu\tau l - \mu\omega\tau l - \omega\tau l\epsilon + \delta\omega + \mu\omega + \omega\epsilon))
\end{aligned}$$

> #Untuk Parameter ω

> $P7 := \text{diff}(R0, \omega)$

$$P7 := (\Lambda \beta (-\mu \tau l - \tau l \epsilon + \delta + \mu + \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) - (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon) (\alpha \delta + \alpha \mu + \alpha \epsilon + \chi \delta + \chi \mu + \chi \epsilon + \delta \mu + \mu^2 + \mu \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2)$$

$$> P77 := P7 \cdot \frac{\omega}{R0}$$

$$P77 := \left(\left((\Lambda \beta (-\mu \tau l - \tau l \epsilon + \delta + \mu + \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) - (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon) (\alpha \delta + \alpha \mu + \alpha \epsilon + \chi \delta + \chi \mu + \chi \epsilon + \delta \mu + \mu^2 + \mu \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2) \right) \omega \mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon) \right) / (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon))$$

> #Untuk Parameter ϵ

> $P8 := \text{diff}(R0, \epsilon)$

$$P8 := (\Lambda \beta (-\omega \tau l + \omega)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) - (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon) (\alpha \mu + \alpha \omega + \chi \mu + \chi \omega + \mu^2 + \mu \omega)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)^2)$$

$$> P88 := P8 \cdot \frac{\epsilon}{R0}$$

$$\begin{aligned}
P88 := & \left(\left((\Lambda \beta (-\omega \tau l + \omega)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \right. \right. \\
& \left. \left. + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) \right) \right. \\
& \left. - \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon \right)^2 \Big) \epsilon \mu (\alpha \delta \mu \\
& + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon \\
& + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon) \Big) / (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega \\
& + \mu \omega + \omega \epsilon))
\end{aligned}$$

> #Untuk Parameter μ

> $P9 := \text{diff}(R0, \mu)$

$$\begin{aligned}
P9 := & (\Lambda \beta (\delta \tau l - \omega \tau l + \omega)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu \\
& + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) \\
& - (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu^2 (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 \\
& + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega \\
& + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) - (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega \\
& + \omega \epsilon) (\alpha \delta + 2\alpha \mu + \alpha \omega + \alpha \epsilon + \chi \delta + 2\chi \mu + \chi \omega + \chi \epsilon + 2\delta \mu + \delta \omega + 3\mu^2 \\
& + 2\mu \omega + 2\mu \epsilon + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon \\
& + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon \\
& + \mu \omega \epsilon)^2)
\end{aligned}$$

> $P99 := P9 \cdot \frac{\mu}{R0}$

$$\begin{aligned}
P99 := & \left(\left((\Lambda\beta(\delta\tau l - \omega\tau l + \omega)) / (\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)) \right. \right. \\
& - (\Lambda\beta(\delta\mu\tau l - \mu\omega\tau l - \omega\tau l\epsilon + \delta\omega + \mu\omega + \omega\epsilon)) / (\mu^2(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega \\
& + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)) - (\Lambda\beta(\delta\mu\tau l - \mu\omega\tau l - \omega\tau l\epsilon + \delta\omega + \mu\omega + \omega\epsilon) (\alpha\delta + 2\alpha\mu + \alpha\omega + \alpha\epsilon + \chi\delta + 2\chi\mu + \chi\omega + \chi\epsilon + 2\delta\mu + \delta\omega + 3\mu^2 \\
& + 2\mu\omega + 2\mu\epsilon + \omega\epsilon)) / (\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon \\
& + \mu\omega\epsilon)^2) \mu^2 (\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon) \Big) / (\Lambda\beta(\delta\mu\tau l \\
& - \mu\omega\tau l - \omega\tau l\epsilon + \delta\omega + \mu\omega + \omega\epsilon))
\end{aligned}$$

> #Untuk Parameter α

> $P10 := \text{diff}(R0, \alpha)$

$$\begin{aligned}
P10 := & -(\Lambda\beta(\delta\mu\tau l - \mu\omega\tau l - \omega\tau l\epsilon + \delta\omega + \mu\omega + \omega\epsilon) (\delta\mu + \delta\omega + \mu^2 + \mu\omega + \mu\epsilon + \omega\epsilon)) / (\mu(\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 \\
& + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega + \mu^2\epsilon + \mu\omega\epsilon)^2)
\end{aligned}$$

> $P110 := P10 \cdot \frac{\alpha}{R0}$

$$\begin{aligned}
P110 := & -((\delta\mu + \delta\omega + \mu^2 + \mu\omega + \mu\epsilon + \omega\epsilon) \alpha) / (\alpha\delta\mu + \alpha\delta\omega + \alpha\mu^2 + \alpha\mu\omega + \alpha\mu\epsilon + \alpha\omega\epsilon + \chi\delta\mu + \chi\delta\omega + \chi\mu^2 + \chi\mu\omega + \chi\mu\epsilon + \chi\omega\epsilon + \delta\mu^2 + \delta\mu\omega + \mu^3 + \mu^2\omega \\
& + \mu^2\epsilon + \mu\omega\epsilon)
\end{aligned}$$

Lampiran 3. Penentuan persamaan bifurkasi

> restart : with(plots) :

>

$\omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 :$
 $N := 1 : \tau 2 := 0.05 : \tau l := 0.8 : \sigma := 0.15 :$

> restart;

> $a := \mu + \sigma + \psi :$

> $b := \mu + \psi :$

> $d := \mu + \epsilon + \delta :$

> $f := \mu + \omega :$

> $g := \mu + \alpha + \chi :$

> $h := \tau l \cdot \beta :$

> $C := \frac{(h \cdot a \cdot b \cdot \Lambda \cdot X + h \cdot \psi \cdot X^2 \cdot (\tau 2 \cdot b \cdot \chi + (1 - \tau 2) \cdot a \cdot \chi + \sigma \cdot \tau 2 \cdot \chi))}{a \cdot b \cdot d \cdot \mu + X \cdot (a \cdot b \cdot d \cdot \beta - b \cdot h \cdot \psi \cdot \epsilon - h \cdot \psi \cdot \sigma \cdot \epsilon)} :$

> $S := \frac{C \cdot d}{h \cdot X} :$

> $Ia := \frac{\beta \cdot (1 - \tau l) \cdot S \cdot X}{f} :$

> $K := \omega \cdot Ia + \delta \cdot C - (\mu + \alpha + \chi) \cdot X = 0 :$

> $J := \text{numer}(\text{lhs}(K)) \cdot \text{denom}(\text{rhs}(K)) = \text{numer}(\text{rhs}(K)) \cdot \text{denom}(\text{lhs}(K)) :$

> $K := \text{lhs}\left(\frac{J}{X}\right) :$

> $K1 := \text{lhs}(J) :$

> implicitplot($K1 = 0, \beta = 0..1, X = 0..0.1$) :

> $y := \text{coeff}(K1, X) :$

> $n := \text{coeff}(K1, X, 0) :$

> $u := \text{coeff}(K1, X^2) :$

>

$R0 := (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2$
 $+ \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega$
 $+ \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$

> $M1 := \text{solve}(R0 - 1 = 0, \beta) :$

Lampiran 4. Sintax simulasi bifurkasi dengan nilai parameter β

$$(\beta = 0.2)$$

```

> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
 $\beta := 0.2 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda$ 
 $:= 0.00005 : \epsilon := 0.3 : N := 1 : \tau_2 := 0.05 : \tau_1 := 0.8 : \sigma := 0.15 :$ 

> P1 :=  $\Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left( \frac{Is}{N} \right) - \mu \cdot S :$ 
> P2 :=  $\left( \frac{\beta \cdot \tau_1 \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$ 
> P3 :=  $\left( \frac{\beta \cdot (1 - \tau_1) \cdot Is}{N} \right) \cdot S - (\mu + \omega) \cdot Ia :$ 
> P4 :=  $Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$ 
> P5 :=  $\epsilon \cdot C + \chi \cdot \tau_2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$ 
> P6 :=  $\chi \cdot (1 - \tau_2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$ 
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, Ia, Is, R1, R2}) :
>
R0 :=  $(\Lambda \beta (\delta \mu \tau_1 - \mu \omega \tau_1 - \omega \tau_1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2$ 
 $+ \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega$ 
 $+ \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$ 
> with(linalg) : with(VectorCalculus) :
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);
-0.00002000000000000000, -0.5000200000000000, -0.6500200000000000,
-0.986197492134063, -0.337733651369526, -0.126128856496411

> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);
-1.15973255775372, 0.00158087934783999, -0.00157849745389581, -0.290319408284038,
-0.500027083163507, -0.6500200000000000

```

$$(\beta = 0.3)$$

```

> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :

```

>

$\beta := 0.3 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda$
 $:= 0.00005 : \epsilon := 0.3 : N := 1 : \tau_2 := 0.05 : \tau_1 := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N} \right) - \mu \cdot S :$

> $P2 := \left(\frac{\beta \cdot \tau_1 \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$

> $P3 := \left(\frac{\beta \cdot (1 - \tau_1) \cdot Is}{N} \right) \cdot S - (\mu + \omega) \cdot Ia :$

> $P4 := Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$

> $P5 := \epsilon \cdot C + \chi \cdot \tau_2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$

> $P6 := \chi \cdot (1 - \tau_2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$

> $T := \text{solve}(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, \{S, C, Ia, Is, R1, R2\}) :$

>

$R0 := (\Lambda \beta (\delta \mu \tau_1 - \mu \omega \tau_1 - \omega \tau_1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2$
 $+ \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega$
 $+ \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$

> *with(linalg) : with(VectorCalculus) :*

> $T1 := T[1] : T2 := T[2] :$

> $\text{jac} := \text{Jacobian}([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :$

> $\text{jac1} := \text{subs}(T1, \text{evalm}(\text{jac})) : \text{eigenvalues}(\text{jac1}) ;$

-0.00002000000000000000, -0.5000200000000000, -0.6500200000000000,
-1.06865294393117, -0.308608607267027, -0.0727984488018024

> $\text{jac2} := \text{subs}(T2, \text{evalm}(\text{jac})) : \text{eigenvalues}(\text{jac2}) ;$

-1.15973438367225, 0.00119060401712791, -0.00119781283223920, -0.290321024853564,
-0.500024048088934, -0.6500200000000000

($\beta = 0.4$)

> *restart :*

> *with(DEtools) :*

> *with(linalg) : with(VectorCalculus) :*

>

$\beta := 0.4 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda$
 $:= 0.00005 : \epsilon := 0.3 : N := 1 : \tau_2 := 0.05 : \tau_1 := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N} \right) - \mu \cdot S :$

> $P2 := \left(\frac{\beta \cdot \tau_1 \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$

```

> P3 := ( (beta * (1 - tau1) * Is) / N ) * S - (mu + omega) * Ia :
> P4 := Ia * omega + delta * C - (mu + alpha + chi) * Is :
> P5 := epsilon * C + chi * tau2 * Is - (mu + psi + sigma) * R1 :
> P6 := chi * (1 - tau2) * Is + sigma * R1 - (mu + psi) * R2 :
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, Ia, Is, R1, R2}) :
>
R0 := (Lambda * beta * (delta * mu * tau1 - mu * omega * tau1 - omega * tau1 * epsilon + delta * omega + mu * omega + omega * epsilon) / (mu * (alpha * delta * mu + alpha * delta * omega + alpha * mu^2 + alpha * mu * omega + alpha * mu * epsilon + alpha * omega * epsilon + chi * delta * mu + chi * delta * omega + chi * mu^2 + chi * mu * omega + chi * mu * epsilon + chi * omega * epsilon + delta * mu^2 + delta * mu * omega + mu^3 + mu^2 * omega + mu^2 * epsilon + mu * omega * epsilon)) :
> with(linalg) : with(VectorCalculus) :
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);
-0.00002000000000000000, -0.5000200000000000, -0.6500200000000000,
-1.13841518316760, -0.0179680903767637, -0.293676726455640
> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);
-1.15973620964772, 0.000589011651680524, -0.000605811119774765,
-0.290322641542702, -0.500021012893889, -0.6500200000000000

```

(beta = 0.45)

```

> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
beta := 0.45 : omega := 0.2 : mu := 0.00002 : alpha := 0.5 : chi := 0.15 : delta := 0.3 : psi := 0.5 : Lambda := 0.00005 : epsilon := 0.3 : N := 1 : tau2 := 0.05 : tau1 := 0.8 : sigma := 0.15 :

```

```

> P1 := Lambda + psi * (R1 + R2) - beta * S * ( Is / N ) - mu * S :
> P2 := ( (beta * tau1 * S * Is) / N ) - (mu + epsilon + delta) * C :
> P3 := ( (beta * (1 - tau1) * Is) / N ) * S - (mu + omega) * Ia :
> P4 := Ia * omega + delta * C - (mu + alpha + chi) * Is :
> P5 := epsilon * C + chi * tau2 * Is - (mu + psi + sigma) * R1 :
> P6 := chi * (1 - tau2) * Is + sigma * R1 - (mu + psi) * R2 :
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, Ia, Is, R1, R2}) :

```

>

$$R0 := (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$$

> *with(linalg) : with(VectorCalculus) :*

> $T1 := T[1] : T2 := T[2] :$

> $jac := \text{Jacobian}([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :$

> $jac1 := \text{subs}(T1, \text{evalm}(jac)) : \text{eigenvalues}(jac1) ;$
 $-0.000020000000000000, -0.500020000000000, -0.650020000000000,$
 $-1.17006043191717, 0.00885703944765298, -0.288856607530478$

> $jac2 := \text{subs}(T2, \text{evalm}(jac)) : \text{eigenvalues}(jac2) ;$
 $-1.15973712258246, -0.0000107974369014347 + 0.000421542677292278 I,$
 $-0.0000107974369014347 - 0.000421542677292278 I, -0.290323449906240,$
 $-0.500019495251188, -0.650020000000000$

($\beta = 0.5$)

> *restart :*

> *with(DEtools) :*

> *with(linalg) : with(VectorCalculus) :*

>

$\beta := 0.5 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda$
 $:= 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau l := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N} \right) - \mu \cdot S :$

> $P2 := \left(\frac{\beta \cdot \tau l \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$

> $P3 := \left(\frac{\beta \cdot (1 - \tau l) \cdot Is}{N} \right) \cdot S - (\mu + \omega) \cdot Ia :$

> $P4 := Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$

> $P5 := \epsilon \cdot C + \chi \cdot \tau 2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$

> $P6 := \chi \cdot (1 - \tau 2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$

> $T := \text{solve}(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, \{S, C, Ia, Is, R1, R2\}) :$

>

$$R0 := (\Lambda \beta (\delta \mu \tau l - \mu \omega \tau l - \omega \tau l \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$$

> *with(linalg) : with(VectorCalculus) :*

> $T1 := T[1] : T2 := T[2] :$
 > $jac := \text{Jacobian}([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :$
 > $jac1 := \text{subs}(T1, \text{evalm}(jac)) : \text{eigenvalues}(jac1);$
 -0.00002000000000000000, -0.5000200000000000, -0.6500200000000000,
 -1.2000200000000000, 0.0350581059358213, -0.285098105935821

 > $jac2 := \text{subs}(T2, \text{evalm}(jac)) : \text{eigenvalues}(jac2);$
 -1.15973803553262, -0.0000131951216027000 + 0.000843969637210361 I,
 -0.0000131951216027000 - 0.000843969637210361 I, -0.290324258320772,
 -0.500017977578366, -0.6500200000000000

$$(\beta = 0.6)$$

> $\text{restart} :$
 > $\text{with}(DEtools) :$
 > $\text{with}(linalg) : \text{with}(VectorCalculus) :$
 >
 $\beta := 0.6 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda$
 $:= 0.00005 : \epsilon := 0.3 : N := 1 : \tau2 := 0.05 : \tau1 := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N}\right) - \mu \cdot S :$
 > $P2 := \left(\frac{\beta \cdot \tau1 \cdot S \cdot Is}{N}\right) - (\mu + \epsilon + \delta) \cdot C :$
 > $P3 := \left(\frac{\beta \cdot (1 - \tau1) \cdot Is}{N}\right) \cdot S - (\mu + \omega) \cdot Ia :$
 > $P4 := Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$
 > $P5 := \epsilon \cdot C + \chi \cdot \tau2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$
 > $P6 := \chi \cdot (1 - \tau2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$
 > $T := \text{solve}(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, \{S, C, Ia, Is, R1, R2\}) :$
 >

$$R0 := (\Lambda \beta (\delta \mu \tau1 - \mu \omega \tau1 - \omega \tau1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$$

> $\text{with}(linalg) : \text{with}(VectorCalculus) :$
 > $T1 := T[1] : T2 := T[2] :$
 > $jac := \text{Jacobian}([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :$
 > $jac1 := \text{subs}(T1, \text{evalm}(jac)) : \text{eigenvalues}(jac1);$
 -0.00002000000000000000, -0.5000200000000000, -0.6500200000000000,
 -1.25580772802631, 0.0854087632049604, -0.279661035178649

 > $jac2 := \text{subs}(T2, \text{evalm}(jac)) : \text{eigenvalues}(jac2);$

-1.15973986153156, -0.0000179904632942017 + 0.00133469905857443 I,
-0.0000179904632942017 - 0.00133469905857443 I, -0.290325875197000,
-0.500014942142357, -0.650020000000000

($\beta = 0.7$)

> restart :

> with(DEtools) :

> with(linalg) : with(VectorCalculus) :

>

$\beta := 0.7 : \omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.15 : \delta := 0.3 : \psi := 0.5 : \Lambda$
 $:= 0.00005 : \epsilon := 0.3 : N := 1 : \tau 2 := 0.05 : \tau 1 := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R1 + R2) - \beta \cdot S \cdot \left(\frac{Is}{N} \right) - \mu \cdot S :$

> $P2 := \left(\frac{\beta \cdot \tau 1 \cdot S \cdot Is}{N} \right) - (\mu + \epsilon + \delta) \cdot C :$

> $P3 := \left(\frac{\beta \cdot (1 - \tau 1) \cdot Is}{N} \right) \cdot S - (\mu + \omega) \cdot Ia :$

> $P4 := Ia \cdot \omega + \delta \cdot C - (\mu + \alpha + \chi) \cdot Is :$

> $P5 := \epsilon \cdot C + \chi \cdot \tau 2 \cdot Is - (\mu + \psi + \sigma) \cdot R1 :$

> $P6 := \chi \cdot (1 - \tau 2) \cdot Is + \sigma \cdot R1 - (\mu + \psi) \cdot R2 :$

> $T := \text{solve}(\{P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0\}, \{S, C, Ia, Is, R1, R2\}) :$

>

$R0 := (\Lambda \beta (\delta \mu \tau 1 - \mu \omega \tau 1 - \omega \tau 1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2$
 $+ \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega$
 $+ \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$

> with(linalg) : with(VectorCalculus) :

> $T1 := T[1] : T2 := T[2] :$

> $jac := \text{Jacobian}([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :$

> $jac1 := \text{subs}(T1, \text{evalm}(jac)) : \text{eigenvalues}(jac1) ;$

-0.0000200000000000000, -0.500020000000000, -0.650020000000000,
-1.30717463530161, 0.133062036791486, -0.275947401489881

> $jac2 := \text{subs}(T2, \text{evalm}(jac)) : \text{eigenvalues}(jac2) ;$

-1.15974168758745, -0.0000227857769430146 + 0.00168834537848090 I,
-0.0000227857769430146 - 0.00168834537848090 I, -0.290327492192871,
-0.500011906585849, -0.650020000000000

($\beta = 0.88$)

```

> restart :
> with(DEtools) :
> with(linalg) : with(VectorCalculus) :
>
β := 0.88 : ω := 0.2 : μ := 0.00002 : α := 0.5 : χ := 0.15 : δ := 0.3 : ψ := 0.5 : Λ
:= 0.00005 : ε := 0.3 : N := 1 : τ2 := 0.05 : τ1 := 0.8 : σ := 0.15 :

> P1 := Λ + ψ·(R1 + R2) - β·S·(Is/N) - μ·S :
> P2 := (β·τ1·S·Is/N) - (μ + ε + δ)·C :
> P3 := (β·(1 - τ1)·Is/N)·S - (μ + ω)·Ia :
> P4 := Ia·ω + δ·C - (μ + α + χ)·Is :
> P5 := ε·C + χ·τ2·Is - (μ + ψ + σ)·R1 :
> P6 := χ·(1 - τ2)·Is + σ·R1 - (μ + ψ)·R2 :
> T := solve({P1 = 0, P2 = 0, P3 = 0, P4 = 0, P5 = 0, P6 = 0}, {S, C, Ia, Is, R1, R2}) :
>
R0 := (Λβ(δμτ1 - μωτ1 - ωτ1ε + δω + μω + ωε)) / (μ(αδμ + αδω + αμ²
+ αμω + αμε + αωε + χδμ + χδω + χμ² + χμω + χμε + χωε + δμ² + δμω
+ μ³ + μ²ω + μ²ε + μωε)) :
> with(linalg) : with(VectorCalculus) :
> T1 := T[1] : T2 := T[2] :
> jac := Jacobian([P1, P2, P3, P4, P5, P6], [S, C, Ia, Is, R1, R2]) :
> jac1 := subs(T1, evalm(jac)) : eigenvalues(jac1);
-0.0000200000000000000, -0.500020000000000, -0.650020000000000,
-1.39122830884031, 0.212773346931893, -0.271605038091584

> jac2 := subs(T2, evalm(jac)) : eigenvalues(jac2);
-1.15974497434063, -0.0000314174380126211 + 0.00218511963365444I,
-0.0000314174380126211 - 0.00218511963365444I, -0.290330403043565,
-0.500006442280444, -0.650020000000000

```

Lampiran 5. Sintax simulasi untuk grafik

> restart :

> with(DEtools) :

> with(linalg) : with(VectorCalculus) :

>

> $\beta := 0.3 :$

$\beta := 0.3 :$

$\omega := 0.2 : \mu := 0.00002 : \alpha := 0.5 : \chi := 0.2 : \delta := 0.3 : \psi := 0.5 : \Lambda := 0.00005 : \epsilon := 0.3 :$

$N := 1 : \tau_2 := 0.05 : \tau_1 := 0.8 : \sigma := 0.15 :$

> $P1 := \Lambda + \psi \cdot (R + U) - \beta \cdot S \cdot \left(\frac{Q}{N}\right) - \mu \cdot S :$

> $P2 := \left(\frac{\beta \cdot \tau_1 \cdot S \cdot Q}{N}\right) - ((\mu + \epsilon + \delta) \cdot C) :$

> $P3 := \left(\frac{\beta \cdot (1 - \tau_1) \cdot Q}{N}\right) \cdot S - (\mu + \omega) \cdot P :$

> $P4 := P \cdot \omega + \delta \cdot C - (\mu + \chi) \cdot Q :$

> $P5 := \epsilon \cdot C + \chi \cdot \tau_2 \cdot Q - ((\mu + \psi + \sigma) \cdot R) :$

> $P6 := (\chi \cdot (1 - \tau_2) \cdot Q) + (\sigma \cdot R) - ((\mu + \psi) \cdot U) :$

> $T := \text{solve}(\{P1=0, P2=0, P3=0, P4=0, P5=0, P6=0\}, \{S, C, P, Q, R, U\}) :$

>

$R0 := (\Lambda \beta (\delta \mu \tau_1 - \mu \omega \tau_1 - \omega \tau_1 \epsilon + \delta \omega + \mu \omega + \omega \epsilon)) / (\mu (\alpha \delta \mu + \alpha \delta \omega + \alpha \mu^2 + \alpha \mu \omega + \alpha \mu \epsilon + \alpha \omega \epsilon + \chi \delta \mu + \chi \delta \omega + \chi \mu^2 + \chi \mu \omega + \chi \mu \epsilon + \chi \omega \epsilon + \delta \mu^2 + \delta \mu \omega + \mu^3 + \mu^2 \omega + \mu^2 \epsilon + \mu \omega \epsilon)) :$

> with(linalg) : with(VectorCalculus) :

> $T1 := T[1] : T2 := T[2] :$

> $\text{jac} := \text{Jacobian}([P1, P2, P3, P4, P5, P6], [S, C, P, Q, R, U]) :$

> $\text{jac1} := \text{subs}(T1, \text{evalm}(\text{jac})) : \text{eigenvalues}(\text{jac1}) :$

> $\text{jac2} := \text{subs}(T2, \text{evalm}(\text{jac})) : \text{eigenvalues}(\text{jac2}) :$

> $T1 := \frac{d}{dt} S(t) = \Lambda + \psi \cdot (R + U) - \beta \cdot S \cdot \left(\frac{Q}{N}\right) - \mu \cdot S :$

> $T21 := \frac{d}{dt} C(t) = \left(\frac{\beta \cdot \tau_1 \cdot S \cdot Q}{N}\right) - ((\mu + \epsilon + \delta) \cdot C) :$

> $T31 := \frac{d}{dt} P(t) = \left(\frac{\beta \cdot (1 - \tau_1) \cdot Q}{N}\right) \cdot S - (\mu + \omega) \cdot P :$

> $T4 := \frac{d}{dt} Q(t) = P \cdot \omega + \delta \cdot C - (\mu + \chi) \cdot Q :$

> $T5 := \frac{d}{dt} R(t) = \epsilon \cdot C + \chi \cdot \tau_2 \cdot Q - ((\mu + \psi + \sigma) \cdot R) :$

> $T6 := \frac{d}{dt} U(t) = (\chi \cdot (1 - \tau_2) \cdot Q) + (\sigma \cdot R) - ((\mu + \psi) \cdot U) :$

>

$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$

>

$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0 ..30, ivs, linecolor = [red, blue], arrows = medium, scene = [t, S(t)], method = rosenbrock, stepsize = 0.5, title = "Susceptible", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$

>

$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$

>

$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0 ..35, ivs, linecolor = [red, blue], arrows = medium, scene = [t, C(t)], method = rosenbrock, stepsize = 0.5, title = "Carrier", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$

>

$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$

>

$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0 ..35, ivs, linecolor = [red, blue], arrows = medium, scene = [t, P(t)], method = rosenbrock, stepsize = 0.5, title = "Infected without Symptoms", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$

>

$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$

>

$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0 ..35, ivs, linecolor = [red, blue], arrows = medium, scene = [t, Q(t)], method = rosenbrock, stepsize = 0.5, title = "Infected with symptoms", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$

>

$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$

>

$DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0 ..35, ivs, linecolor = [red, blue], arrows = medium, scene = [t, R(t)], method = rosenbrock, stepsize = 0.5, title = "Recovery without Disability", labels = [Bulan(t), Populasi], titlefont = ["ARIAL", 15], labelfont = ["HELVETICA", 10]) :$

>

$ivs := [[S(0) = 1500, C(0) = 300, P(0) = 30, Q(0) = 150, R(0) = 40, U(0) = 30], [S(0) = 1000, C(0) = 150, P(0) = 10, Q(0) = 50, R(0) = 10, U(0) = 10]] :$

>

```
DEplot([T1, T21, T31, T4, T5, T6], [S(t), C(t), P(t), Q(t), R(t), U(t)], t = 0..35, ivs, linecolor  
= [red, blue], arrows = medium, scene = [t, U(t)], method = rosenbrock, stepsize = 0.5,  
title = "Recovery with Disability", labels = [Bulan(t), Populasi], titlefont = ["ARIAL",  
15], labelfont = ["HELVETICA", 10]) :
```

Lampiran 6. Penjabaran untuk Untuk Memperoleh Persamaan (4.37)

$$\omega i_A^* + \delta c^* - (\mu + \alpha + \chi) i_s^* = 0$$

$$\begin{aligned} &\Leftrightarrow \omega \left(\frac{abdj\lambda i_s^* + d\psi j(i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abdf\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} \right) + \\ &\quad \delta \left(\frac{ab\Lambda\beta\tau_1 i_s^* + \psi\delta\beta\tau_1 (i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abd\mu + i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} \right) - (\mu + \alpha + \chi) i_s^* = 0 \\ &\Leftrightarrow \frac{abdj\omega\lambda i_s^* + d\omega\psi j(i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abdf\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} + \\ &\quad \frac{ab\Lambda\delta\beta\tau_1 i_s^* + \psi\delta\beta\tau_1 (i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abd\mu + i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} - (\mu + \alpha + \chi) i_s^* = 0 \\ &\Leftrightarrow \frac{abdj\omega\lambda i_s^* + d\omega\psi j(i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abdf\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} + \\ &\quad \frac{abf\Lambda\delta\beta\tau_1 i_s^* + f\psi\delta\beta\tau_1 (i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abd\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} - \\ &\quad \frac{abdf\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)}{abdf\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} (\mu + \alpha + \chi) i_s^* = 0 \end{aligned}$$

misalakan $g = (\mu + \alpha + \chi)$, maka diperoleh:

$$\begin{aligned} &\Leftrightarrow \frac{abdj\omega\lambda i_s^* + d\omega\psi j(i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abdf\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} + \\ &\quad \frac{abf\Lambda\delta\beta\tau_1 i_s^* + f\psi\delta\beta\tau_1 (i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2]}{abd\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} - \\ &\quad \frac{abdf\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)}{abdf\mu + f i_s^* (abd\beta - \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)} i_s^* = 0 \\ &\Leftrightarrow abdj\omega\lambda i_s^* + d\omega\psi j(i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2] + abf\Lambda\delta\beta\tau_1 i_s^* + \\ &\quad f\psi\delta\beta\tau_1 (i_s^*)^2 [b\chi\tau_2 + a\chi(1-\tau_2) + \sigma\chi\tau_2] - (abdfg\mu + f g i_s^* (abd\beta - \\ &\quad \psi b\epsilon\beta\tau_1 - \psi\sigma\epsilon\beta\tau_1)) i_s^* = 0 \\ &\Leftrightarrow (bd\omega\psi j\chi\tau_2 + ad\omega\psi j\chi(1-\tau_2) + d\omega\psi j\sigma\chi\tau_2 + bf\psi\delta\beta\tau_1\chi\tau_2 \\ &\quad + af\psi\delta\beta\tau_1\chi(1-\tau_2) + f\psi\delta\beta\tau_1\sigma\chi\tau_2 + fg\psi b\epsilon\beta\tau_1 \\ &\quad + fg\psi\sigma\epsilon\beta\tau_1 - abdfg\beta) [i_s^*]^2 \\ &\quad + (abdj\omega\lambda + abf\Lambda\delta\beta\tau_1 - abdfg\mu) i_s^* = 0 \end{aligned} \tag{4.37}$$

